MATLAB基礎學習與應用

教學投影片

Part 3
Symbolic Math Toolbox

【Q】利用Symbolic做 微分 (Differentiate) 運算

- **Symbolic Math Toolbox**

- **Ex1:** 求 $3x^4 - x^3 + 2x^2 + x + 1$ 之微分式

>>syms x

>>f=3*x^4-x^3+2*x^2+x+1

>>diff(f)

ans =

12*x^3-3*x^2+4*x+1

Ex2: 求 $\sin(x^2)$ 之微分式

```
>>syms x
>>f=sin(x^2)
>>diff(f)
ans=
    2*cos(x^2)*x
```

- 二次微分 **diff(f,2)**

```
% differentiate with respect to a
>>syms a x
>>f=sin(a*x^2)
>>diff(f,a,2)
```

【Q】利用Symbolic做 積分 (Integrate) 運算

- **Ex1:** 求 $\frac{-2x}{(1+x^2)^2}$ 之積分式，即 $\int \frac{-2x}{(1+x^2)^2} dx = ?$

>>syms x

>>f=-2*x/(1+x^2)^2

>>int(f)

ans=

1/(1+x^2)

- **Ex1:** 求 $\int_0^1 x\ln(1+x)\,dx$ 之值

```
>>syms x
>>f= x*log(1+x)
>>int(f,0,1)
ans=
    1/4
```

【Q】利用Symbolic求 級數之和 (Summation of series)

Ex1: 求 $\sum_{n=0}^{k} n = 0+1+2+3+...+k = ?$

>>syms n k

>>symsum(n,0,k)

ans=

1/2*(k+1)^2-1/2*k-1/2

若所求為 $\sum_{n=0}^{10} n = 0+1+2+3...+10 = ?$

>>k=10

>>1/2*(k+1)^2-1/2*k-1/2

ans=

- **Ex2:** 求 $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$ 之值

```
>>syms n
>>symsum(1/(n*(n+1)*(n+2)),1,inf)

ans=
    1/4
```

【Q】利用Symbolic 展開多項式

Ex1: $$(x-2)(x-4)=x^2-6x+8$$

```
>>syms x
>>expand((x-2)*(x-4))
ans=
    x^2-6*x+8
```

Ex2:

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

```
>>syms x y
>>expand(cos(x+y))
ans=
    cos(x)*cos(y)-sin(x)*sin(y)
```

【Q】利用Symbolic 簡化多項式

Ex1:

$$x^3+3x^2+3x+1=(x+1)^3$$

>>syms x

>>simple(x^3+3*x^2+3*x+1)

ans=

(x+1)^3

Ex2:

$$2\cos^2(x)-2\sin^2(x)=?$$

>>syms x

>>simple(2*cos(x)^2-sin(x)^2)

ans=

3*cos(x)^2-1

【Q】利用Symbolic求 多項式和聯立方程之解

Ex1: $ax^2+bx+c=0,\ x=?$

```
>>syms a b c x
>>y=solve(a*x^2+b*x+c)
y=
   1/2/a*(-b+(b^2-4*a*c)^(1/2))
   1/2/a*(-b-(b^2-4*a*c)^(1/2))
```

⇨ $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

% pretty 漂亮的表示式

pretty(y)

- **Ex2: 求解** $\begin{cases} x+y=1 \\ x-11y=5 \end{cases}$

>>syms x y

>>S=solve('x+y-1','x-11*y-5')

```
S =
    x: [1x1 sym]
    y: [1x1 sym]
S.x = 4/3
S.y = -1/3
```

註：請比較**[x,y]=solve(‘x+y-1’,‘x-11*y-5’)**之指令

- **Ex3: 求解** $\begin{cases} x+y+z=1 \\ x-y+z=2 \\ x+y-z=-1 \end{cases}$

>>syms x y z

>>S=solve('x+y+z=1','x-y+z=2','x+y-z=-1')

```
S =
    x: [1x1 sym]
    y: [1x1 sym]
    z: [1x1 sym]
```

S.x=1/2

S.y=-1/2

S.z=1

註：**請比較[x,y,z]=solve('x+y+z=1','x-y+z=2','x+y-z=-1')**
之指令

- **Ex4: 求解** $\begin{cases} x^2 y^2 = 0 \\ x - \dfrac{1}{2} y = \alpha \end{cases}$

>>syms x y alpha

>>[x,y]=solve('x^2*y^2','x-y/2-alpha')

- **Ex5:求解** $\begin{cases} u^2 - v^2 = a^2 \\ u + v = 1 \\ a^2 - 2a = 3 \end{cases}$

>>syms u v a

>>[u,v,a]=solve('u^2-v^2=a^2','u+v=1','a^2-2*a=3')

