DEVELOPMENT OF AN EVOLUTIONARY STREAM MERGING METHOD IN HEAT EXCHANGER NETWORK DESIGN

C. T. Chang* and T. P. Yu

Based on the feasibility criterion and the generalized heat load graph analogy presented in this paper, evolutionary stream merging techniques have been developed to reduce the number of units in a heat exchanger network without energy penalty and without violating the given constraint of minimum temperature approach. One example is provided to demonstrate the capability of the proposed method.
(Key words: stream merging, heat exchanger network)

INTRODUCTION

In chemical processes, there sometimes exist situations where two or more streams are allowed to be merged. For example, hydrogen and naphtha or hydrogen and distillate may be merged and heated to the reaction temperature in the hydrotreating plants of a refinery; the hot contaminated waste water from different areas in a chemical plant may be merged to be sent to treatment units before disposal; the overhead product streams of a multi-effect distillation unit may be combined and cooled to the storage temperature, etc.. In general, the feasibility of merging process streams can be explored whenever multiple streams are fed to the same unit for storage or for further processing.

Although numerous studies have been published in the literature for designing low-cost heat exchanger networks [1-8], none of them can be used to systematically identify the possibility of reducing extra exchanger units when merging some of the process streams is allowed. More specifically, an extension of the conventional heat exchanger network design problem was considered in this paper:

> Given a set of stream data, the corresponding maximum energy recovery (MER) network [4,5] and the set(s) of hot (or cold) process streams which are allowed to be merged, reduce the number of exchanger units in the MER network, by merging and splitting

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process streams, as much as possible without increasing the utility usage, and without violating the minimum temperature driving force, ΔT_{min}.

Since the given stream data will be changed after stream merging, a new pinch point may be created and, furthermore, the resulting energy targets may exceed those of the given MER network. Based on a rigorous equation derived in this work, a quantitative criterion is recommended to determine the feasibility of merging any number of streams without energy penalty. The opportunities of reducing extra units in a given network can be easily identified in the generalized heat load graph proposed in this study. In this graph, a node was interpreted as an imaginary stream which may or may not represent one single physical stream.

Based on the above concepts, an evolutionary stream merging procedure has been developed to minimize the number of heat exchanger units in a reference structure, i.e. the MER network obtained by the Pinch Design Method [4,5]. First, the possibility of merging two or more process streams entirely is explored by using the proposed feasibility criterion. Second, if whole-stream merging is not possible, a part-stream merging procedure can be used to shift the heat loads among exchanger units so that one or more units may be eliminated from the reference network without energy penalty. An example problem is provided in this paper to demonstrate the capability of the proposed procedure.

THE FEASIBILITY CRITERION

The feasibility criterion is, of course, a condition that the heat exchange system has to satisfy in order to avoid an increase in the utility usage after merging streams for a given minimum temperature driving force, ΔT_{min}. Let's first examine a specific case of merging two process streams. The extension to more complicated merging schemes will become apparent after the quantitative feasibility criterion for this case has been derived.

It is assumed in this paper that the readers are familiar with the Problem Table algorithm [3,4,5]. An alternative mathematical formulation of this algorithm is presented in the Appendix. Now, consider a general heat exchange system with m hot streams and n cold streams. Without stream merging, an expression for the cumulative heat flow, Q_{out}, at any (shifted) temperature, t, can be obtained by using the alternative formulation, Eq. (A20).

$$Q_{out}|_t = Q_{in,1} + \{\sum_{l=1}^{k}[(\sum_{i=1}^{m} CP_{h,i})_l(t_{C,l-1} - t_{C,l})] + (\sum_{i=1}^{m} CP_{h,i})_{k+1}(t_{C,k} - t)\}$$

$$-\{\sum_{l=1}^{k}[(\sum_{j=1}^{n} CP_{C,j})_l(t_{C,l-1} - t_{C,l})] + (\sum_{j=1}^{n} CP_{C,j})_{k+1}(t_{C,k} - t)\} \qquad (1)$$

where $t_{C,k}>t>t_{C,k+1}$, $k=1,2,3,\ldots,N-1$, and $t_{C,k}$'s are the Problem Table interval temperatures, i.e. the shifted corner point temperatures on the composite curves. Without loss of generality, the actual temperatures of the hot streams are shifted down by an amount of ΔT_{min} while the cold stream temperatures are unchanged in this study. The symbols $CP_{h,i}$ and $CP_{C,j}$ refer to heat capacity flow rates of the (i)th hot stream and the (j)th cold stream respectively. The index l refers to the (l)th temperature interval partitioned in the Problem Table calculation. A plot of Q_{out} vs. t is the grand composite curve [4]. The value of the hot utility consumption rate, $Q_{in,1}$, should maintain a non-negative cumulative heat flow at any (shifted) temperature in the system. A more detailed explanation of Eq. (1) can be found in the Appendix.

Assume that among the m hot streams in the heat exchange system only the two streams with heat capacity flow rates, $CP_{h,i1}$ and $CP_{h,i2}$, where $1 \le i1, i2 \le m$, can be merged. The supply temperatures are t_{l1} and t_{l2} $(0 \le l1 \le l2 < N)$ respectively. The target temperatures of both streams are the same, t_{l3}, where $0 \le l1 \le l2 \le l3 \le N$. The merging temperatures are t_{l4} and t_{l5}, where $t_{l1} > t_{l4} > t_{l3}$ and $t_{l2} > t_{l5} > t_{l3}$.

