Pergamon

Computers chem. Engng Vol. 18, No. 4, pp. 345–368, 1994

0098-1354/94 $6.00 + 0.00

APPLICATION OF THE GENERALIZED STREAM STRUCTURE IN HEN SYNTHESIS

C.-T. CHANG,† K.-K. CHU and J.-R. HWANG
Department of Chemical Engineering, National Cheng Kung University, Tainan, Taiwan 70101, Republic of China

(Received 3 December 1992; final revision received 13 August 1993; received for publication 31 August 1993)

Abstract—A systematic procedure is proposed in this paper to incorporate the options of merging and/or splitting streams from multiple origins in process synthesis. In particular, design problems associated with the heat exchanger networks (HENs) are discussed in detail. First, an input–output system structure is described to facilitate the derivation of the modified LP and MILP models for calculating the design targets corresponding to the operating and capital costs. Next, construction procedure of a generalized stream structure is presented and the modified NLP model for generating the optimal networks is formulated accordingly. To accelerate the convergence rate of the corresponding iterative solution process, an evolutionary short-cut method is provided to produce feasible networks for use as the initial guesses. Two examples are also presented to demonstrate the effectiveness of the proposed approach. The results show that, without increasing the operating costs, the capital costs of HENs can be reduced significantly by considering the proposed stream structure in design.

INTRODUCTION

Over the past three decades, various design problems in process synthesis, e.g. those concerning the heat exchanger networks (HENs), the mass exchanger networks (MENs), the heat-integrated distillation systems, the separation sequences, etc. have been studied extensively by academic researchers and practitioners in the industries. One of the design techniques routinely used by the process engineers for reducing the overall capital cost associated with a flowsheet is stream-splitting. For example, Ponton and Donalson (1974) and Linnhoff and Hindmarsh (1983) have found that better HENs can be obtained via proper implementation of parallel processing schemes and also El-Halwagi and Manousiouthakis (1989) followed essentially the same approach to design MENs. On the other hand, the possibilities of merging process streams from multiple origins have seldom been investigated systematically in synthesizing process flow diagrams.

In many existing chemical processes, there are situations where two or more streams are allowed to be merged. For example, the overhead product streams of a multi-effect distillation unit may be combined and cooled to the storage temperature; the hot contaminated wastewater from different areas in a food additive plant may be merged and sent to the treatment unit before disposal; naphtha and hydrogen may be merged and heated to the reaction temperature in the hydrotreating plant of a refinery. In designing the HENs of these systems, merging process streams originated from different units is often a viable alternative in addition to the traditional synthesis techniques. Notice also that this option can be considered in designing systems other than HENs, e.g. the MENs. In the coke oven gas (COG) sweetening process (el-Halwagi and Manousiouthakis, 1989), the possibility of combining the sour COG and the tail gases from the claus unit may be considered in MEN design. Also, in the coal conversion process described by El-Halwagi and Manousiouthakis (1990), offgas streams from the gasifier and the ebullated reactor must be sent to spray washers to remove ammonia and phenol. Conceivably, the waste water streams from these washers can be merged and sent to mass exchangers before treatment.

From the above examples, one can observe that there are a large number of potential applications of merging and splitting schemes in process synthesis. Although stream-splitting techniques have been discussed extensively in the past, very little was published on methods that take advantage of the additional opportunties created by merging process streams. In a preliminary study, Chang and Yu (1988) showed that such techniques can be used in an evolutionary synthesis procedure to reduce the number of heat exchangers in a maximum energy recovery network (Linnhoff and Hindmarsh, 1983) without energy penalty. Since this procedure must

† To whom all correspondence should be addressed.

be applied manually, only simple structures can be included in the resulting design, i.e. the possibilities of mixing/splitting of multiple (more than two) streams were neglected completely. Further, due to the inherent nature of the evolutionary approach, synthesis options evaluated in the proposed design procedure are far from comprehensive.

One logical approach to develop systematic implementation strategies of the stream merging and splitting schemes is to treat both of them in a unified framework using mathematical programming techniques. In this work, a group of subproblems of process synthesis, i.e. those associated with the HENs, have been studied in detail. Traditionally, the mathematical programming approach to solve these problems is divided into three steps (Cerda *et al.*, 1983; Cerda and Westerberg, 1983; Papoulias and Grossmann, 1983; Grossmann, 1985; Duran and Grossmann, 1986; Floudas *et al.*, 1986; Floudas and Grossmann, 1987; Ciric and Floudas, 1989; Gundersen and Naess, 1988):

- Solve a linear programming (LP) model to determine the pinch points and the minimum consumption rates of utilities;
- Solve a mixed inter linear programming (MILP) model to determine the minimum number of exchanger units;
- Solve a nonlinear programming (NLP) model to obtain an optimal network, i.e. the one with minimum operating cost and minimum capital investment.

Notice that the possibility of stream mixing was explored in only very few of the related publications. Although the superstructure proposed by Floudas *et al.* (1986) was derived for each process stream in such a way so as to include alternatives on stream split, bypass, matches in series, matches in parallel, matches in series–parallel, etc. options of combining multiple superstructures were not considered. In a more recent study on retrofitting existing HENs, Yee and Grossmann (1991) developed another superstructure in which the possibility of mixing different process streams was examined. However, in their stream structure, certain potentially useful features are still missing, e.g. the bypass lines from inlet splitters to the exit mixers. Further, on the basis of practical considerations, it is often desirable to remove some of the restrictions implied in their formulations, e.g. the number of the exit mixers equals that of the inlet splitters. Thus, there is clearly a need to develop a systematic procedure to incorporate all options of merging and/or splitting process streams (from multiple origins to different destinations) for the grass-root design of HENs.

In this study, a generalized stream structure has been developed for such a purpose. If the corresponding network configurations are allowed in design, it becomes unnecessary to require the numbers of inputs and outputs of the HEN system to be the same. Any form of merging and/or splitting may take place among the input streams that are allowed to be mixed. The only requirement is that the operating conditions, e.g. temperature, concentration and flowrate, etc. of the corresponding output streams must satisfy the constraints governed by the needs of the downstream units. This type of input–output system structure is described in detail later in this paper and has been adopted as the basis of the modified LP and MILP formulations. In addition to the modified mathematical models, an effective procedure was developed to enhance the computational efficiency in solving the NLP problems. Based on the solutions of LP and MILP problems, this procedure can be applied manually in a systematic way to generate a set of feasible solutions within a short period of time. These solutions can then be used as initial guesses in the iterative search process for the optimal network configurations.

To demonstrate the feasibility of the proposed synthesis procedure, the results of two application examples are presented in this paper. One can clearly observe that, as a result of incorporating the proposed new techniques in process synthesis, better alternatives may be generated in solving the same design problem. More specifically, the number of matches determined by the modified MILP model is less than that obtained with the traditional method and, thus, the capital investment of a HEN can be further reduced without increasing the overall operating costs. In certain cases, due to the extra degree of flexibility introduced by the generalized stream structure, it is even possible to produce networks with exchanger units fewer than those predicted by the modified MILP model. Finally, it should also be noted that, since the structures of the MEN models are almost identical to those of the HEN models (El-Halwagi and Manousiouthakis, 1990), the techniques described in this paper are directly applicable to MEN design problems as well.

THE INPUT–OUTPUT SYSTEM STRUCTURE

To facilitate our later discussions, the input–output system structure of a typical HEN must be clearly defined first. In our study, this structure is described by the block diagram presented in Fig. 1. Notice that a number l ($l = 1, 2, \ldots, M$) is assigned

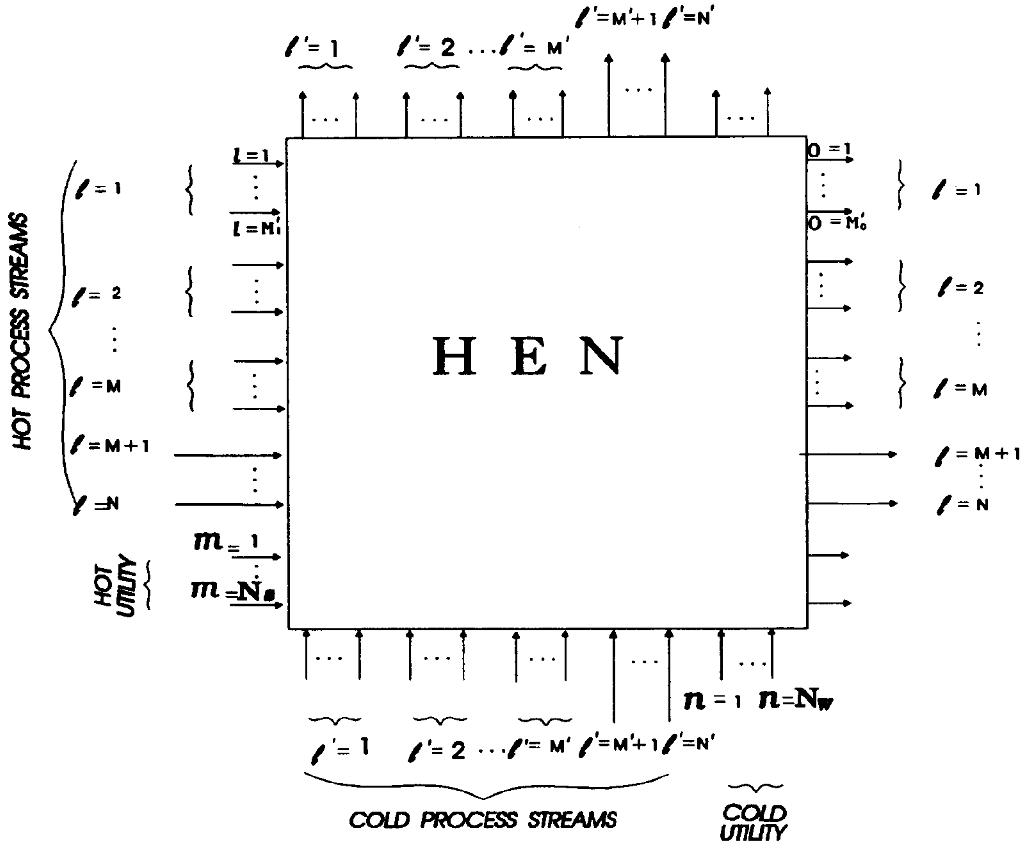

Fig. 1. The input–output system structure.

to each group of hot process streams in which the streams are allowed to be mixed with one another. Corresponding to a specific group l, the streams entering the system may be originated from M_I^l different units and those leaving the HEN may be required to be delivered to M_O^l destinations. In general, $M_I^l \neq M_O^l$. Also, there may be another $N-M$ hot streams which are not allowed to be merged with other streams. Each of these streams is again given a label l and $l = M+1, M+2, \ldots, N$. Finally, let us assume that there are N_S hot utility streams and they are labeled separately by m ($m = 1, 2, \ldots, N_S$). Thus, the hot streams in a typical HEN can be classified according to the definitions of the following stream sets:

$\mathbf{H}' = \{l \mid l$ is the label of the l-th group of hot process streams in which the streams are allowed to be merged with one another$\}$,

$\mathbf{H}'' = \{l \mid l$ is the label of a hot process stream which is not allowed to be mixed with other streams$\}$,

$\mathbf{HU} = \{m \mid m$ is the label of a hot utility stream$\}$.

