# Dynamic surfing and trapping of charged colloids in a traveling-wave electrophoretic ratchet

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The author theoretically demonstrates a gel-free electrophoretic ratchet under a *nearly* unidirectional traveling electric field whose wavelength is much longer than the transverse dimension. Because of length scale separation, a charged particle can migrate synchronously or asynchronously with the field as if it was surfing on the wave. The author shows, with a dynamical phase portrait, that if the wave speed is slower than the characteristic electrophoretic velocity, a suspension of charged particles can be trapped into distinct particle bands synchronizing with the field. A tunable sieving capability of this ratchet provides the potential for continuous fractionation and characterization of colloidal suspensions. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2740176]

Electrophoresis (EP) is the motion of charged particles under the actions of an applied electric field. It has been widely used in separation of charged species and recently explored under the theme of microfluidics.[1] The electrophoretic velocity $U$ of a charged particle in response to an electric field $E$ is described by

$$U = \mu E. \tag{1}$$

Here $\mu$ is the particle mobility of a scale $\varepsilon\zeta/\eta$, where $\varepsilon$ and $\eta$ represent, respectively, dielectric constant and viscosity of the solution and $\zeta$ is the surface zeta potential.

In this letter, we demonstrate an alternative EP scheme based on ac electric fields for separation of charged colloids. Our strategy is to utilize traveling-wave fields to produce an electrophoretic ratchet capable of trapping charged particles selectively, called *traveling-wave electrophoresis* (TWEP). In contrast to conventional EP ratchets[2,3] which often requires working with gel for rendering field/size dependent mobilities, a TWEP ratchet can sieve charged particles purely on a free-EP basis. The core of this ratchet invokes a length scale disparity in the field whose wavelength $\ell$ is sufficiently long compared to the transverse dimension $d$. Because Gauss's law with electroneutrality demands that the field must be divergence-free (it is obvious that a rigorously one-dimensional traveling field is impossible because it will inevitably lead to space charges), the field in the traveling direction and hence the electrophoretic velocity are much higher than those in the transverse direction. In analogy to hydrodynamic lubrication flow,[4] the resulting particle movement looks nearly unidirectional. As this character can suppress lateral drift in a much smaller dimension, it allows the particle to move synchronously or asynchronously with the field as if it were "surfing" on the wave.[5] We thus refer to such motion as a *nearly one-dimensional TWEP* despite its two-dimensional origin. We should also emphasize that our strategy differs fundamentally from those in previous studies[5–8] using traveling-wave signals. In these early studies, there involve *non-equilibrium* dynamics of charged toner,[5,6] space charge injection,[7] or high-frequency ac polarization,[8] so their systems are not electro-neutral and hence involve significant Coulombic forces. In our study, on the contrary, common colloids are typically dielectric and hence the charges are usually confined within the thin (10–100 nm) Debye screening layer at Poisson-Boltzmann equilibrium. There is virtually no electrostatic attraction/repulsion between particles even if they aggregate.[9] As we shall demonstrate, our ratchet can permit non-Coulombic aggregation of like-charge colloids in the context of electrophoresis; it seems unlikely to realize like-charge aggregation in these previous studies.