【Q】利用Symbolic解微分方程式

Ex1: solve $\frac{dy}{dt} = 1 + y^2$

```
>>dsolve('Dy=1+y^2')
```

Ex2: solve $\frac{dy}{dt} = 1 + y^2$, I.C. $y(0) = 1$

```
>>dsolve('Dy=1+y^2','y(0)=1')
```

Ex2: solve $\left(\frac{dx}{dt}\right)^2 + x^2 = 1$, I.C. $x(0) = 1$

```
>>dsolve('(Dx)^2+x^2=1','x(0)=0')
```

- **Ex 4: solve** $\frac{d^2y}{dx^2} = \cos(2x) - y$ **,** $y(0) = 1, \quad y'(0) = 0$

```
>>y=dsolve('D2y=cos(2*x)-y','y(0)=1','Dy(0)=0','x')
```

- **Ex 5: solve** $\frac{d^3u}{dx^3} = u, \quad u(0) = 1, \quad u'(0) = -1, \quad u''(0) = \pi$

```
>>u=dsolve('D3u=u','u(0)=1', 'Du(0)=-1', 'D2u(0)=pi', 'x')
```

- **Ex 6: solve** $\begin{cases} \frac{df}{dt} = 3f + 4g \\ \frac{dg}{dt} = -4f + 3g \end{cases}$ **, I.C.** $f(0) = 0, \quad g(0) = 1$

```
>>[f,g]=dsolve('Df=3*f+4*g','Dg=-4*f+3*g','f(0)=0','g(0)=1')
```

【Q】利用Symbolic求 Laplace 轉換

$$L[f]=\int_0^\infty f(t)e^{-ts}dt$$

Ex1: $f(t)=t^4$ ⇨ $F(s)=\frac{24}{s^5}$

```
>>syms t
>>laplace(t^4)
ans=
   24/s^5
```

- **Ex2:** $f(t)=e^{-at}$ ⇒ $F(s)=\dfrac{1}{(s+a)}$

```
>>syms t a
>>laplace(exp(-a*t))
ans=
    1/(s+a)
```

【Q】利用Symbolic求反Laplace轉換

$$L^{-1}[f] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} f(s)e^{st}\,ds$$

Ex1: $f(s) = \dfrac{1}{s^2}$ ⇨ $F(t) = t$

```
>>syms s
>>ilaplace(1/s^2)
ans =
      t
```

- **Ex2:** $f(s) = \frac{1}{(s-a)^2}$ ⇒ $F(t) = te^{at}$

>>syms s a

>>ilaplace(1/(s-a)^2)

ans =

t*exp(a*t)

【Q】利用Symbolic求 Fourier轉換

$$F(\omega)=\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx$$

Ex1: $f(x)=e^{-x^2}$ ⇨ $F(\omega)=\sqrt{\pi}e^{-\omega^2/4}$

>>syms x

>>fourier(exp(-x^2))

ans=

pi^(1/2)*exp(-1/4*w^2)

【Q】如何求積分 $\int_{\beta}^{\alpha} f(x)dx = ?$ 之數值解

- **A. 數據點的積分,不知函數f(x)**

方法1.

>>x=[0 10 20 30 40];

>>y=[0.5 0.7 0.9 0.6 0.4];

>>area=trapz(x,y)   %梯形法

area =

26.5000

- **B.已知f(x), 求** $\int_{\beta}^{\alpha} f(x)dx$

- Ex: $\int_0^1 e^{-x}\cos(x)dx = ?$

1. edit fun.m

```
function  y=fun(x)
y=exp(-x).*cos(x);
```

2. 求積分(回到*Matlab Command Window*)

```
area=quadl('fun',0,1)  %newton-cote  3/8rule
```

NOTE: 亦可使用

```
area=quadl(‘exp(-x).*cos(x)’,0,1)
```

註：重積分

```
dblquad(‘sqrt(x.^2+y.^2)’,0,1,0,2)
triplequad(‘sqrt(x.^2+y.^2+z.^2)’,0,1,0,2,0,3)
```