After merging the two streams, the values of heat capacity flow rates and supply and target temperatures of all the process streams remain unchanged except those associated with the (i1)th and (i2)th hot streams. Three "new streams" are formed. Let's label the heat capacity flow rates of these new streams as $CP_{h,i3}$, $CP_{h,i4}$ and $CP_{h,i5}$ respectively. Their corresponding supply temperatures are t_{l1}, t_{l2} and t_{l6} and target temperatures are t_{l4}, t_{l5} and t_{l3}. These stream data can be determined by the following equations:

$$CP_{h,i3} = CP_{h,i1} \tag{2}$$

$$CP_{h,i4} = CP_{h,i2} \tag{3}$$

$$CP_{h,i5} = CP_{h,i1} + CP_{h,i2} \tag{4}$$

$$t_{l6} = [(t_{l4})(CP_{h,i1})+(t_{l5})(CP_{h,i2})]/(CP_{h,i1}+CP_{h,i2}) \tag{5}$$

Thus, the corresponding heat flow expression can be written as

$$Q_{out}|_t = Q_{in,1} + \{\sum_{p=1}^{q}[(CP_{h,i3} + CP_{h,i4} + CP_{h,i5} + \sum_{i=1,i\neq i1,i\neq i2}^{m} CP_{h,i})_p(t_{C,p-1} - t_{C,p})]$$
$$+(CP_{h,i3} + CP_{h,i4} + CP_{h,i5} + \sum_{i=1,i\neq i1,i\neq i2}^{m} CP_{h,i})_{q+1}(t_{C,q} - t)\}$$
$$-\{\sum_{p=1}^{q}[(\sum_{j=1}^{n} CP_{C,j})_p(t_{C,p-1} - t_{C,p})] + (\sum_{j=1}^{n} CP_{C,j})_{q+1}(t_{C,q} - t)\} \tag{6}$$

where, q = 1,2,3,.......,N+2. Note, that three new corner points are generated, i.e., $t_{C,14}$, $t_{C,15}$, and $t_{C,16}$. They should be included in determining the temperature intervals for the new system. The indices p and q are used to reflect this increase in temperature intervals.

It is clear that Eq. (6) will be unchanged if a quantity,

$$\sum_{p=1}^{q}[(CP_{h,i1}+CP_{h,i2})_p(t_{C,p-1}-t_{C,p})]+(CP_{h,i1}+CP_{h,i2})_{q+1}(t_{C,q}-t)$$

is added to and then substracted from the right hand side. Thus Eq. (6) can be rearranged to become

$$Q_{out}|_t = -Q^{(1)} + Q^{(2)} \tag{7}$$

where

$$Q^{(1)} = \sum_{p=1}^{q}[(-CP_{h,i3}-CP_{h,i4}-CP_{h,i5}+CP_{h,i1}+CP_{h,i2})_p(t_{C,p-1}-t_{C,p})]$$
$$+(-CP_{h,i3}-CP_{h,i4}-CP_{h,i5}+CP_{h,i1}+CP_{h,i2})_{q+1}(t_{C,q}-t) \tag{8}$$

$$Q^{(2)} = Q_{in,1}+\{\sum_{p=1}^{q}[(\sum_{i=1}^{m}CP_{h,i})_p(t_{C,p-1}-t_{C,p})]+(\sum_{i=1}^{m}CP_{h,i})_{q+1}(t_{C,q}-t)\}$$
$$-\{\sum_{p=1}^{q}[(\sum_{j=1}^{n}CP_{C,j})_p(t_{C,p-1}-t_{C,p})]+(\sum_{j=1}^{n}CP_{C,j})_{q+1}(t_{C,q}-t)\} \tag{9}$$

$Q^{(2)}$ in Eq. (9) is essentially the cumulative heat flow of the original system before stream merging. That is, Eq. (9) is equivalent to Eq. (1). The increase in the number of temperature intervals does not affect the outcome. This point is well illustrated in the Appendix.

The implication of Eq. (7) is that the feasibility of merging streams can be determined by calculating $Q^{(1)}$ only. Since the heat flow $Q^{(2)}$ has been obtained already for the original system, feasibility can be determined by simply comparing $Q^{(1)}$ with $Q^{(2)}$. If at any temperature the value of $Q^{(1)}$ is larger than that of $Q^{(2)}$, the merging scheme under consideration is infeasible. From Eq. (8), we can see that $Q^{(1)}$ can be obtained simply by a routine Problem Table calculation with ΔT_{min} set to be zero. Note, the three "new" streams formed after merging should be regarded as the "cold" streams. Since only those stream data involved in merging are needed, this calculation should be simple and straight forward. Further, from Eq. (2) to (5), it can be concluded that the value of $Q^{(1)}$ should be zero at temperatures outside the interval (t_{14},t_{15}) or (t_{15},t_{14}). Thus, this calculation needs to be performed only at temperatures between the merging temperatures. Also, since only corner points on composite curves may be the candidates for a pinch point, only corner points on the two cumulative heat flow curves, $Q^{(1)}$ and $Q^{(2)}$, between the merging temperatures need to be checked for feasibility, i.e. $Q^{(2)}-Q^{(1)}\geq 0.0$.

To illustrate the use of the feasibility criterion, the following example problem is provided.

Stream Type and Number	Heat Capacity Flow Rates (kW/K)	TS(K)	TT(K)
Hot 1 (H1)	2.0	453	313
Hot 2 (H2)	4.0	423	313
Cold 1 (C1)	3.0	333	453
Cold 2 (C2)	2.6	303	378

$$\Delta T_{min} = 10 \text{ K}$$

The pinch point of the given system is 423 K in terms of the hot stream temperature and the hot and cold utility consumption rates are 60.0 and 225.0 KW respectively. Assume that H1 and H2 are allowed to be merged. If the two are combined at 420 K and 320 K respectively, the feasibility criterion will be violated and a new pinch point will be created at 353.3 K (Figure 1(a)). On the other hand, if these streams are mixed at 320 K and 400 K, the merging scheme will not cause an increase in utility consumption (Figure 1(b)).

Finally, the approach to derive the feasibility criterion described here is general. Although only the criterion for a specific two-stream merging scheme is presented, the same derivation can be followed for more complex merging schemes and for merging more than two process streams. The values of $Q^{(1)}$ can be obtained by carrying out the Problem Table Algorithm with ΔT_{min} set to be zero. Again, in the calculation, the "new" streams formed after merging should be considered as "cold" streams if hot streams are combined and otherwise if cold streams are combined. By comparing the values of $Q^{(1)}$ and $Q^{(2)}$ at corner points, the feasibility of applying any merging scheme without increasing the utility usage can be determined by using the same criterion, i.e. $Q^{(2)}-Q^{(1)}\geq 0.0$.