Let $x$ and $y$ stand, respectively, for the wave propagating and transverse directions with $y=0$ defined at the channel bottom surface embedded with electrodes. Define the wave number $k=2\pi/\ell(\ll 1/d)$. The longitudinal, traveling field is taken in a simple sinusoidal form satisfying Laplace equation: $E_x=E_0\cosh(k(y-d))\cos(k(x-ct))$, with the corresponding transverse field $E_y=E_0\sinh(k(y-d))\sin(k(x-ct))$ that satisfies zero normal field penetration $E_y=0$ at the top boundary $y=d$, where $E_0$ is the characteristic field strength, $c$ the wave speed, and $t$ time. Owing to much smaller transverse dimension $ky\sim kd\ll 1$, $E_x=E_0\cosh(k(x-ct)))$ which does not vary in $y$. In addition, $E_y$ now becomes $O(E_0\,kd)\ll E_x$ in magnitude and hence the velocity $U_y\ll U_x$, making transverse drift confined in a much smaller dimension. These two characters deduced by length scale separation stipulate that the particle motion will be dictated by the longitudinal field and hence can be viewed as nearly one dimensional. While the particle migrates in response to the traveling field, the particle position changes with time and hence the spatial influence of the field on the particle. To capture this effect, the local field $E$ $(=E_x)$ in Eq. (1) must be evaluated at the particle instant position $x_p(t)$, viz., $x=x_p(t)\equiv x_0+\int_0^t U(t_1)dt_1$, where $x_0$ is the initial position of the particle. This yields the following equation governing the particle motion under nearly one-dimensional (1D) TWEP:

$$U = \mu E_0\cos(kL), \quad \text{with } L=\int_0^t U(t_1)dt_1 - ct + x_0. \tag{2}$$

As a result, $U$ is determined simultaneously by the particle displacement *relative* to the wave, $L$. $L>0$ $(<0)$ corresponds to the particle ahead of (behind) the wave. As the local field

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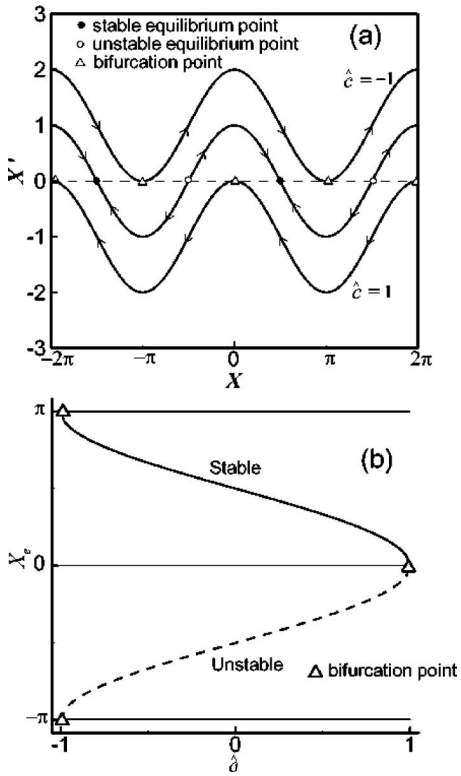


FIG. 1. (a) Phase portrait for a particle with $\mu>0$. The arrows indicate the directions of how the dynamical state of the particle motion evolves. (b) A saddle-node bifurcation.

exerting on the particle depends on the history of the time-varying particle-field phase difference, the *apparent* particle mobility $\mu\cosh(kL)$ now becomes *nonlinear* in $U$. Hence the particle motion is *not* reversible even though $\mu$ is independent of the applied field—it is this irreversibility that furnishes the unique ratchet capable of trapping/separating charged particles, as we will show shortly.

Without loss of generality, hereinafter we consider only the motion of a positively charged ($\mu>0$) particle. Letting $(X,X_0)=k(L,x_0)$, $V=U/U_0$, and $\tau=kU_0t$, with $U_0=\mu E_0$ being the velocity at dc fields, we rewrite Eq. (2) in the dimensionless form,

$$X' = \cos(X) - \hat{c}, \quad \text{with } X = \int_0^\tau V(\tau_1)d\tau_1 - \hat{c}\tau + X_0, \qquad (3)$$

where $\hat{c}=c/U_0$ is the ratio of the wave speed to the characteristic electrophoretic velocity. Prime denotes the derivative with respect to $\tau$. Instead of directly observing the particle trajectory in space, we adopt an alternative approach to identify the nature of the particle motion. Here we employ a dynamical phase diagram[10] to elicit the evolution of the motion in the displacement-velocity space. This approach does not require solving the solution in detail. More importantly, it can immediately reveal the stabilities of various stationary solutions (i.e., fixed points), so that the ultimate fate of the particle can be graphically identified without needs in carrying out integration or numerical simulations.