Corresponding to each stream group $l (l \in \mathbf{H}')$, two stream sets can be defined, i.e.

$\mathbf{HI}_l = \{\iota \mid \iota$ represents the label of the ιth hot input stream in group $l\}$,

$\mathbf{HO}_l = \{o \mid o$ is the label of the oth hot output stream in group $l\}$.

Based on the principle of conservation of mass and the assumption that the heat capacity of every process stream is constant, the heat-capacity flowrates associated with the input streams in set $\mathbf{HI}_l$ and the output streams in set $\mathbf{HO}_l$ can be related by:

$$\sum_{\iota \in \mathrm{HI}_l} F_{Cp_\iota}^{\mathrm{HI}_l} = \sum_{o \in \mathrm{HO}_l} F_{Cp_o}^{\mathrm{HO}_l}, \qquad l \in \mathbf{H}' \tag{1}$$

Similarly, three sets of cold streams can be defined in the same way:

$\mathbf{C}' = \{l' \mid l'$ is the label of the l'th group of cold process streams in which the streams are allowed to be merged with one another$\}$,

$\mathbf{C}'' = \{l' \mid l'$ is the label of a cold process stream which is not allowed to be mixed with other streams},

$\mathbf{CU} = \{n \mid n$ is the label of a cold utility stream$\}$.

Corresponding to each group l' ($l' \in \mathbf{C}'$), two stream sets are used to describe the input and output conditions, i.e.

$\mathbf{CI}_{l'} = \{\iota' \mid \iota'$ represents the label of the ι'th cold input stream in group $l'\}$,

$\mathbf{CO}_{l'} = \{o' \mid o'$ is the label of the o'th cold output stream in group $l'\}$.

Again, a similar relationship exists among the heat-capacity flowrates associated with the input streams in set $\mathbf{CI}_{l'}$ and the output streams in set $\mathbf{CO}_{l'}$:

$$\sum_{\iota' \in \mathbf{CI}_{l'}} F_{Cp_{\iota'}}^{\mathbf{CI}_{l'}} = \sum_{o' \in \mathbf{CO}_{l'}} F_{Cp_{o'}}^{\mathbf{CO}_{l'}}, \qquad l' \in \mathbf{C}'. \tag{2}$$

In this work, it is assumed that the stream data associated with this input–output system structure have already been extracted from the flowsheet. In other words, the values of M, M', N, N', M_{I}^{l}s, $M'^{l'}_{\mathrm{I}}$s, M_{O}^{l}s, $M'^{l'}_{\mathrm{O}}$s and the temperatures and heat-capacity flowrates of all input and output streams are needed before solving our design problem. From this assumption and the previous description about the input–output system structure, one can see clearly that the conventional methods for determining the energy targets, i.e. the minimum utility consumption rates and the pinch temperatures, are no longer directly applicable in this situation. There is thus a need for the development of a modified calculation procedure.

THE MINIMUM UTILITY COSTS

If a process stream is split into several branches and each branch is cooled (or heated) individually to the target temperature, the shape of the corresponding hot (or cold) composite curve will not be affected. Similarly, the curve remains unchanged if multiple streams are merged at the *same* temperature and, then, delivered to exchangers or other processing units. However, if multiple hot (or cold) streams are mixed at different temperatures, the trajectory of the corresponding composite curve must deviate from that of the original curve. Furthermore, the position of the former at the temperature after mixing must be closer to the other, i.e. cold (or hot), composite curve (Chang and Yu, 1988). Thus, the minimum utility consumption rates of a HEN which includes stream-merging configurations should be *no less than* those of the HEN obtained by not considering such options in design.

On the basis of the above arguments, a modified LP model has been developed to determine the lower bound of the energy cost corresponding to a HEN with the input–output system structure of Fig. 1. More specifically, a fictitious hot stream is assumed for each pair of ι and o ($\iota \in \mathbf{HI}_l$ and $o \in \mathbf{HO}_l$) in stream group l and $l \in \mathbf{H}'$. An example of such a concept is presented in Fig. 2. Also, for illustration purposes, another stream set is defined to represent these fictitious hot streams as a whole:

$$\mathbf{HIO}_l = \{(\iota, o) \mid \iota \in \mathbf{HI}_l, o \in \mathbf{HO}_l, T_\iota^{\mathrm{HI}_l} > T_o^{\mathrm{HO}_l}\},$$

where $T_\iota^{\mathrm{HI}_l}$ and $T_o^{\mathrm{HO}_l}$ denote, respectively, the temperatures of the ιth input stream and the oth output stream of the lth stream group in set $\mathbf{H}'$. Notice that the input temperature of a fictitious hot stream is assumed to be higher than the output temperature. If this constraint is violated, the corresponding pair (ι, o) must be excluded from HIO_l. The heat-capacity flowrates of these fictitious streams are related to those of the input and output streams by the following equations:

$$\sum_{o \in \mathbf{HO}_l, (\iota, o) \in \mathbf{HIO}_l} F_{Cp_{\iota o}}^{\mathrm{HIO}_l} = F_{Cp_\iota}^{\mathrm{HI}_l}, \qquad \iota \in \mathbf{HI}_l, \qquad l \in \mathbf{H}', \tag{3}$$

$$\sum_{\iota \in \mathbf{HI}_l, (\iota, o) \in \mathbf{HIO}_l} F_{Cp_{\iota o}}^{\mathrm{HIO}_l} = F_{Cp_o}^{\mathrm{HO}_l}, \qquad o \in \mathbf{HO}_l, \qquad l \in \mathbf{H}'. \tag{4}$$

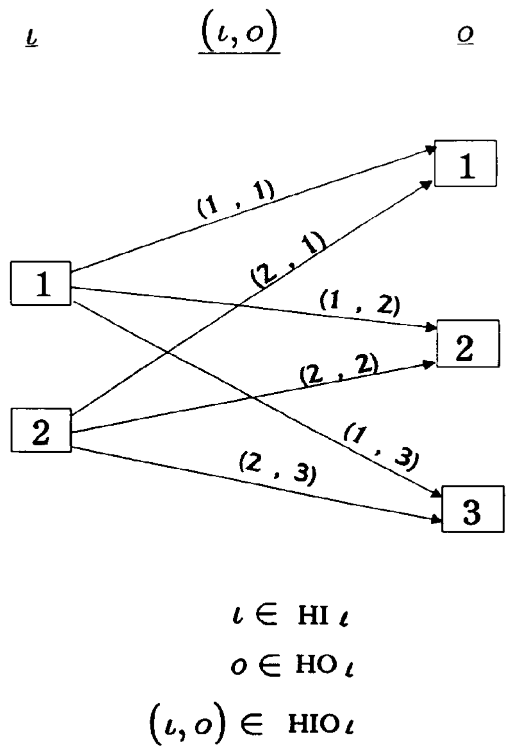

Fig. 2. An example of the fictitious streams ($M_{\mathrm{I}}^{l}=2$, $M_{\mathrm{O}}^{l}=3$).

Corresponding to each ι ($\iota \in \mathbf{HI}_l$), the heat capacity flowrates of the fictitious streams in equation (3), i.e. $F_{Cp_{\iota o}}^{\mathrm{HIO}_l}$, should be summed over all o's that satisfy two conditions, i.e. $o \in \mathbf{HO}_l$ and $(\iota, o) \in \mathbf{HIO}_l$. Similarly, the summation over ι's in equation (4) should be carried out in such a way that the two constraints, $\iota \in \mathbf{HI}_l$ and $(\iota, o) \in \mathbf{HIO}_l$, are satisfied for any given o ($o \in \mathbf{HO}_l$). Notice that the same approach can be taken to define the set of all fictitious cold streams in group l' of set $\mathbf{C}'$, i.e. $\mathbf{CIO}_{l'}$. Relations similar to equations (3) and (4) also exist between their heat-capacity flowrates and those of the input and output cold streams. Due to the limitation of space, their presentations are omitted in this paper.

If the hot streams in set $\mathbf{H}'$ and the cold streams in set $\mathbf{C}'$ are replaced by the fictitious streams described above, the system in Fig. 1 can be transformed into one with the same number of inputs and outputs. To this transformed input–output system structure, the traditional LP model is applicable. As a matter of fact, this structure can be viewed as a special case of Fig. 1. Each of the M_I^l inputs in group l ($l \in \mathbf{H}'$) can be split into M_O^l streams and cooled individually to their respective target temperatures and, then, these split streams can be combined isothermally into M_O^l outputs. Notice that the minimum energy cost obtained from the LP model corresponding to the transformed structure should be the lower bound of that of the general system described in Fig. 1. This is due to the facts that the transformed system can be regarded as a HEN with special stream-merging schemes (i.e. the streams are merged at the same temperatures) and, also, the introduction of any other stream-merging schemes in the HEN design can only increase the demand for additional consumption in utilities. But, on the other hand, if model MP1 is based on only a special case of the general input–output system structure, then the corresponding minimum utility consumption rates should not be lower than those of the general case which includes additional stream merging/splitting alternatives. Therefore, we conclude that the minimum of MP1 can actually be considered as the solution to the general problem.