THE GENERALIZED HEAT LOAD GRAPH

The Euler's network relation in graph theory has been widely used for determining the number of heat transfer units in a heat exchange system [9]. In a heat load graph, the nodes represent streams, while the arcs symbolize heat transfer units. An example of the graph analogy to a heat recovery system which contains two hot streams (H1 and H2), two cold streams (C1 and C2), one cooling utility (C) and hot utility (H) is presented in Fig. 2(a). Notice that only enthalpy balance is considered here. There is no restriction on the heat duty of any of the arc, except that the sum of the heat duties associated with those connected to a particular node must be equal to the heat load of the node. This special nature of the heat load graph representation suggests the interpretation of a node may be expanded when stream merging is allowed in a heat exchanger network design problem.

In the generalized heat load graph, a node can be considered as an imaginary stream which may or may not represent one single physical stream. The heat duties of the arcs attached to different nodes are not allowed to be shifted. However, the heat duties of the arcs attached to the same node can be distributed arbitrarily, as long as the sum is equal to the heat load of the node. If all the streams that are allowed to be merged can be regarded as one node, the total number of nodes will be less than that of the graph analogy generated from the conventional definition of a node. The total number of matches, i.e., arcs, will not be reduced due to an increase in the number of loops or a decrease in components caused by the modified interpretation. For example, if streams C1 and C2 are allowed to be merged, according to the "new" definition of nodes, Fig. 2(a) becomes Fig. 2(b).

If there are paths between any pair of streams that are allowed to merged, e.g., C1-H-C2 or C1-H1-C2 in Fig. 2(a), extra loops will form in the new graph analogy. This result is especially useful, since a loop can be "broken" by shifting the heat duties of the arcs in the loop without changing the heat loads of the nodes. Thus, the identification of such paths is an important task, because they represent opportunities for eliminating matches in a given network. For the purpose of comparison, the conventional definition of nodes will be used to generate the graph analogy to the initial structure. The proposed evolutionary procedure will be performed on such graphs with the understanding that the heat loads on the two nodes which are connected by paths can be redistributed.

For the purpose of later discussion, a term "Full String" (FS) will be used in this paper to represent a sequence of units (UT) and streams (SM), i.e. ($SM(i_1)$, $UT(i_2)$, $SM(i_3)$, $UT(i_4)$.....$SM(i_n)$). Here, $SM(i_1)$ and $SM(i_n)$ are any two process streams that are allowed to be merged. All the streams in this list are connected by the units between them. A Stream String (SS) can then be defined as a full string with the unit names deleted. A Unit String (US) can be defined in a similar way. For example, let's consider the structure in Fig. 3. If streams H1 and H3 are allowed to be merged, the stream strings can be identified in this graph as (H1, C2, H2, C, H3) and (H1, C2, H3). The unit strings in this case are (H1-C2, C2-H2, H2-C, C-H3) and (H1-C2, C2-H3), where the heat transfer unit between streams H1 and C2 is represented by H1-C2, and so on.

Also defined are the corresponding stream heat load strings, SHLS, and unit heat load strings, UHLS. They are:

(H1, C2, H2, CW, H3) : (20, 90, 60, 40, 80)
(H1, C2, H3) : (20, 60, 80)
(H1-C2, C2-H2, H2-CW, CW-H3) : (20, 20, 10, 30)
(H1-C2, C2-H3) : (20, 50)

If the constraint of ΔT_{min} is not imposed, the heat duties of the exchangers may be shifted freely along the UHLS. This is because the UHLS can be regarded as a loop in the generalized heat load graph. Consequently, a standard operation, unit heat load redistribution (UHLR), is defined as

> Starting from the left-hand side (or right-hand side) of the UHLS, identify the lowest heat load value among the odd position members; substract it from all the odd position members and add the same amount to all the even position members in the same string.

This operation will be regularly performed in the proposed design procedure described in the next section.

THE EVOLUTIONARY STREAM MERGING PROCEDURE

The evolutionary procedure consists of two basic parts, i.e., whole-stream merging procedure (Fig. 4(a)) and part-stream merging procedure (Fig. 4(b)). The "whole-stream merging" scheme refers to the case when two streams are mixed completely at their supply temperatures. "Part-stream merging" refers to the case when only parts of two streams are mixed. The purpose of whole-stream merging is to reduce the stream number as much as possible before carrying out a standard Pinch Design for the MER network. The part-stream merging procedure is used to redistribute the heat duties along the UHLS so that the number of exchanger units can be further reduced from the MER network without energy penalty.

In this work, part-stream merging is accomplished exclusively by the general merging scheme described below: Let's consider a merging scheme shown by the grid representation in Fig. 5. Here a fraction η_1 is split from Stream 1 at temperature TM1 and a fraction η_2 is split from Stream 2 at temperature TM2. The split branch from Stream 1 is merged with the residual of Stream 2 and the split branch from Stream 2 is merged with the residual of Stream 1.

Certainly, there exist many other complex two-stream merging schemes. Based on our experience, the merging scheme presented in Fig. 5 is flexible enough for solving all the problems we have studied so far. Therefore, it was decided to be unnecessary to consider other more complex schemes in the proposed evolutionary design method.

AN EXAMPLE

Let's apply the proposed procedures in Fig. 4 to the example problem described previously in the feasibility criterion section:

A. The whole-stream merging procedure

It can be determined by applying the suggested feasibility criterion that merging streams H1 and H2 completely will cause an increase in utility consumption. Therefore, we need to generate an initial MER structure by using a standard Pinch Design method and apply the part-stream merging procedure. The initial structure is shown in Fig. 6(a) with a total of six (6) heat transfer units.