Figure 1(a) shows the phase diagram for Eq. (3). The path of the dynamical state for $X'>0$ is toward right, whereas that for $X'<0$ goes left. At $X'=0$, there are a number of fixed points $X_e$ at which the particle stays stationary. However, these points are not all dynamically stable, depending on how the two paths intercept at the $X'=0$ axis. For a field with negative gradients, the fixed points toward which the two paths converge are stable; otherwise they are unstable. Therefore, when the particle is close to the stable (unstable) fixed points, it will be trapped toward (repelled from) there. Since the fixed points are critical to the fate of the particle and their locations depend on $\hat{c}$, below we discuss three cases: (i) $|\hat{c}|<1$, (ii) $|\hat{c}|>1$, and (iii) $|\hat{c}|=1$.

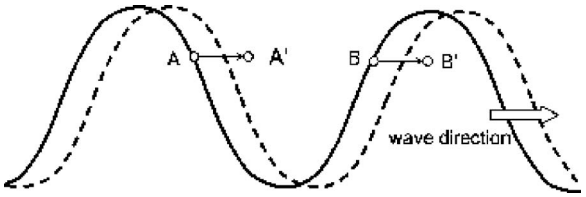


FIG. 2. Surfing and trapping of a charged particle under a traveling-wave electric field. Consider a particle starting with some speed faster than the wave speed. Suppose its initial position (at $A$) ahead of the wave (solid curve). At the next instant, the particle (at $A'$) will be slowed down since it experiences a weaker field (dashed curve). This process continues until the particle speed matches the wave's, after which the particle movement will be synchronized with the wave. If the particle trails behind the wave (at $B$), it will first accelerate (at $B'$) before reaching the maximum of the field, and then decelerate afterwards as in the $A$-$A'$ process.

For $|\hat{c}|<1$ there are two fixed points within the wavelength: $X_e^{\pm}=\pm\cos^{-1}(\hat{c})+2(n-1)\pi$ ($n$ is an integer). $X_e^+$ is a stable fixed point and hence will trap the particle from any positions $X\in(X_e^+-2\cos(\hat{c}),X_e^+-2\cos(\hat{c})+2\pi)$. On the contrary, $X_e^-$ is an unstable fixed point, so the particle near which will be repelled; yet it will take a time of at least $O(\ln|\sin(X_e^-)|)$ to be arrested again by one of the neighboring stable fixed points. In the special case of $\hat{c}=0$, the field is stationary, so all the particles will be trapped at the zero-field points: $X_e^+=\pi/2+2n\pi$, regardless of the amounts of the charges.

As for $|\hat{c}|>1$, there are no fixed points at all, so the particle is never able to be trapped by the field. Also since $X'=\cos(X)-\hat{c}<0(>0)$ for $\hat{c}>1(<-1)$ from Eq. (3), the particle always lags (leads) the wave; it will move back and forth with a nonzero net displacement during a cycle.

Finally, for $|\hat{c}|=1$ case, we have fixed points: $X_e=2n\pi$, where $X'$ has *zero* maxima/minima. In this case, as indicated by Fig. 1(a), if $|\hat{c}|$ is slightly different from unity, either a fixed point can split into two when $|\hat{c}|<1$ or there is no fixed point at all when $|\hat{c}|>1$. As this bifurcation phenomenon always occurs at the neighborhood of $|\hat{c}|=1$ and the associated fixed points, it is identified to be *saddle-point bifurcation*, as shown in Fig. 1(b).