Usually, the first step in deriving the mathematical programming models is to partition the entire temperature range of all streams into K temperature intervals for which any suitable partition method can be adopted, e.g. Linnhoff and Hindmarsh (1983) and Cerda *et al.* (1983). The intervals are labeled from the highest level ($k = 1$) down to the lowest level ($k = K$) of temperature, with each interval k having a temperature change ΔT_k. This research followed the same approach. Depending upon whether to impose restrictions on the matches in HEN, two different modified LP models were established. First, let us consider the model without restrictions:

$$\min \left[\sum_{m \in \mathrm{HU}} PH_m F_m^{\mathrm{HU}} + \sum_{n \in \mathrm{CU}} PC_n F_n^{\mathrm{CU}} \right], \qquad (\mathrm{MP1}). \ (5)$$

Subject to

$$\begin{aligned} R_k - R_{k-1} - \sum_{m \in \mathrm{HU}_k} F_m^{\mathrm{HU}} \Delta h_{mk} + \sum_{n \in \mathrm{CU}_k} F_n^{\mathrm{CU}} \Delta h_{nk}, \\ = \sum_{l \in \mathrm{H}'} \sum_{(\iota, o) \in \mathrm{HIO}_l} Q_{\iota o k}^{\mathrm{HIO}_l} + \sum_{\iota \in H_k''} Q_{\iota k}^{\mathrm{H}''} \\ - \sum_{l' \in \mathrm{C}'} \sum_{(\iota' o') \in \mathrm{CIO}_{l'}} Q_{\iota' o' k}^{\mathrm{CIO}_{l'}} - \sum_{j \in \mathrm{C}_k''} Q_{jk}^{\mathrm{C}''}, \\ k = 1, 2, \ldots, K, \end{aligned} \qquad (6)$$

$$\sum_{o \in \mathrm{HO}_l, (\iota, o) \in \mathrm{HIO}_l} F_{Cp_{\iota o}}^{\mathrm{HIO}_l} = F_{Cp_\iota}^{\mathrm{HI}_l}, \qquad \iota \in \mathbf{HI}_l, \qquad l \in \mathbf{H}', \quad (7)$$

$$\sum_{\iota \in \mathrm{HI}_l, (\iota, o) \in \mathrm{HIO}_l} F_{Cp_{\iota o}}^{\mathrm{HIO}_l} = F_{Cp_o}^{\mathrm{HO}_l}, \qquad o \in \mathbf{HO}_l, \qquad l \in \mathbf{H}', \quad (8)$$

$$\sum_{o' \in \mathrm{CO}_{l'}, (\iota', o') \in \mathrm{CIO}_{l'}} F_{Cp_{\iota' o'}}^{\mathrm{CIO}_{l'}} = F_{Cp_{\iota'}}^{\mathrm{CI}_{l'}}, \qquad \iota' \in \mathbf{CI}_{l'}, \qquad l' \in \mathbf{C}', \quad (9)$$

$$\sum_{\iota' \in \mathrm{CI}_{l'}, (\iota', o') \in \mathrm{CIO}_{l'}} F_{Cp_{\iota' o'}}^{\mathrm{CIO}_{l'}} = F_{Cp_{o'}}^{\mathrm{CO}_{l'}}, \qquad o' \in \mathbf{CO}_{l'}, \qquad l' \in \mathbf{C}', \quad (10)$$

$$R_0 = R_K = 0, \qquad R_k \geq 0, \qquad k = 1, 2, \ldots, K-1, \quad (11)$$

$$F_m^{\mathrm{HU}} \geq 0, \qquad m \in \mathbf{HU}, \qquad F_n^{\mathrm{CU}} \geq 0, \qquad n \in \mathrm{CU}, \quad (12)$$

$$F_{Cp_{\iota o}}^{\mathrm{HIO}_l} \geq 0, \qquad (\iota, o) \in \mathbf{HIO}_l, \qquad F_{Cp_{\iota' o'}}^{\mathrm{CIO}_{l'}} \geq 0, \qquad (\iota', o') \in \mathbf{CIO}_{l'} \quad (13)$$

where PH_m and PC_n denote the unit costs of the mth hot and nth cold utilities, respectively. Equation (6) represents the energy balance around the temperature interval k. The heat inputs to interval k is coming from several sources (Papoulias and Grossmann, 1983), i.e. the residual heat flow from interval $k-1$ (R_{k-1}) and the hot streams and heating utilities whose temperature range includes interval k. There are two types of hot process streams. For a stream l in set $\mathbf{H}''$, the heat input to the interval can be *calculated* by:

$$Q_{lk}^{\mathrm{H}''} = F_{Cp_{lk}}^{\mathrm{H}''} \Delta T_k, \qquad (14)$$

where

$$F_{Cp_{lk}}^{\mathrm{H}''} = \begin{cases} F_{Cp_l}^{\mathrm{H}''} & [T_{k+1}, T_k] \subset [T_l^{\mathrm{O}}, T_l^{\mathrm{I}}], \\ 0 & \text{otherwise.} \end{cases} \qquad (15)$$

In the above equation, $F^{\mathrm{H''}}_{Cp_l}$ is the heat-capacity flowrate of stream l in set $\mathbf{H''}$, and T^{I}_l and T^{O}_l are the corresponding input and output temperature. On the other hand, the heat input from a fictitious hot stream,, $Q^{\mathrm{HIO}_l}_{\iota o k}$, can be expressed by:

$$Q^{\mathrm{HIO}_l}_{\iota o k} = = F^{\mathrm{HIO}_l}_{Cp_{\iota o k}} \Delta T_k, \quad (16)$$

where

$$F^{\mathrm{HIO}_l}_{Cp_{\iota o k}} = \begin{cases} F^{\mathrm{HIO}_l}_{Cp_{\iota o}} & [T_{k+1}, T_k] \subset [T^{\mathrm{HIO}_l}_o, T^{\mathrm{HIO}_l}_\iota] \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

Notice that the heat-capacity flowrates, $F^{\mathrm{HIO}_l}_{Cp_{\iota o}}$s, are the unknowns in model MP1 and they are required to satisfy the constraints imposed by equations (7) and (8). In addition to the residual heat flow R_k, the heat outputs from interval k are transported to the cold process streams and cooling utilities whose temperature range includes interval k. They can be calculated or expressed according to equations similar to equations (14–17) and, thus, are omitted for the sake of brevity. Based on the above discussions, it can be concluded that the variables in model MP1 are R_ks, F^{HU}_ms, F^{CU}_ns, $F^{\mathrm{HIO}_l}_{Cp_{\iota o}}$s and $F^{\mathrm{CIO}_{l'}}_{Cp_{\iota' o'}}$s.

Next, to illustrate the formulations of the modified LP model with restricted matches, the following stream sets must be introduced:

$$\mathbf{H} = \{i \mid i \in \mathbf{H'} \text{ or } i \in \mathbf{H''}\},$$

$$\mathbf{C} = \{j \mid j \in \mathbf{C'} \text{ or } i \in \mathbf{C''}\}.$$

If several pairs of hot and cold process streams are not allowed to be matched due to practical reasons, they are grouped into a set $\mathbf{F}$ in this study, i.e.

$\mathbf{F} = \{(\xi, \eta) \mid \xi \in \mathbf{H}, \eta \in \mathbf{C}$ and the match between ξ and η is forbidden$\}$.

The hot and cold streams involved in these forbidden matches are also collected in individual subsets, i.e.

$$\mathbf{HF} = \{\xi \mid (\xi, \eta) \in \mathbf{F}\}, \qquad \mathbf{CF} = \{\eta \mid (\xi, \eta) \in \mathbf{F}\}.$$

Similarly, a set $\mathbf{P}$ is defined in this study to include all pairs of hot and cold process streams that must exchange a specific amount of heat, i.e.

$\mathbf{P} = \{(\xi, \eta) \mid \xi \in \mathbf{H}, \eta \in \mathbf{C}$ and the match between ξ and η is compulsory$\}$.

The corresponding subsets of the hot and cold streams involved in these compulsory matches are:

$$\mathbf{HP} = \{\xi \mid (\xi, \eta) \in \mathbf{P}\}, \qquad \mathbf{CP} = \{\eta \mid (\xi, \eta) \in \mathbf{P}\}.$$

The other unrestricted hot process and utility streams are viewed as a combined hot stream with the label h and, similarly, the cold combined stream c can be obtained in the same way. Thus, all hot streams can be included in the following stream set:

$$\tilde{\mathbf{H}} = \{\xi \mid \xi = h \text{ or } \xi \in \mathbf{HF} \text{ or } \xi \in \mathbf{HP}\},$$

and, also, all cold streams can be collected in:

$$\tilde{\mathbf{C}} = \{\eta \mid \eta = c \text{ or } \eta \in \mathbf{CF} \text{ or } \eta \in \mathbf{CP}\}.$$

Finally, corresponding to any temperature interval k, subsets of $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{C}}$ can be defined:

$\tilde{\mathbf{H}}_k = \{\xi \mid \xi \in \tilde{\mathbf{H}}$ and stream ξ is present in interval $\bar{k} \leq k\}$,

$\tilde{\mathbf{C}}_k = \{\eta \mid \eta \in \tilde{\mathbf{C}}$ and stream η is present in interval $k\}$.