B. Part-stream merging procedure

i) Identification of strings:

Based on the heat load graph analogy to the initial network shown in Fig. 6(b), the related strings can be identified as:

FS	:	(a):	(H1, H1-C1, C1, C1-H2, H2)
		(b):	(H1,H1-C,C,C-H2,H2)
US	:	(a):	(H1-C1, C1-H2)
		(b):	(H1-C, C-H2)
UHLS	:	(a):	(60, 240)
		(b):	(25, 200)

ii) Unit heat load redistribution:

For string (a), the UHLS becomes (0, 300) after UHLR. The grid representation of the resulting structure is presented in Fig. 7(a), which shows an infeasible temperature difference at the hot-end of Unit 2 on Hot Stream 2.

iii) Stream merging:

a) Correct the infeasible unit by merging the two streams at the outlet temperature of the first member of Unit String (US), i.e. Unit 1 in Fig. 7(a), and the inlet temperature of the last member of the US, i.e. Unit 2. These temperatures are 453 K on H1 and 423 K on H2. Based on a heuristic guideline presented elsewhere [10], the split fractions of H1 and H2 can be determined to be 1.0 and 0.75 respectively. Thus, the resulting heat capacity flow rate of the combined stream is 3.0 KW/K. The corrected network structure is shown in Fig. 7(b).

b) Restore heat duties on downstream units by applying the merging scheme described in Fig. 5 again. The split fraction on H1 should be 0.815. The corrected network structure after heat duty restoration is shown in Fig.7(c). Note, that restoration of heat duties is not really necessary for this particular case. But, for the sake of consistency, this operation is included as one of the design steps.

iv) Constraint checking:

The constraint of ΔT_{min} is checked for every unit in the network (see Fig. 7(c)).

After the part-stream merging procedure has been completed for one string, the same operations can be applied to the other strings which connect H1 and H2 in the resulting heat load graph. Since there is still one string left, the part-stream merging techniques are applied in a similar way.

i) Identification of strings (Fig. 8(a)):

FS : (H1, H1-C, C, C-H2, H2)
US : (H1-C, C-H2)
UHLS : (25, 200)

ii) Unit heat load redistribution:

UHLS : (0, 225)

iii) Stream merging (Fig. 8(b)):

$\eta_1 = 1.0$
$\eta_2 = 0.0$

iv) Constraint Checking:

All units in the network satisfy the constraint of 10 K minimum temperature approach constraint (Fig. 8(b)).

Since no more strings can be found in the graph analogy (Fig. 8(c)) of the resulting structure, the structure shown in Fig. 8(b) is the final network design. Thus, if stream merging is allowed between H1 and H2, additional two units can be eliminated from the MER network without causing an increase in the utility consumption and without violating the given constraint of minimum temperature approach. This is accomplished by using the proposed procedure.

CONCLUSIONS

Solution techniques for heat exchanger network synthesis problems have been advanced significantly since they first appeared. At the present time, there are many methods available in the literature for designing an energy efficient structure. If some of the process streams are allowed to be merged, the evolutionary method developed in this work can be used to reduced the number of exchanger units in a given network without energy penalty. Depending on the stream data, the design problem can be solved by the whole-stream merging procedure and/or the part-stream merging procedure. By applying the proposed strategy to an example problem, it is found that the resulting network has fewer exchanger units than the initial structure does.

It should be pointed out that the effect of stream merging on the total heat transfer area of the network has been ignored in this work. If the network generated by the proposed procedures requires significantly larger area, further modification of the network structure becomes necessary. This problem will be the subject of our future research.

NOMENCLATURE

CP : heat capacity flow rate, KW/K
H : enthalpy flow rate, KW
P : an arbitrarily chosen temperature range, K
Q : cumulative heat flow, KW
t : shifted temperature, K
T : actual temperature, K
TS : supply temperature, K
TT : target temperature, K
TM : merging temperature, K
: split fraction from a process stream
ΔT_{min} : minimum temperature approach, K

Subscript

c : cold stream
h : hot stream
in : inlet to a subsystem
out : outlet to a subsystem

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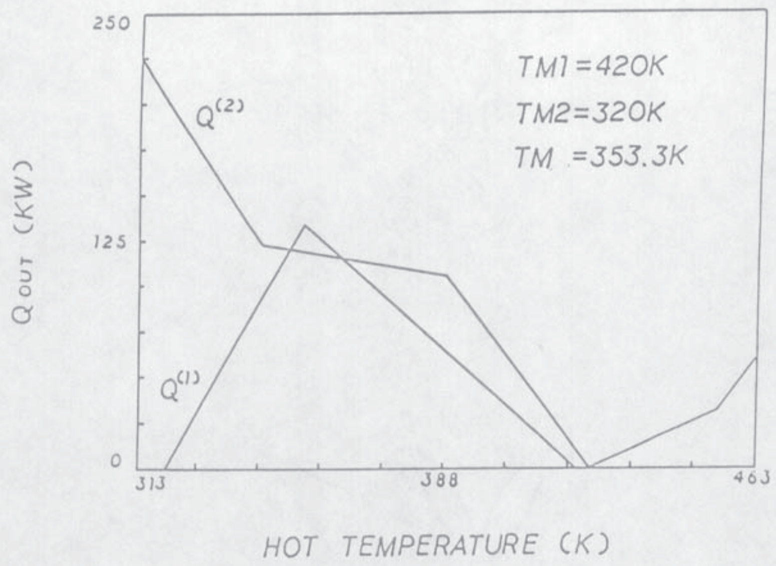
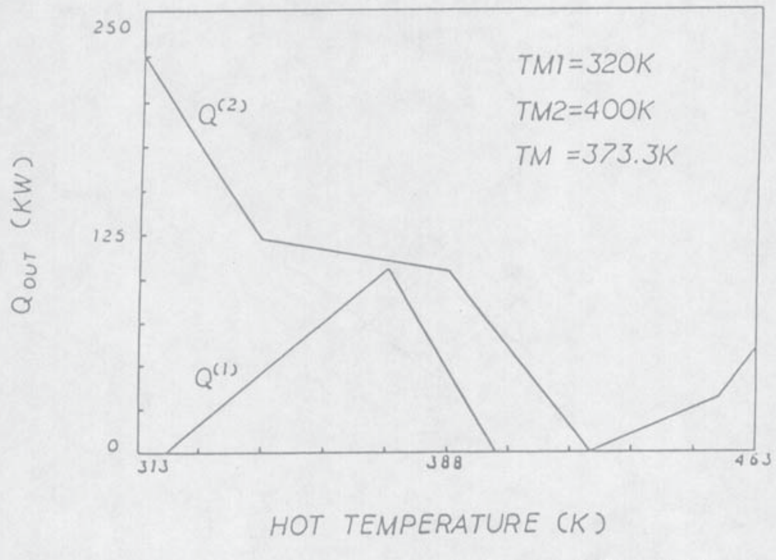