【Q】如何求微分之數值解

A. 給數據求各點微分

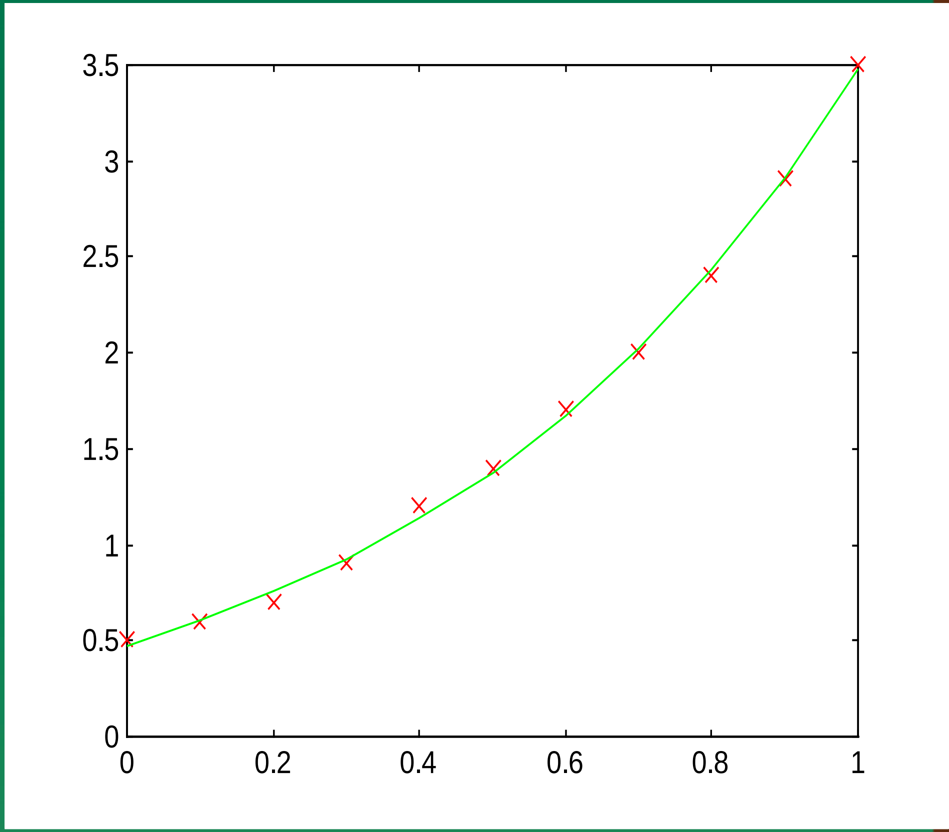
方法1.

```
>>x=0:0.1:1;
>>y=[0.5 0.6 0.7 0.9 1.2 1.4 1.7 2.0 2.4 2.9 3.5];
>>dx=diff(x);
>>dy=diff(y);
>>dydx=diff(y)./diff(x)
dydx =
  Columns 1 through 7
   1.0000    1.0000    2.0000    3.0000   2.0000    3.0000    3.0000
  Columns 8 through 10

   4.0000    5.0000    6.0000
```

方法2.

```
>>x=0:0.1:1;
>>y=[0.5 0.6 0.7 0.9 1.2 1.4 1.7 2.0 2.4 2.9 3.5];
>>p=polyfit(x,y,3);
>>dp=polyder(p);
>>dydx=polyval(dp,x)
>>plot(x,y,'x',x,polyval(p,x))
dydx =
  Columns 1 through 7
    1.2537    1.3883    1.5987    1.8848
    2.2467   2.6843    3.1977
  Columns 8 through 11
    3.7869    4.4518    5.1925    6.0089
```



- **B.給f(x) , 求f'(x)**

Ex. $f(x)=e^{-x}\cos(x), \quad f'(x)=?$ for $x\in[0,1]$

```
>>x=linspace(0,1,100);
>>y=exp(-x).*cos(x);
```

再用A.法

【Q】如何求微分方程式之數值解

Ex: $\begin{cases} x'_1 = x_1 + x_2 e^{-t} \\ x'_2 = -x_1 x_2 + \cos(t) \end{cases}$, $x_1(0)=0$，$x_2(0)=1$

1. edit fun.m

```
function  y=fun(t,x)
y=[ x(1)+x(2)*exp(-t);
       -x(1)*x(2)+cos(t) ];
```

2.求解ode45 ode23 % 回到matlab command window

```
[t,x]=ode45('fun', [0 10],[0 1]') %[0 10]- t上下限, [0 1]'-x的起始值
x1=x(:,1);
x2=x(:,2);
plot(t,x1,'r',t,x2,'y')
```

【Q】如何解高階微分方程式

Ex: $y'' + e^{-t} y' + \cos(t) y = \sin(t) e^{-t}, \quad y(0) = 1, y'(0) = 0$

Step 1.將原式轉成聯立的一階為分方程式，令

$$x_1 = y \quad \Rightarrow \quad x_1' = y'$$

$$x_2 = x_1' = y' \quad \Rightarrow \quad x_2' = \sin(t) e^{-t} - e^{-t} x_2 - \cos(t) x_1$$

$$\Rightarrow \quad \begin{cases} x'_1 = x_2 \\ x'_2 = \sin(t) e^{-t} - e^{-t} x_2 - \cos(t) x_1 \end{cases}, \quad x_1(0) = 1, x_2(0) = 0$$

Step 2. 用上例的方法求解

Ex: $y''' + 2y'' + 3y' + ty = \cos(t)$

$$\Rightarrow \begin{cases} x'_1 = x_2 \\ x'_2 = x_3 \\ x'_3 = \cos(t) - 2x_3 - 3x_2 - tx_1 \end{cases}$$