Based on the above definitions, the modified LP model with restricted matches can be developed, i.e.

$$\min \left[\sum_{m \in \mathrm{HU}} PH_m F^{\mathrm{HU}}_m + \sum_{n \in \mathrm{CU}} PC_n F^{\mathrm{CU}}_n \right], \quad \text{(MRP1)}, \quad (18)$$

subject to:

$$R_{\xi,k} - R_{\xi,k-1} + \sum_{\eta \in \tilde{C}_k} Q_{\xi\eta k} = Q^{\tilde{\mathrm{H}}}_{\xi k}, \qquad \xi \in \tilde{\mathbf{H}}_k, \quad (19)$$

$$\sum_{\xi \in \tilde{H}_k} Q_{\xi\eta k} = Q^{\tilde{C}}_{\eta k}, \qquad \eta \in \tilde{\mathbf{C}}_k, \quad (20)$$

$$Q^{\tilde{\mathrm{H}}}_{hk} = \sum_{\xi \in \mathrm{H'}, \xi \notin \mathrm{HF}, \xi \notin \mathrm{HP}} \; \sum_{(\iota, o) \in \mathrm{HIO}_\xi} F^{\mathrm{HIO}_\xi}_{Cp_{\iota o k}} \Delta T_k + \sum_{\xi \in \mathrm{H''}, \xi \notin \mathrm{HF}, \xi \notin \mathrm{HP}} F^{\mathrm{H''}}_{Cp_{\xi k}} \Delta T_k + \sum_{m \in \mathrm{HU}_k} F^{\mathrm{HU}}_m \Delta h_{mk}, \quad (21)$$

$$Q^{\tilde{C}}_{ck} = \sum_{\eta \in \mathrm{C'}, \eta \notin \mathrm{CF}, \eta \notin \mathrm{CP}} \; \sum_{(\iota', o') \in \mathrm{CIO}_\eta} F^{\mathrm{CIO}_\eta}_{Cp_{\iota' o' k}} \Delta T_k + \sum_{\eta \in \mathrm{C''}, \eta \notin \mathrm{CF}, \eta \notin \mathrm{CP}} F^{\mathrm{C''}}_{Cp_{\eta k}} \Delta T_k + \sum_{n \in \mathrm{CU}_k} F^{\mathrm{CU}}_n \Delta h_{nk}, \quad (22)$$

$$Q_{\xi\eta k} = 0, \; (\xi, \eta) \in \mathbf{F}, \quad (23)$$

$$Q_{\xi\eta k} = Q^{\mathrm{com}}_{\xi\eta k}, \; (\xi, \eta) \in \mathbf{P}, \quad (24)$$

$$\sum_{o \in \mathrm{HO}_l, (\iota, o) \in \mathrm{HIO}_l} F^{\mathrm{HIO}_l}_{Cp_{\iota o}} = F^{\mathrm{HI}_l}_{Cp_\iota}, \qquad \iota \in \mathbf{HI}_l, \qquad l \in \mathbf{H'}, \quad (25)$$

$$\sum_{\iota \in \mathrm{HI}_l, (\iota, o) \in \mathrm{HIO}_l} F^{\mathrm{HIO}_l}_{Cp_{\iota o}} = F^{\mathrm{HO}_l}_{Cp_o}, \qquad o \in \mathbf{HO}_l, \qquad l \in \mathbf{H'}, \quad (26)$$

$$\sum_{o' \in \mathrm{CO}_{l'}, (\iota', o') \in \mathrm{CIO}_{l'}} F^{\mathrm{CIO}_{l'}}_{Cp_{\iota' o'}} = F^{\mathrm{CI}_{l'}}_{Cp_{\iota'}}, \qquad \iota' \in \mathbf{CI}_{l'}, \qquad l' \in \mathbf{C'} \quad (27)$$

$$\sum_{\iota' \in \mathrm{CI}_{l'}, (\iota', o') \in \mathrm{CIO}_{l'}} F^{\mathrm{CIO}_{l'}}_{Cp_{\iota' o'}} = F^{\mathrm{CO}_{l'}}_{Cp_{o'}}, \qquad o' \in \mathbf{CO}_{l'}, \qquad l' \in \mathbf{C'}, \quad (28)$$

$$Q_{\xi\eta k} \geq 0, \qquad \xi \in \tilde{\mathbf{H}}_k, \qquad \eta \in \tilde{\mathbf{C}}_k, \quad (29)$$

$$R_{\xi,0} = R_{\xi,K} = 0, \qquad R_{\xi,k} \geq 0, \qquad \xi \in \tilde{\mathbf{H}}_k, \quad (30)$$

$$F_m^{\mathrm{HU}} \geq 0, \qquad m \in \mathbf{HU}, \qquad F_n^{\mathrm{CU}} \geq 0, \qquad n \in \mathbf{CU}, \quad (31)$$

$$F_{Cp_{\iota o}}^{\mathrm{HIO}_l} \geq 0, \qquad (\iota, o) \in \mathbf{HIO}_l, \qquad l \in \mathbf{H}', \quad (32)$$

$$F_{Cp_{\iota' o'}}^{\mathrm{CIO}_{l'}} \geq 0, \qquad (\iota', o') \in \mathbf{CIO}_{l'}, \qquad l' \in \mathbf{C}', \quad (33)$$

where, the subscript k in the above model is associated with the kth temperature interval ($k = 1, 2, \ldots, K$), and $Q_{\xi\eta k}^{\mathrm{com}}$ is the amount of heat that must be exchanged between streams ξ and η in interval k. Notice that equation (19) is the heat balance equation for a hot stream ξ in set $\tilde{\mathbf{H}}$ around interval k. If $\xi = h$, $Q_{\xi k}^{\tilde{\mathrm{H}}}$ on the right-hand side of this equation can be replaced by equation (21). If $\xi \in \mathbf{HF}$ or $\xi \in \mathbf{HP}$, then, depending upon $\xi \in \mathbf{H}'$ or $\xi \in \mathbf{H}''$, different expressions should be substituted for $Q_{\xi k}^{\tilde{\mathrm{H}}}$, i.e. $\Sigma_{(\iota, o) \in \mathrm{HIO}_\xi} F_{Cp\iota ok}^{\mathrm{HIO}_\xi} \Delta T_k$ in the former case and $F_{Cp_{\xi k}}^{\mathrm{H}''} \Delta T_k$ for the latter. Note also that the latter expression can be computed directly. Similar expressions can be used to replace the term $Q_{\eta k}^{\tilde{\mathrm{C}}}$ on the right-hand side of equation (20). Due to the limitation of space, the corresponding presentations are not repeated in this paper.

Finally, it should be pointed out that, corresponding to the same minimum utility consumption rates, it is possible to identify more than one set of suitable $F_{Cp_{\iota o}}^{\mathrm{HIO}_l}$s and $F_{Cp_{\iota' o'}}^{\mathrm{CIO}_{l'}}$s in solving MP1 and MRP1. This situation is acceptable since only the energy targets will be used in the modified MILP model for computing the minimum number of exchangers. On the other hand, it should also be noted that, although the stream data of the fictitious streams are not needed in formulating the MILP and NLP models, these informations are still quite useful. In particular, they can be adopted in our evolutionary procedure for generating the initial feasible solutions for solving the modified NLP problem.

THE DESIGN TARGETS FOR NETWORK SYNTHESIS

To synthesize an optimal HEN with mathematical programming techniques, it is necessary to determine first the design targets for use in the NLP model, i.e. the minimum number of exchangers, the corresponding heat duty in each exchanger unit and the hot and cold streams involved in every match. Usually, they are determined with an MILP model. For the system in which process streams originated from different units are not allowed to be merged, a model has already been developed (Papoulias and Grossmann, 1983). If, however, stream-merging schemes can be considered in the synthesis of HENS, then additional exchangers can be eliminated from such networks and, thus, the capital investment can be lowered significantly. To facilitate illustration of the modified MILP model, the following two stream sets need to be defined:

$$\hat{\mathbf{H}} = \{\mu \mid \mu \in \mathbf{H} \text{ or } \mu \in \mathbf{HU}\},$$

$$\hat{\mathbf{C}} = \{\nu \mid \nu \in \mathbf{C} \text{ or } \nu \in \mathbf{CU}\}.$$

From the above definitions and the definitions of $\mathbf{H}'$ and $\mathbf{C}'$, one can see that, if a group of process streams are allowed to be mixed, these streams should be regarded as *one* member in set $\hat{\mathbf{H}}$ or $\hat{\mathbf{C}}$. In other words, every group of mixable streams is viewed as one combined heat source or sink in the modified MILP model.

The entire temperature range of the process streams is again divided into K intervals by the same method described in the previous section and they are numbered by an index k. In addition, if the number of pinch points determined by the model MP1 is S, the HEN can be separated into $S+1$ subnetworks. Starting from the high-temperature end, a label s ($s = 1, 2, \ldots, S+1$) is assigned to each subnetwork. Within any subnetwork s, there may be several temperature intervals. The set of these intervals is represented by $\mathbf{SN}_s$ in this study.

Using the definitions stated above, further classification of the streams can be achieved:

$$\hat{\mathbf{H}}_s = \{\mu \mid \mu \in \hat{\mathbf{H}} \text{ and stream } \mu \text{ is present in } \mathbf{SN}_s\},$$

$$\hat{\mathbf{C}}_s = \{\nu \mid \nu \in \hat{\mathbf{C}} \text{ and stream } \nu \text{ is present in } \mathbf{SN}_s\},$$

and

$$\hat{\mathbf{H}}_{sk} = \{\mu \mid \mu \in \hat{\mathbf{H}}_s,\ k \in \mathbf{SN}_s \text{ and stream } \mu \text{ is present in interval } \tilde{k} \leq k\},$$

$$\hat{\mathbf{C}}_{sk} = \{\nu \mid \nu \in \hat{\mathbf{C}}_s,\ k \in \mathbf{SN}_s \text{ and stream } \nu \text{ is present in interval } k\}.$$

The modified MILP model can then be formulated accordingly:

$$\min \sum_{s=1}^{S+1} \sum_{\mu \in \hat{\mathrm{H}}_s} \sum_{\nu \in \hat{\mathrm{C}}_s} Y_{\mu\nu s}, \qquad (\mathrm{MP2}), \qquad (34)$$

Subject to:

$$R_{\mu,k} - R_{\mu,k-1} + \sum_{\nu \in \hat{\mathrm{C}}_{sk}} Q_{\mu\nu k} = Q_{\mu k}^{\hat{\mathrm{H}}_s}, \qquad \mu \in \hat{\mathbf{H}}_{sk}, \quad (35)$$

$$\sum_{\mu \in \hat{\mathrm{H}}_{sk}} Q_{\mu\nu k} = Q_{\nu k}^{\hat{\mathrm{C}}_s}, \qquad \nu \in \hat{\mathbf{C}}_{sk}, \qquad (36)$$

$$\sum_{k \in \mathrm{SN}_s} Q_{\mu\nu k} - U_{\mu\nu s} Y_{\mu\nu s} \leq 0, \qquad \mu \in \hat{\mathbf{H}}_s, \qquad \nu \in \hat{\mathbf{C}}_s, \quad (37)$$

$$U_{\mu\nu s} = \min\left(\sum_{k \in \mathrm{SN}_s} Q_{\mu k}^{\hat{\mathrm{H}}_s}, \sum_{k \in \mathrm{SN}_s} Q_{\nu k}^{\hat{\mathrm{C}}_s}\right), \qquad \mu \in \hat{\mathbf{H}}_s, \quad \nu \in \hat{\mathbf{C}}_s, \qquad (38)$$