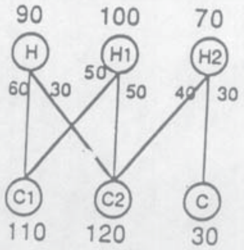
(a)



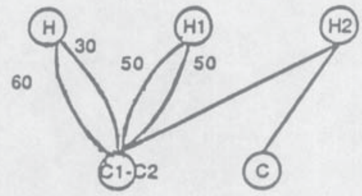
(b)

Figure 1 Application of the Feasibility Criterion

(a) infeasible (b) feasible



(a)



(b)

Figure 2 The Graph Analogy to Heat Exchange System
(a) Conventional
(b) Generalized

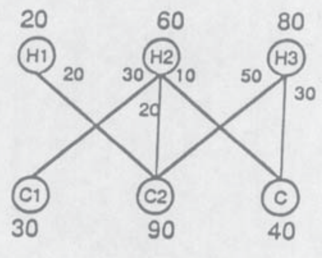


Figure 3 The Strings

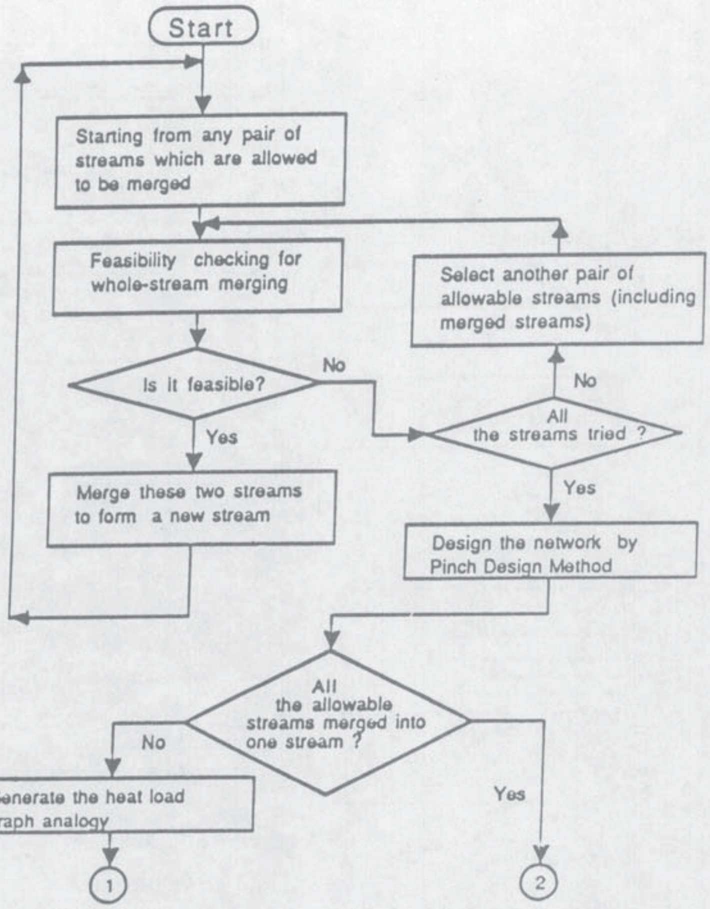


Figure 4 - The Flowchart of Evolutionary Stream Merging Method

(a) The Whole-Stream Merging Procedure

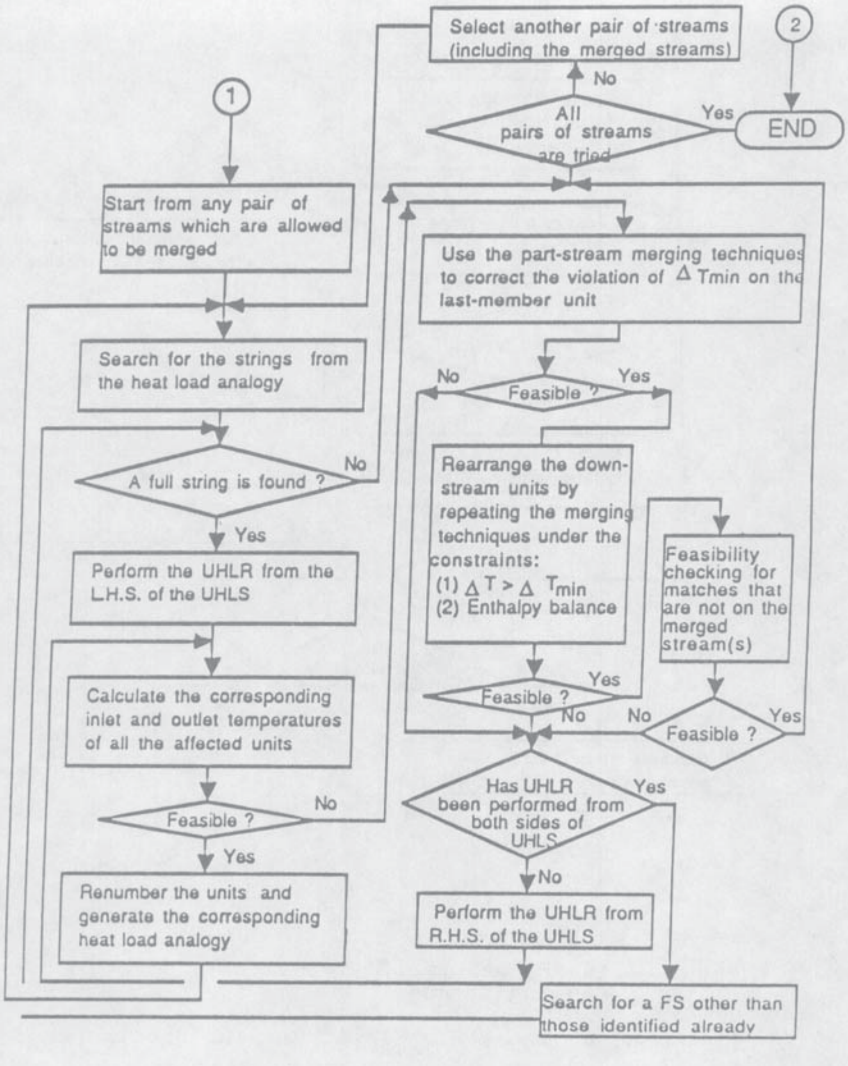


Figure 4 - The Flowchart of Evolutionary Stream Merging Method

(b) The Part-Stream Merging Procedure

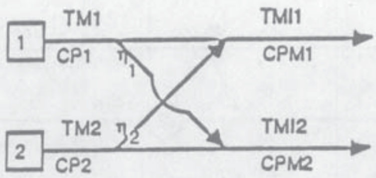


Figure 5 Grid Representation of A General Merging Scheme

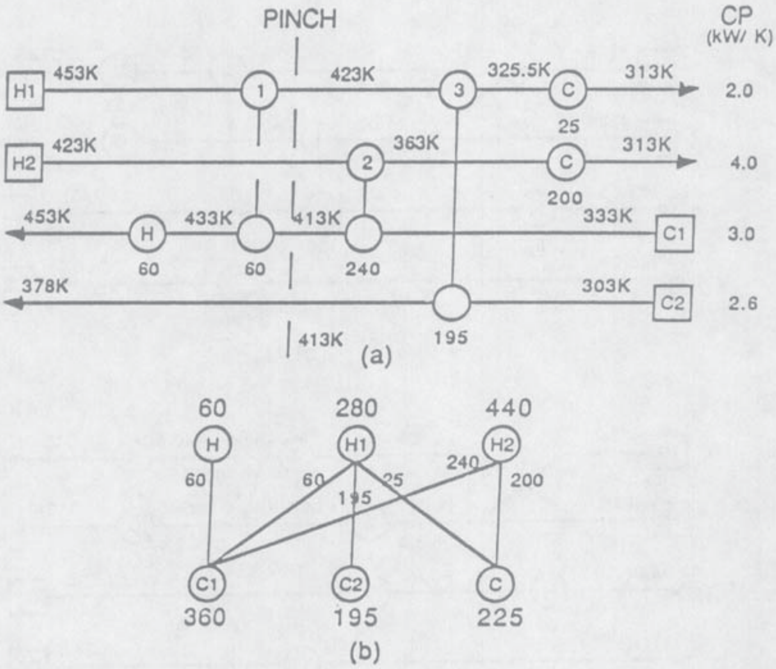


Figure 6 Initial Network Structure of The Example Problem
(a) Grid Representation
(b) Graph Analogy