How a charged particle surfs in a traveling-wave field and is trapped by the ratchet can be pictured in Fig. 2. In short, the key to trapping a particle is that the particle must have a speed not slower than the wave speed during its journey. Consequently, such trapping is feasible only when $U_0>c$. As such, it is possible to sort charged particles by choosing an appropriate wave speed of the applied field. The existence of stable fixed points suggests that a suspension of uniformly charged particles will be attracted toward these points, forming an organized band structure. Since these fixed points are viewed in the frame moving with the field, the particle band will synchronize with the field once it forms. As the band locates according to the particle charge (via $U_0$), particles with different charges will sooner or later be locked into their designated positions at a given wave

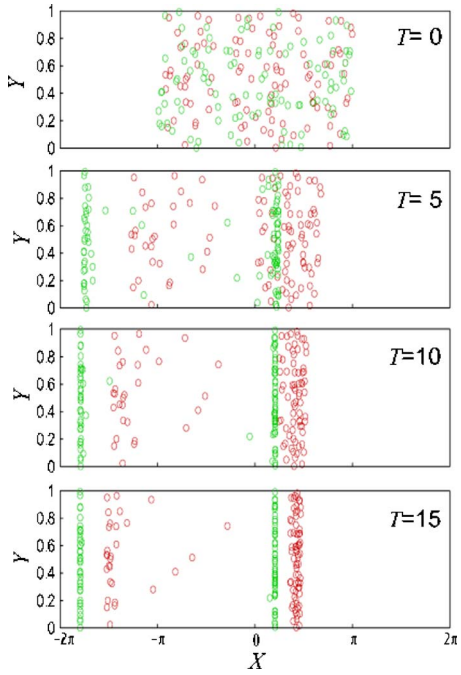


FIG. 3. (Color online) Selective trapping of charged particles in a nearly 1d TWEP ratchet shown in the frame moving with the traveling field. The suspension consists of two types of positively charged particles: red ($U_0=0.2c$) and green ($U_0=0.8c$), and 100 particles for each. Initially, the particles are randomly distributed with $-\pi<X<\pi$. Upon the field is applied, the particles gradually migrate toward their own trapping locations, forming respective band structures. $T=t(kc)/(2\pi)$ measures the time normalized by the cycling period of the field.

speed of the field, leading to a number of distinct particle clouds moving with the field. We carry out numerical simulations for Eq. (3) to identify this phenomenon in Fig. 3. Such a ratchet can be applied to particle sorting or detection for a solution consisting of a variety of particle populations. As a colloidal suspensions can be characterized by correlating band positions/intensities to amounts of charges/particles, a fingerprint will be readily obtained for the suspension.

In previous studies,[5,6] the particle motion can exhibit various drifting behaviors due to two-dimensional traveling fields. Our work is distinct from these studies. Our nearly 1D TWEP ratchet possesses a length scale disparity in the field and hence significantly reduces lateral drift during the trapping—it is this scale/field separation that can sustain the trapped particle clouds without being dispersed indefinitely.

It is possible to trap particles using nearly 1D TWEP in microdevices. Similar to the setup in traveling-wave dielectrophoresis,[8] a traveling field can be generated by an array of interdigitated electrodes subject to sequential phase-shifted voltages. With the electrodes of $\ell\sim10^2$ $\mu$m (or $k\sim10^{-2}$ $\mu m^{-1}$), the device needs 10 $\mu$m or smaller in depth $d$ for minimizing lateral drift. A typical electrophoretic mobility $\mu$ for colloids is about $10^4$ $\mu m^2/(s\,V)$. At an applied voltage of $V_e=10$ V, one can trap particles with the wave speed $c$ slower than $U_0=\mu E_0\sim\mu V_e/\ell\sim10^3$ $\mu$m/s. Therefore, the applied frequency $\omega=kc$ must be not faster than $kU_0\sim10$ Hz for the effect at work. With the conditions above and from Fig. 3, it takes about 10 cycles (less than 100 s) for a colloidal suspension to form discrete particle bands. Such a ratchet does not require a field-dependent mobility for realizing biased trapping. In addition, it has a tunable sieving capability which can be controlled by the applied frequency and voltage. This nearly 1D TWEP ratchet not only provides a renewed strategy for electrophoretic separation on a gel-free basis but also offers an added advantage for continuous particle sorting and functional fractionation of colloidal suspensions.

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