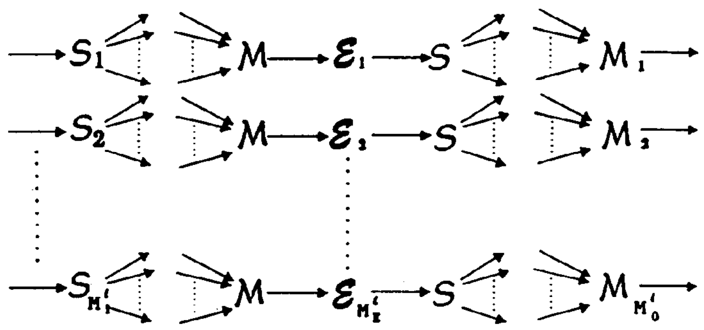

Fig. 3. The generalized stream structure.

$$Y_{\mu\nu s}=0 \text{ or } 1, \quad \mu\in\hat{\mathbf{H}}_s, \quad \nu\in\hat{\mathbf{C}}_s, \quad (39)$$

$$R_{\mu,k}\geq 0, \quad Q_{\mu\nu k}\geq 0, \quad \mu\in\hat{\mathbf{H}}_s, \quad \nu\in\hat{\mathbf{C}}_s, (40)$$

where $k\in \mathbf{SN}_S$ and $s=1,2,\ldots,S+1$. Notice that the terms $Q^{\hat{H}}_{\mu k}$ and $Q^{\hat{C}}_{\nu k}$ in this model can be computed directly from the stream data of the problem and the solutions of the modified LP models. Also, the values of the residual heats $R_{\mu k}$s entering or leaving all subnetworks should be set to zero. Thus, the unknowns in model MP2 are $Q_{\mu\nu k}$s, $Y_{\mu\nu k}$s and $R_{\mu k}$s (except the ones already set to zeros).

From the above description, one can clearly see that solving MP2 should produce the fewest number of matches *within* each subnetwork. If there is only one subnetwork, then the resulting number of exchangers in the network is truely at its minimum. If, however, pinch points exist, then it may be possible to reduce the total number of matches predicted by MP2 further. In this work, the evolutionary procedure suggested by Linnhoff and Hindmarsh (1983) has been utilized to "break" the loops identified in the solutions of MP2. Details of this approach are provided in Example 1.

THE GENERALIZED STREAM STRUCTURE

To integrate all options of merging/splitting process streams into the HEN design strategy, a generalized stream structure has been developed in this study. This structure is a modified version of the superstructures derived by Floudas *et al.* (1986) and Yee and Grossmann (1991). Corresponding to each group of process streams in $\mathbf{H}'$ or $\mathbf{C}'$, it can be constructed according to the following procedure:

- List all the exchangers between this group of streams and other streams.
- Install a mixer before every exchanger listed above.
- Install a mixer on each output stream in the group.
- Install a splitter on each input stream in the group and the split branches are connected to all the mixers mentioned above.
- Install a splitter at the exit of each exchanger and connect the split branches to all the previously mentioned mixers *except* the one before the same exchanger.

The structure of this generalized scheme can be represented by Fig. 3, in which the symbol $\mathcal{S}_\iota$ denotes the ιth entrance splitter ($\iota=1, 2, \ldots, M^l_{\mathrm{I}}$), the symbol $\mathcal{M}-\mathcal{E}_e-\mathcal{S}$ represents the eth exchanger ($e=1, 2, \ldots, M^l_{\mathrm{E}}$) and the mixer and splitter attached before and after this exchanger, and $\mathcal{M}_o$ denotes the oth exit mixer ($o=1, 2, \ldots, M^l_{\mathrm{O}}$). An example corresponding to the case $M^l_{\mathrm{I}}=M^l_{\mathrm{E}}=M^l_{\mathrm{O}}=2$ is presented in Fig. 4. Notice that all possible configurations in connecting multiple process streams are imbedded in this scheme, e.g. stream split, bypass, matches in series, matches in parallel, matches in series–parallel, etc. Further, the generalized stream structure presented in Fig. 3 reduces to the superstructure suggested by Floudas *et al.* (1986) if $M^l_{\mathrm{I}}=M^l_{\mathrm{O}}=1$ and, thus, is also suitable for streams which are not allowed to be mixed with other streams, i.e. the streams in set $\mathbf{H}''$ or $\mathbf{C}''$.

THE OPTIMUM NETWORK STRUCTURE

The generalized stream merging/splitting configurations described in the previous section can be

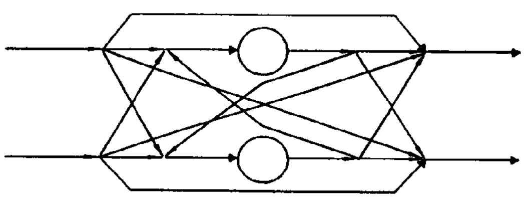

Fig. 4. The generalized stream structure corresponding to the case $M^l_{\mathrm{I}}=M^l_{\mathrm{E}}=M^l_{\mathrm{O}}=2$.

incorporated in an NLP model to generate the optimal HEN structure. Within a specific subnetwork s, the set of matches between hot and cold streams can be written as:

$$\mathbf{MA} = \{(i,j) \mid Y_{ijs} = 1, \quad i \in \hat{\mathbf{H}}_s, \quad j \in \hat{\mathbf{C}}_s\}$$

The heat exchanged in each match $(i,j) \in \mathbf{MA}$ can be calculated directly from the solutions of the modified MILP problem, i.e. $Q_{ij} = \Sigma_{k \in \mathrm{SN}_s} Q_{ijk}$. Let us group all the hot and cold streams in a **HEN** into one single stream set **HCT**, i.e.

$$\mathbf{HCT} = \hat{\mathbf{H}} \cup \hat{\mathbf{C}} = \{\kappa\}.$$

Notice that there may be several splitters and mixers on a stream (or a group of streams) corresponding to a particular κ in the set **HCT**. The branches before and after each splitter, mixer or exchanger can be viewed as different streams. In this study, each of them is given a label j and collected in the stream set $\mathcal{N}_\kappa$, i.e. $\mathcal{N}_\kappa = \{j\}$. The temperature and heat-capacity flowrate of stream j are represented by T_j^κ and f_j^κ, respectively. Also, the sets of splitters and mixers on stream κ (or group of streams with label κ) are denoted by $\mathcal{S}_\kappa = \{\sigma\}$ and $\mathcal{M}_\kappa = \{\rho\}$, respectively. The input and output stream sets associated with any of these splitters can be defined as:

$\mathcal{S}_\kappa^{\mathrm{I}}(\sigma) = \{j \mid j$ is the label of an input stream to the splitter σ, $j \in \mathcal{N}_\kappa$, $\sigma \in \mathcal{S}_\kappa\}$,

$\mathcal{S}_\kappa^{\mathrm{O}}(\sigma) = \{j \mid j$ is the label of an output stream from the splitter σ, $j \in \mathcal{N}_\kappa$, $\sigma \in \mathcal{S}_\kappa\}$.

Similarly, the input and output streams of a mixer can be collected in two stream sets, i.e.

$\mathcal{M}_\kappa^{\mathrm{I}}(\rho) = \{j \mid j$ is the label of an input stream to the mixer ρ, $j \in \mathcal{N}_\kappa$, $\rho \in \mathcal{M}_\kappa\}$,

$\mathcal{M}_\kappa^{\mathrm{O}}(\rho) = \{j \mid j$ is the label of an output stream from the mixer, ρ, $j \in \mathcal{N}_\kappa$, $\rho \in \mathcal{M}_\kappa\}$.

In addition, the streams connected to exchangers in the generalized structure are classified according to the definitions of the following stream sets:

$\mathbf{E}_{ij}^{\mathrm{HI}} = \{\alpha \mid \alpha$ is the label of a hot input stream to exchanger (i,j), $\alpha \in \mathcal{N}_i$, $(i,j) \in \mathbf{MA}\}$,

$\mathbf{E}_{ij}^{\mathrm{HO}} = \{\beta \mid \beta$ is the label of a hot output stream from exchanger (i,j), $\beta \in \mathcal{N}_i$, $(i,j) \in \mathbf{MA}\}$,

$\mathbf{E}_{ij}^{\mathrm{CI}} = \{\gamma \mid \gamma$ is the label of a cold input stream to exchanger (i,j), $\gamma \in \mathcal{N}_j$, $(i,j) \in \mathbf{MA}\}$,

$\mathbf{E}_{ij}^{\mathrm{CO}} = \{\delta \mid \delta$ is the label of a cold output stream from exchanger (i,j), $\delta \in \mathcal{N}_j$, $(i,j) \in \mathbf{MA}\}$.

The material and energy balance equations associated with the splitters, the mixers and the exchangers in the HEN should be adopted as the constraints of the modified NLP model, i.e.