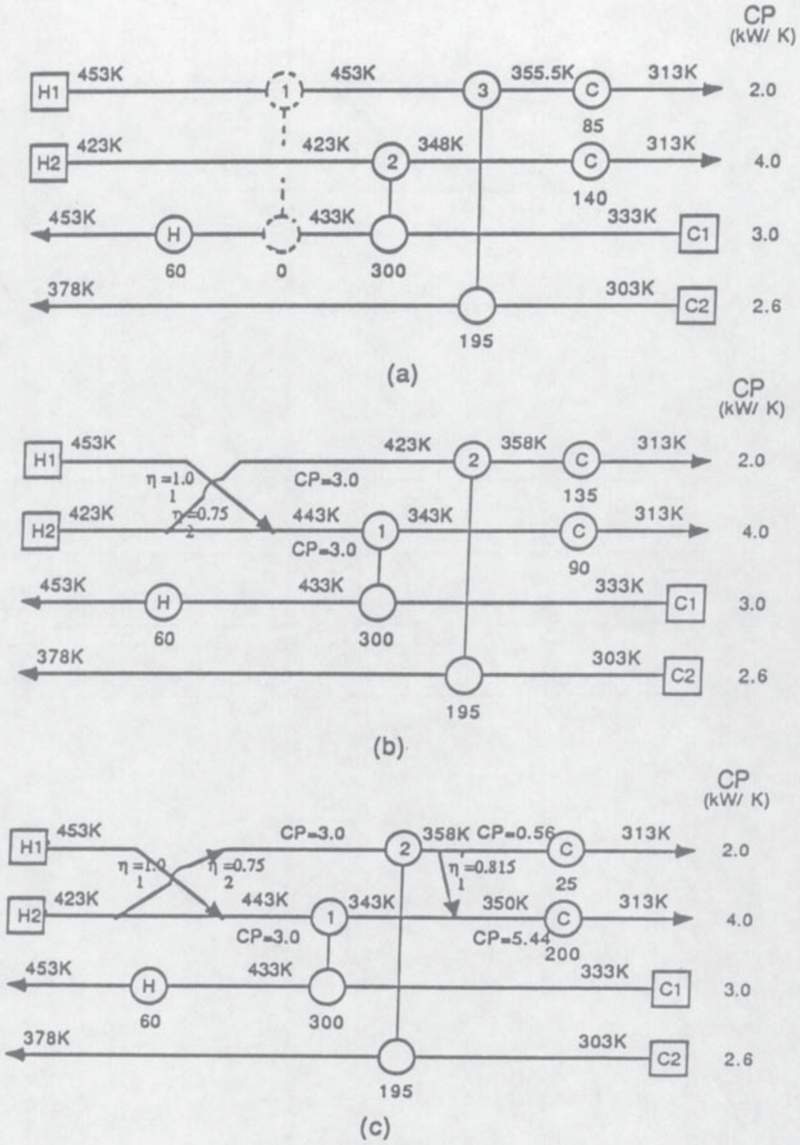


Figure 7 Evolution of Network Structures - I

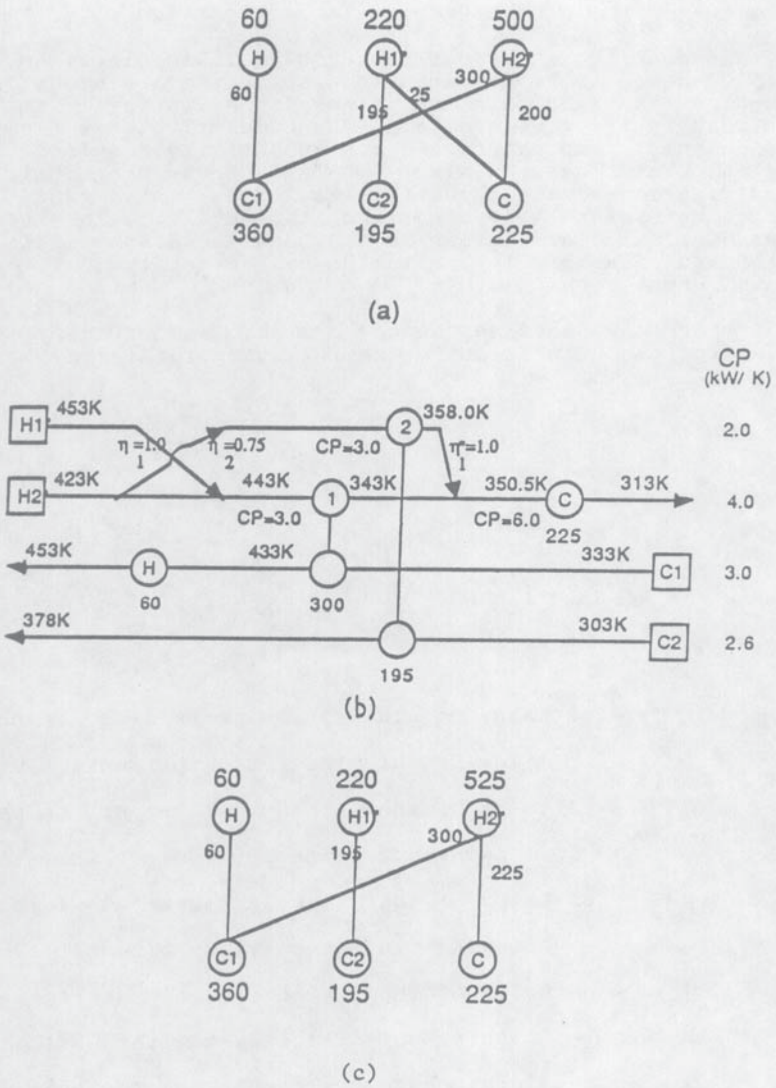


Figure 8 Evolution of Network Structures - II

APPENDIX

Alternative Mathematical Formulation of the Problem Table Algorithm

In order to carry out the Problem Table Algorithm, a heat table needs to be prepared to display the net amount of heat surplus in each of the temperature interval implicitly defined by the stream data. The cumulative heat flows, the pinch point temperature and minimum utility requirements can be calculated accordingly. Since stream merging implies a modification of the cumulative heat flow, it is essential for us to understand this algorithm thoroughly. An alternative mathematical formulation is provided in this appendix. The proposed feasibility criterion can be easily derived from this formulation.

For any heat exchange system, an enthalpy balance equation can be written in the following form:

$$Q_{in} + \sum_{i=1}^{m} H_{h,i}^{in} + \sum_{j=1}^{n} H_{C,j}^{in} = Q_{out} + \sum_{i=1}^{m} H_{h,i}^{out} + \sum_{j=1}^{n} H_{C,j}^{out} \qquad \text{(A1)}$$

or

$$Q_{out} - Q_{in} = (\sum_{i=1}^{m} H_{h,i}^{in} - \sum_{j=1}^{n} H_{C,j}^{out}) - (\sum_{i=1}^{m} H_{h,i}^{out} - \sum_{j=1}^{n} H_{C,j}^{in}) \qquad \text{(A2)}$$

where, $H_{h,i}^{in}$ - enthalpy of the ith hot stream at temperature $T_{h,i}^{in}$ flowing into the system per unit time

$H_{h,i}^{out}$ - enthalpy of the ith hot stream at temperature $T_{h,i}^{out}$ flowing out of the system per unit time

$H_{c,j}^{in}$ - enthalpy of the jth cold stream at temperature $T_{c,j}^{in}$ flowing into the system per unit time