- Material balance for splitters:

$$\sum_{j \in \mathcal{S}_\kappa^{\mathrm{I}}(\sigma)} f_j^\kappa - \sum_{j \in \mathcal{S}_\kappa^{\mathrm{O}}(\sigma)} f_j^\kappa = 0, \quad \sigma \in \mathcal{S}_\kappa, \quad \kappa \in \mathbf{HCT}. \quad (41)$$

- Energy balance for splitters:

$$T_i^\kappa = T_j^\kappa, \quad i \in \mathcal{S}_\kappa^{\mathrm{I}}(\sigma), \quad j \in \mathcal{S}_\kappa^{\mathrm{O}}(\sigma), \quad \sigma \in \mathcal{S}_\kappa, \quad \kappa \in \mathbf{HCT}. \quad (42)$$

- Material balance for mixers:

$$\sum_{j \in \mathcal{M}_\kappa^{\mathrm{I}}(\rho)} f_j^\kappa - \sum_{j \in \mathcal{M}_\kappa^{\mathrm{O}}(\rho)} f_j^\kappa = 0, \quad \rho \in \mathcal{M}_\kappa, \quad \kappa \in \mathbf{HCT}. \quad (43)$$

- Energy balance for mixers:

$$\sum_{j \in \mathcal{M}_\kappa^{\mathrm{I}}(\rho)} f_j^\kappa T_j^\kappa - \sum_{j \in \mathcal{M}_\kappa^{\mathrm{O}}(\rho)} f_j^\kappa T_j^\kappa = 0, \quad \rho \in \mathcal{M}_\kappa, \quad \kappa \in \mathbf{HCT}. \quad (44)$$

- Material balance for exchangers:

  Corresponding to any match $(i,j) \in \mathbf{MA}$, the input and output flowrates should be equal, i.e.

$$f_\alpha^i = f_\beta^i, \quad \alpha \in \mathbf{E}_{ij}^{\mathrm{HI}}, \quad \beta \in \mathbf{E}_{ij}^{\mathrm{HO}}, \quad (45)$$

$$f_\gamma^j = f_\delta^j, \quad \gamma \in \mathbf{E}_{ij}^{\mathrm{CI}}, \quad \delta \in \mathbf{E}_{ij}^{\mathrm{CO}} \quad (46)$$

- Energy balance for exchangers:

  Corresponding to any match $(i,j) \in \mathbf{MA}$, the heat exchanged Q_{ij} should be the same as that released by hot stream i or that absorbed by cold stream, j, i.e.

$$Q_{ij} - f_\alpha^i (T_\alpha^i - T_\beta^i) = 0, \quad \alpha \in \mathbf{E}_{ij}^{\mathrm{HI}}, \quad \beta \in \mathbf{E}_{ij}^{\mathrm{HO}}, \quad i \notin \mathbf{HU}, \quad (47)$$

$$Q_{ij} - f_\gamma^j (T_\gamma^j - T_\delta^j) = 0, \quad \gamma \in \mathbf{E}_{ij}^{\mathrm{CI}}, \quad \delta \in \mathbf{E}_{ij}^{\mathrm{CO}} \quad j \notin \mathbf{CU}, \quad (48)$$

$$Q_{ij} - f_\alpha^i \Delta h_i^{\mathrm{HU}} = 0, \quad \alpha \in \mathbf{E}_{ij}^{\mathrm{HI}}, \quad i \in \mathbf{HU}, \quad (49)$$

$$Q_{ij} - f_\gamma^j \Delta h_j^{\mathrm{CU}} = 0, \quad \gamma \in \mathbf{E}_{ij}^{\mathrm{CI}}, \quad j \in \mathbf{CU}. \quad (50)$$

In addition to the above equality constraints, there are two types of inequality constraints in the NLP model, i.e.

- The minimum temperature approach required in every exchanger:

  Corresponding to each $(i,j) \in \mathbf{MA}$, the following relations should be satisfied:

$$T_\alpha^i - T_\delta^j \geq \Delta T_{\min}, \quad \alpha \in \mathbf{E}_{ij}^{\mathrm{HI}}, \quad \delta \in \mathbf{E}_{ij}^{\mathrm{CO}}, \quad (51)$$

$$T_\beta^i - T_\gamma^j \geq \Delta T_{\min}, \quad \beta \in \mathbf{E}_{ij}^{\mathrm{HO}}, \quad \gamma \in \mathbf{E}_{ij}^{\mathrm{CI}}. \quad (52)$$

- Non-negative heat-capacity flowrates:

$$f_j^\kappa \geq 0, \quad j \in \mathcal{N}_\kappa, \quad \kappa \in \mathbf{HCT}. \quad (53)$$

The above constraints are, in essence, the same as those suggested by Floudas *et al.* (1986) in formulating the original NLP model. Since the input–output system structure of the HEN adopted in this study is modified to accommodate the possibility of stream-merging, the inlet and outlet temperatures and heat-capacity flowrates must be specified differently. Notice that, corresponding to each $\kappa \in (\mathbf{H}' \cup \mathbf{C}')$, there may be multiple initial splitters (each on one input stream) and multiple final mixers (each on one output stream) associated with the generalized scheme (Fig. 3). In this paper, these splitters are denoted by σ_τ^0 ($\tau \in \mathbf{HI}_\kappa$ or $\tau \in \mathbf{CI}_\kappa$) and the mixers by ρ_λ^0 ($\lambda \in \mathbf{HO}_\kappa$ or $\lambda \in \mathbf{CO}_\kappa$). On the other hand, if $\kappa \in (\mathbf{H}'' \cup \mathbf{C}'')$, the symbols σ^0 and ρ^0 are used to represent, respectively, the only initial splitter and final mixer on the generalized scheme for stream κ. As a result, the inlet and outlet conditions can be imposed according to the above notations:

- Specifications for the inlet heat-capacity flowrates:

$$f_j^\kappa = F_\kappa, \tag{54}$$

where

$$j \in \mathcal{S}_\kappa^{\mathrm{I}}(\sigma^0), \qquad \kappa \in (\mathbf{H}'' \cup \mathbf{C}''),$$
$$F_\kappa = \{F_i, i \in \mathbf{H}''; F_j, j \in \mathbf{C}''\},$$

and F_i and F_j represent the heat-capacity flowrates of hot stream i and cold stream j, i.e. $F_{Cp_i}^{\mathrm{H}''}$ and $F_{Cp_j}^{\mathrm{C}''}$, respectively:

$$f_j^\kappa = F_\kappa^\tau, \tag{55}$$

where

$$j \in \mathcal{S}_\kappa^{\mathrm{I}}(\sigma_\tau^0), \qquad \kappa \in (\mathbf{H}' \cup \mathbf{C}'),$$
$$\tau \in \mathbf{HI}_\kappa \quad \text{or} \quad \tau \in \mathbf{CI}_\kappa,$$
$$F_\kappa^\tau = \{F_i^\iota, i \in \mathbf{H}', \iota \in \mathbf{HI}_i; F_{i'}^{\iota'}, i' \in \mathbf{C}', \iota' \in \mathbf{CI}_{i'}\}$$

and

$$F_i^\iota = F_{Cp_\iota}^{\mathrm{HI}_i}, \tag{56}$$
$$F_{i'}^{\iota'} = F_{Cp_{\iota'}}^{\mathrm{CI}_{i'}}, \tag{57}$$

- Specifications for the outlet heat-capacity flowrates:

$$f_j^\kappa = F_\kappa, \tag{58}$$

where

$$j \in \mathcal{M}_\kappa^{\mathrm{O}}(\rho^0), \qquad \kappa \in (\mathbf{H}'' \cup \mathbf{C}''),$$
$$F_\kappa = \{F_i, i \in \mathbf{H}''; F_j, j \in \mathbf{C}''\},$$

and F_i and F_j represent the heat-capacity flowrates of hot stream i and cold stream j, i.e. $F_{Cp_i}^{\mathrm{H}''}$ and $F_{Cp_j}^{\mathrm{C}''}$, respectively:

$$f_j^\kappa = F_\kappa^\lambda, \tag{59}$$

where

$$j \in \mathcal{M}_\kappa^{\mathrm{O}}(\rho_\lambda^0), \qquad \kappa \in (\mathbf{H}' \cup \mathbf{C}'),$$
$$\lambda \in \mathbf{HO}_\kappa \quad \text{or} \quad \lambda \in \mathbf{CO}_\kappa,$$
$$F_\kappa^\lambda = \{F_i^o, i \in \mathbf{H}', o \in \mathbf{HO}_i; F_{i'}^{o'}, i' \in \mathbf{C}', o' \in \mathbf{CO}_{i'}\}$$

and

$$F_i^o = F_{Cp_o}^{\mathrm{HO}_i}, \tag{60}$$
$$F_{i'}^{o'} = F_{Cp_{o'}}^{\mathrm{CO}_{i'}}. \tag{61}$$

- Specifications for the inlet temperatures:

$$T_j^\kappa = T_\kappa^{\mathrm{I}}, \tag{62}$$

where

$$j \in \mathcal{S}_\kappa^{\mathrm{I}}(\sigma^0), \qquad \kappa \in (\mathbf{H}'' \cup \mathbf{C}''),$$
$$T_\kappa^{\mathrm{I}} = \{T_i^{\mathrm{I}}, i \in \mathbf{H}''; T_j^{\mathrm{I}}, j \in \mathbf{C}''\},$$
$$T_j^\kappa = T_\kappa^\tau, \tag{63}$$

where

$$j \in \mathcal{S}_\kappa^{\mathrm{I}}(\sigma_\tau^0), \qquad \kappa \in (\mathbf{H}' \cup \mathbf{C}'),$$
$$\tau \in \mathbf{HI}_\kappa \quad \text{or} \quad \tau \in \mathbf{CI}_\kappa,$$
$$T_\kappa^\tau = \{T_i^\iota, i \in \mathbf{H}', \iota \in \mathbf{HI}_i; T_{i'}^{\iota'}, i' \in \mathbf{C}', \iota' \in \mathbf{CI}_{i'}\}$$

and

$$T_i^\iota = T_\iota^{\mathrm{HI}_i} \tag{64}$$
$$T_{i'}^{\iota'} = T_{\iota'}^{\mathrm{CI}_{i'}}. \tag{65}$$

- Specifications for the outlet temperatures:

$$T_j^\kappa = T_\kappa^{\mathrm{O}}, \tag{66}$$

where

$$j \in \mathcal{M}_\kappa^{\mathrm{O}}(\rho^0), \qquad \kappa \in (\mathbf{H}'' \cup \mathbf{C}''),$$
$$T_\kappa^{\mathrm{O}} = \{T_i^{\mathrm{O}}, i \in \mathbf{H}''; T_j^{\mathrm{O}}, j \in \mathbf{C}''\},$$
$$T_j^\kappa = T_\kappa^\lambda, \tag{67}$$

where

$$j \in \mathcal{M}_\kappa^{\mathrm{O}}(\rho_\lambda^0), \qquad \kappa \in (\mathbf{H}' \cup \mathbf{C}'),$$
$$\lambda \in \mathbf{HO}_\kappa \quad \text{or} \quad \lambda \in \mathbf{CO}_\kappa,$$
$$T_\kappa^\lambda = \{T_i^0, i \in \mathbf{H}', o \in \mathbf{HO}_i; T_{i'}^{0'}, i' \in \mathbf{C}', o' \in \mathbf{CO}_{i'}\},$$

and

$$T_i^o = T_0^{\mathrm{HO}_i}, \tag{68}$$
$$T_{i'}^{o'} = T_{o'}^{\mathrm{CO}_{i'}}, \tag{69}$$

The objective function for minimizing the investment cost of subnetwork s of a HEN can be expressed by

$$\min\left[\sum_{(i,j)\in MA} Ct_{ij} A_{ij}^{Cb_{ij}}\right], \tag{70}$$

where A_{ij} is the area of the exchanger corresponding to match $(i, j) \in \mathbf{MA}$ and Ct_{ij} and Cb_{ij} are cost coefficients. Also, the log-mean temperature difference needed for determining the heat transfer area A_{ij} can be computed by the method suggested by Paterson (1984). Thus, the modified NLP model

(MP3) for the HENs can be formulated with equations (70) and (41)–(69). The final HEN design can be obtained by combining the results corresponding to all subnetwork.