$H_{c,j}^{out}$ - enthalpy of the jth cold stream at temperature $T_{c,j}^{out}$ flowing out of the system per unit time

Q_{in} - the total heat rate taken from the heat source

Q_{out} - the total heat rate rejected to the heat sink

The process streams in the heat exchange system covers a temperature range that is bounded by the hottest and coldest process stream temperatures, t_{max} and t_{min}. Let's consider an artificial subsystem in which the temperatures of all the hot streams are within (t_h-P,t_h) and the temperatures of all the cold streams are within (t_c-P,t_c). The values of t_h, t_c, and P are arbitrarily chosen. However, the two temperatures, t_h and t_c, have to satisfy the following constraints:

$$t_{min} < t_c < t_h < t_{max} \tag{A3}$$

$$t_h = t_c + \Delta T_{min} \tag{A4}$$

Also, the number of streams entering and leaving the subsystem should remain unchanged. For this subsystem, Eq. (A2) is valid if we make the following substitutions:

$$T_{h,i}^{in} = t_h \qquad i = 1,2,\ldots\ldots\ldots,m \tag{A5}$$

$$T_{h,i}^{out} = t_h - P \qquad i = 1,2,\ldots\ldots\ldots,m \tag{A6}$$

$$T_{C,j}^{out} = t_C \qquad j = 1,2,\ldots\ldots\ldots,n \tag{A7}$$

$$T_{C,j}^{in} = t_C - P \qquad j = 1,2,\ldots\ldots,n \tag{A8}$$

The heat input Q_{in} to this subsystem should be supplied by hot utilities or "hotter" subsystem outside this subsystem, i.e., from hot process streams with temperature higher than t_h. It should be pointed out that Q_{in} is a non-negative value, since, by definition, it is impossible to transfer heat from hot to cold stream when the temperature difference is less than ΔT_{min}. Similarly, the same argument can be applied to $Q_{out} \geq 0$.