It should be noted that MP3 is formulated according to the solutions of the modified MILP problem (MP2). Notice also that the residual heats entering or leaving all subnetworks are set to zero in MP2. If this restriction is removed, i.e. heat is allowed to flow across the pinch, it may be possible to construct a network with exchanger number lower than that predicted by the MILP model, e.g. see Yee and Grossmann (1990a–c) and Ciric and Floudas (1991). In our study, this is done by formulating an NLP model which includes all subnetworks. The design targets adopted in this model can be obtained by modifying the MILP results with the loop-breaking techniques (Linnhoff and Hindmarsh, 1983) described previously. Such an approach will certainly increase the complexity of the mathematical model and, also, create computational difficulties. But, on the other hand, opportunities in eliminating some of the matches predicted by the modified MILP model may be identified in the combined system and the corresponding network configurations may be generated due to the extra degree of flexibility introduced by the generalized stream structure.

GENERATION OF THE INITIAL FEASIBLE SOLUTIONS

Although all possible configurations are included in the generalized stream structure and, thus, comprehensiveness is ensured in the synthesis procedure, the present approach suffers one major drawback, i.e. the computation load of the iterative solution process associated with the NLP problem is often overwhelming. This is due to the fact that, by allowing the additional options created by merging process streams in HEN design, the number of undetermined variables becomes extremely large even for a system with relatively few process streams. Generally speaking, the rate of convergence in solving a nonlinear programming problem is strongly dependent upon the initial guess. This is especially true in our case. If the initial values of the variables in model MP3 are assigned arbitrarily, the corresponding numerical solution process, carried out with a well-developed commercial software, e.g. MINOS (Murtagh and Saunders, 1983) and GINO (Liebman *et al.*, 1986), is usually divergent. Presently, one of the most popular approaches to produce initial feasible solutions is to introduce slack variables into the equality constraints and use their sum as the objective function. Similar to other NLP problems, convergence still cannot be guaranteed in this initialization process. Further, even such an initial feasible network can be identified, it is highly possible to locate a local optimum instead of a global one in the subsequent optimization problem due to the nonlinear nature of the model MP3. Thus, if the present approach is to be adopted for practical applications, a systematic short-cut method must be developed to quickly generate, as many as possible, feasible network structures to complement the existing numerical initialization procedure. Furthermore, there are other incentives for using the proposed evolutionary approach. In particular, engineering judgement can be exercised in synthesizing the initial HENs. Thus, the solution of the NLP model (MP3) may not be needed since the initial feasible solution is already a practical design with a cost considerably lower than most other networks obtained with the conventional methods. This feature can be very useful when the solution of MP3 is difficult to obtain.

Since the modified LP and MILP models can be solved easily with LINDO (Schrage, 1990), information concerning matches in the system, i.e. their heat duties and the streams $(i, j) \in \mathbf{MA}$ involved in heat exchange, should be available before the synthesis of the actual HEN configuration. On the basis of these informations, an evolutionary procedure can be followed to construct feasible networks for use as the initial guesses of the numerical iteration process:

1. Treat each fictitious stream as an individual process stream and use the Pinch Design Method (Linnhoff and Hindmarsh, 1983) to produce the maximum energy recovery (MER) networks. Notice that more than one design may be generated in this step.
2. Compare the matches in an MER network with those obtained from solving MP2, i.e. the matches in set **MA**. If all the matches in the latter case are included in the former, then consider the corresponding MER network as one of the candidates. Notice that, since both the MER designs and the solutions to the MILP problem (MP2) may not be unique, more than one candidate may be found.
3. Select one of the candidates. List all matches in the candidate network and, then, remove the ones belonging to set **MA** from the list. The resulting list is referred to as list **L** in this procedure. Treat each stream (or each group of streams) in set **HCT** as a node and each match in the candidate MER network as an edge in an undirected graph. In this graph,

identify the loops that contain edges corresponding to the matches in list **L**. If there exists a match which is in list **L** but not on any of the above loops, then discard the present candidate network and repeat the current step. If there exists a loop which does not contain nodes corresponding to stream group $l \in \mathbf{H}'$ (or $l' \in \mathbf{C}'$), then also discard the present candidate and repeat the current step.

4. Select one of the matches in list **L** and one of its corresponding loops. Note that it is better to consider loops with fewer nodes first. Shift heat duties of the exchangers on this loop to eliminate the selected match. Then, revise the inlet and outlet temperatures of the exchangers in the network accordingly. Notice that, at this point, the target temperatures of the fictitious streams may not be the same as those specified in the problem statements. This is acceptable and can be corrected later.
5. Identify all the matches with increased heat duties on the loop selected in the previous step and, then, divide them into two groups, i.e. the matches with one or two fictitious streams involved and those without. The matches in the latter group are examined in this step. If, in any of these matches, the minimum temperature approach is smaller than the given limit, then go to Step 4 to work on another loop. If all loops are exhausted, then discard the present candidate and go to Step 3. Notice that, if a loop contains only two nodes, the present step is not required.
6. Examine the first group of matches identified in the previous step. Notice that they are connected with the neighboring matches in the selected loop via nodes corresponding to the fictitious streams. In general, each of these nodes is associated with two fictitious streams having the same group number $l \in \mathbf{H}'$ (or $l' \in \mathbf{C}'$). Notice that the heat duties of the neighboring matches must have been reduced or shifted completely in Step 4. If the minimum temperature approach in one of the matches examined is smaller than the given limit, then label this match as **exchanger I**, its corresponding fictitious stream as **stream A**, its neighboring match as **exchanger II**, and the fictitious stream associated with its neighboring match as **stream B**.The minimum temperature approach in **exchanger I** can be corrected by implementing a simplified version of the generalized stream structure to **stream A** and **stream B**. Specifically, this structure can be constructed with the following subprocedure:
   - Install a mixer on **stream A** before **exchanger I**.
   - Install a mixer on **stream B** before the *downstream* match of **exchanger II**.
   - Install a splitter before the mixer on **stream A**. The temperature of this splitter is the outlet temperature of the *upstream* match of **exchanger I**. The split branches are connected to the two mixers mentioned above.
   - Install a splitter before the mixer on **stream B**. The temperature of the splitter is the inlet temperature of **exchanger II**. The split branches are also connected to the two mixers mentioned above.
   - Adjust the flowrates of the four split branches in such a way that the minimum temperature approaches in the two downstream matches of both mixers are higher than the given limit.

   Notice that there may be two (one hot and one cold) fictitious streams involved in **exchanger I**. In such cases, the above procedure can be applied to either one or both of them to raise the corresponding temperature approach. In addition, the number of matches identified as **exchanger I** in this step may be greater than one. Thus, it may be necessary to repeat the subprocedure to modify the inlet and outlet conditions of these matches so that the constraints of $\Delta T_{\min}$ can be satisfied.

   If any of the tasks described in this step cannot be accomplished, go to Step 4 and try to eliminate the match through another loop. If all alternative loops are exhausted, discard the present candidate and go to Step 3.
7. Examine the remaining exchangers in the resulting network. If, in any of the exchanger, the constraint of $\Delta T_{\min}$ is violated, the simplified stream structure can also be utilized for correcting this situation. If this step fails, go to Step 4 and try to eliminate the match through another loop. If all alternative loops are exhausted, discard the present candidate and go to Step 3.
8. Repeat Steps 4–7 until all matches in list **L** are eliminated.
9. The desired output conditions can be achieved by implementing another version of the simplified stream structure:
   - Install mixers on output streams corresponding to all $o \in \mathbf{HO}_l$ ($l \in \mathbf{H}'$) and all $o' \in$

Table 1. Data needed in formulating the LP, MILP and NLP models for Example 1

Stream type	Group No.	Input stream No.	Heat-capacity flowrate (kW/°C)	Temperature (K)	Output stream No.	Heat-capacity flowrate (kW/°C)	Temperature (°C)
		1′	11.4	37.8	1′	24.4	204.4
Cold	1′	2′	12.9	65.6	2′	12.9	182.2
		3′	13.0	93.3			
Hot	1		16.6	248.9		16.6	121.1
Hot	2		13.3	204.4		13.3	65.6

$\Delta T_{min} = 11.1$°C.

$\mathbf{CO}_{l'}$ ($l' \in \mathbf{C}'$).

- Install splitters on all fictitious streams at the exits of the network obtained in the previous step. The split branches of the fictitious stream $(\iota, o) \in \mathbf{HIO}_l$ [or $(\iota', o') \in \mathbf{CIO}_{l'}$] are connected to the mixers on all output streams in set $\mathbf{HO}_l$ (or $\mathbf{CO}_{l'}$). Notice that both $\mathbf{HIO}_l$ (or $\mathbf{CIO}_{l'}$) and $\mathbf{HO}_l$ (or $\mathbf{CO}_{l'}$) are corresponding to the same group number.
- Adjust the flowrates of the split branches in such way that the desired output temperatures and heat-capacity flowrates are achieved.

If these tasks can be accomplished, then the resulting network is an initial feasible solution.

10. Repeat Steps 3–9 until all candidates are exhausted.

Finally, it should be noted that this procedure is still applicable if the matches in MA and their corresponding heat duties have been modified with the loop-breaking techniques described previously for the purpose of reducing the total exchanger number predicted by MP2.