Let

$$\Delta H|_{tC} = \sum_{i=1}^{m} H_{h,i}|_{tC+\Delta Tmin} - \sum_{j=1}^{n} H_{C,j}|_{tC} \tag{A9}$$

then,

$$Q_{out} - Q_{in} = \Delta H|_{tC} - \Delta H|_{tC-P} \tag{A10}$$

or

$$Q_{out} = Q_{in} + (\Delta H|_{tC} - \Delta H|_{tC-P}) \tag{A11}$$

Under the restriction that the number of process streams (hot or cold) entering and leaving the subsystem remains unchanged, the temperature interval (t_c-P,t_c) can be chosen arbitrarily. Let's use a set of conveniently chosen temperatures to divide the entire system temperature range into N subsystems. Note, in this derivation, the hot-stream tempteratures are shifted by an amount of ΔT_{min}. The most frequently selected temperatures are the corner-point and end-point temperatures on the hot and cold composite curves. However, any set of temperatures can be used as long as each temperature interval satisfies the requirements described above.

Let those temperatures be

$$t_{max} = t_{C,0} > t_{C,1} > t_{C,2} > \dots\dots\dots\dots > t_{C,N-1} > t_{C,N} = t_{min} \qquad (A12)$$

then, Eq. (A11) can be applied to each of the following temperature intervals: $(t_{C,N}, t_{C,N-1}), (t_{C,N-1}, t_{C,N-2})$, ..., $(t_{C,2}, t_{C,1}), (t_{C,1}, t_{C,0})$, i.e.

$$Q_{out,k} = Q_{in,k} + (\Delta H|_{tC^-,k-1} - \Delta H|_{tC^+,k}) \qquad (A13)$$

where, k = 1,2,........,N. The superscripts "-" and "+" of $t_{C,k-1}$ and $t_{C,k}$ are used to reflect the fact that the end point temperatures are not included in the interval. For convenience, they will be neglected in the later formulations. For the kth subsystem, $Q_{out,k}$ is determined by $Q_{in,k}$. Since it is desirable to make use of the available energy in the system as much as possible, we assign

$$Q_{in,k} = Q_{out,k-1} \qquad k = 2, 3, \dots\dots\dots, N \qquad (A14)$$

This can always be achieved due to the artificial partitioning of the entire system. That is, the hot-stream temperatures in the (k - 1)th subsystem are always higher than the cold-stream temperatures in the kth subsystem by at least ΔT_{min}. Thus,

$$\begin{aligned} Q_{out,k} &= Q_{in,k} + (\Delta H|_{tC,k-1} - \Delta H|_{tC\ ,k}) \\ &= Q_{in,k-1} + (\Delta H|_{tC,k-2} - \Delta H|_{tC\ ,k-1}) \\ &\qquad + (\Delta H|_{tC,k-1} - \Delta H|_{tC\ ,k}) \\ &\quad \vdots \\ &= Q_{in,1} + \sum_{l=1}^{k} (\Delta H|_{tC,l-1} - \Delta H|_{tC\ ,l}) \end{aligned} \qquad (A15)$$

where, k = 1,2,3,.......,N. At any temperature t, where $t_{C,k} > t > t_{C,k+1}$, we can obtain the following equation by using Eq. (A13) and Eq. (A15), i.e.

$$Q_{out}|_t = Q_{in,1} + \sum_{l=1}^{k} (\Delta H|_{tC,l-1} - \Delta H|_{tC\ ,l}) + (\Delta H|_{tC,k} - \Delta H|_t) \qquad (A16)$$

where, k = 0,1,2,........,N-1. In Eq. (A16), the second term on the right hand side vanishes when k = 0. Since the heat flow, $Q_{out}|_t$, should be always non-negative, the minimum heat input to the entire system, $Q_{in,1}$, should be such a value that

$$Q_{out}|_t \geq O \qquad \text{(A17)}$$

for $t_{C,N} \leq t \leq t_{C,0}$. Note that Eq. (A16) is valid despite variable heat capacities (temperature dependent) and phase changes.

Let's consider the special case where the heat capacity of each process stream is constant. Since

$$\Delta H|_{tC} = \sum_{i=1}^{m} H_{h,i}|_{tC+\Delta Tmin} - \sum_{j=1}^{n} H_{C,j}|_{tC}$$

$$= \sum_{i=1}^{m} [H^o_{h,i} + CP_{h,i}(t_C + \Delta T_{min} - t^o)]$$

$$- \sum_{j=1}^{n} [H^o_{C,j} + CP_{C,j}(t_C - t^o)] \qquad \text{(A18)}$$

where, t^o refers to the reference temperature, $H^o_{h,i}$ and $H^o_{c,j}$ refer to the reference enthalpies of the (i)th hot stream and the (j)th cold stream respectively. Next, substitute Eq. (A18) into Eq. (A15):

$$Q_{out,k} = Q_{in,1} + \sum_{l=1}^{k} [(\sum_{i=1}^{m} CP_{h,i} - \sum_{j=1}^{n} CP_{C,j})_l (t_{C,l-1} - t_{C,l})] \qquad \text{(A19)}$$

Here, the index l refers to the temperature interval, $t_{C,l} < t < t_{C,l-1}$. Note, that some of the values of $CP_{h,i}$'s or $CP_{C,j}$'s may be zero, since not all the process streams exist in the (l)th interval.

Finally, substituting Eq. (A18) into Eq. (A16) yields

$$Q_{out}|_t = Q_{in,1} + \sum_{l=1}^{k} [(\sum_{i=1}^{m} CP_{h,l} - \sum_{j=1}^{n} CP_{C,j})_l (t_{C,l-1} - t_{C,l})]$$

$$+ (\sum_{i=1}^{m} CP_{h,i} - \sum_{j=1}^{n} CP_{C,j})_{k+1} (t_{C,k} - t) \qquad \text{(A20)}$$

where, $t_{C,k} > t > t_{C,k+1}$. Eq. (A16) and (A20) are essentially alternative statements of the Problem Table Algorithm. Since they are in equation form, the variation in cumulative heat flow due to partial change in stream data can be easily formulated.