APPLICATION EXAMPLES

To show the effectiveness of the proposed approach, two examples are presented here. The first one is a simple HEN design problem with only one subnetwork. The proposed synthesis procedure is explained in detail using this example. The second problem is concerned with the design of heat recovery network in a food additive processing plant. The possibility of eliminating additional units from the "optimal" network is clearly demonstrated in this case study.

Example 1

Let us consider the HEN design problem specified in Table 1. There are two hot streams in this system and they are not allowed to be mixed with each other. On the other hand, the three cold input streams in this problem can be merged in any form and, after leaving the HEN, the mixture is sent to two different downstream units for further processing.

Minimum utility cost without restricted matches— According to the definition of the stream set $\mathbf{CIO}_{1'}$, the supply and target temperatures of the fictitious cold streams in this case are:

Fictitious stream No.	*Supply temperature (°C)*	*Target temperature (°C)*
1′1′	37.8	204.4
2′1′	65.6	204.4
3′1′	93.3	204.4
1′2′	37.8	182.2
2′2′	65.6	182.2
3′2′	93.3	182.2

The heat-capacity flowrates of these fictitious streams are the undetermined variables in model MP1. They must satisfy the following constraints:

$$\sum_{\iota'=1'}^{3'} F_{Cp_{\iota'1'}}^{\mathrm{CIO}_{1'}} = 24.4,$$

$$\sum_{\iota'=1'}^{3'} F_{Cp_{\iota'2'}}^{\mathrm{CIO}_{1'}} = 12.9,$$

$$\sum_{o'=1'}^{2'} F_{Cp_{1'o'}}^{\mathrm{CIO}_{1'}} = 11.4,$$

$$\sum_{o'=1'}^{2'} F_{Cp_{2'o'}}^{\mathrm{CIO}_{1'}} = 12.9,$$

$$\sum_{o'=1'}^{2'} F_{Cp_{3'o'}}^{\mathrm{CIO}_{1'}} = 13.0.$$

Based on any of the standard partition method, the temperature intervals of the transformed system can be defined (Fig. 5). The corresponding LP model (MP1) was solved by LINDO (Schrage, 1991). The minimum consumption rates of hot and cold utilities of this problem were found to be 880.1 and 0 kW,

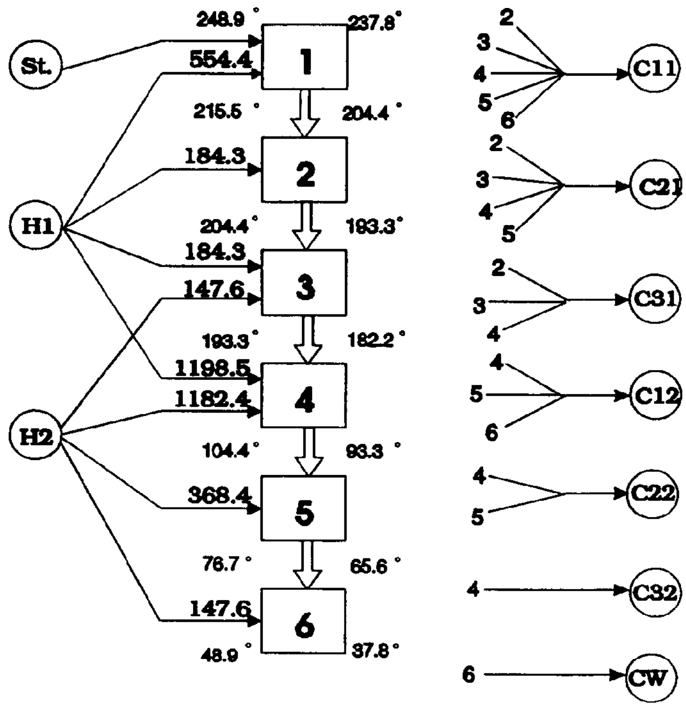

Fig. 5. The heat flow pattern specified in the modified LP model of Example 1.

respectively, and, thus, there is no pinch point. The heat-capacity flowrates of the fictitious streams were also determined. Their values (in kW/°C) are:

$$F^{\mathrm{ClO}_{1'}}_{Cp_{1'1'}} = 11.4, \quad F^{\mathrm{ClO}_{1'}}_{Cp_{2'1'}} = 0.0, \quad F^{\mathrm{ClO}_{1'}}_{Cp_{3'1'}} = 13.0,$$

$$F^{\mathrm{ClO}_{1'}}_{Cp_{1'2'}} = 0.0, \quad F^{\mathrm{ClO}_{1'}}_{Cp_{2'2'}} = 12.9, \quad F^{\mathrm{ClO}_{1'}}_{Cp_{3'2'}} = 0.0.$$

Notice that the solution to MP1 may not be unique. thus, it is possible to find that the minimum utility cost can be achieved with more than one set of fictitious heat-capacity flowrates.

3Minimum utility cost with restricted matches—Let us consider the same problem presented in Table 1, except that the match between hot stream 2 and any of the cold streams is forbidden. The corresponding model MRP1 has been solved by LINDO. In this case, the minimum hot and cold utility consumption rates are 2726.1 and 1846.0 kW, respectively.

The design targets for network synthesis—The energy targets, i.e. the minimum utility consumption rates and the pinch temperatures, obtained by solving MP1 were used to determine the design targets here. According to model MP2 and the definition of $\hat{\mathbf{C}}$, all the cold process streams in this problem should be viewed as one combined heat sink. Thus, its corresponding heat flow pattern (Fig. 6) is different from that of model MP1 (Fig. 5). The solution of MP2 can also be obtained by LINDO (Table 2). Since there is only one subnetwork, these results will be adopted in the NLP model without modifications. Also, notice that the heat-capacity flowrates of the fictitious streams computed with MP1 were *not* adopted to construct MP2. On the other hand, if the possibility of merging the cold streams in Table 1 is excluded in HEN design, then the MILP problem should be formulated as one with three individual cold process streams. Their heat-capacity flowrates can be chosen to be the same as the solutions of MP1, i.e. those of the fictitious streams. The corresponding minimum number of exchangers is 5. However, from Table 2, one can see that the actual minimum value should be 3 if stream-merging is allowed in HEN synthesis.

The initial feasible solutions—The purpose of identifying feasible solutions of model MP3 is to accelerate the numerical iteration process for determining the optimal network configuration. Since a global optimum cannot be guaranteed in solving this nonlinear programming problem, it is desirable to generate several feasible networks for use as the initial guesses. Notice also that constructing MER networks for the transformed input–output system structure is the starting step of the proposed evolutionary synthesis procedure. Therefore, it is actually quite helpful to identify more than one set of heat-

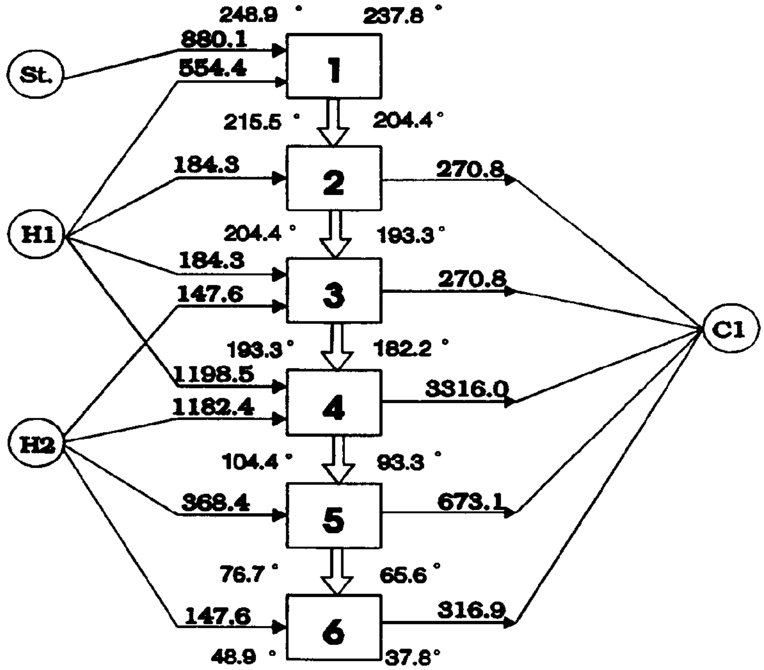

Fig. 6. The heat flow pattern specified in the modified MILP model of Example 1.

capacity flowrates for the fictitious streams in solving MP1.

A number of candidate HENs have been constructed by following the proposed procedure:

Step 1—Based on the heat-capacity flowrates of the fictitious streams obtained from solving MP1, two MER networks can be produced by implementing the Pinch Design Method (Fig. 7a, b).

Step 2—From the fact the matches in Table 2 are included in the HENs presented in Fig. 7a and b, one can conclude that both of them can be used as the candidates for generating the initial network configurations.

Step 3—Select the MER network in Fig. 7a as the candidate. The list of matches to be eliminated can be determined to be:

$$\mathbf{L} = \{(1, 1')(2, 1')\}.$$

Table 2. The optimum matches for Example 1

Subnetwork No.	Match		Heat duty (kW)
	Hot stream No.	Cold stream No.	
1	utility	1′	880.1
1	1	1′	2121.5
1	2	1′	1846.0

The corresponding loops are:

$$1 \rightarrow 1' \rightarrow 1, \qquad (a)$$

$$2 \rightarrow 1' \rightarrow 2. \qquad (b)$$

Step 4—Select loop (b). After shifting the heat duties in this loop, the inlet and outlet temperatures of the exchangers in the HEN were revised accordingly (Fig. 8a).

Step 5—This step is not required.

Step 6—The constraint of ΔT_{min} in exchanger 1 of the revised HEN in Fig. 8a is violated. The simplified stream structure was then applied to correct this situation. The resulting network is presented in Fig. 8b.

Step 7—From Fig. 8b, one can observe that none of the exchangers violate the constraint of ΔT_{min}.

Step 8—Repeat Steps 4–7 to eliminate exchanger 3 in Fig. 8b. The resulting network can be found in Fig. 8c.

Step 9—By implementing the proposed procedure to produce the desired output conditions, an initial feasible solution can be obtained (Fig. 9a).

Step 10—By repeating Steps 3–9, several other feasible solutions can be generated. Some of them are presented in Figs 9b–d.

The optimal heat exchanger network—Using a constant hot-utility temperature of 300°C and the above feasible solutions as the initial guesses, the optimal heat exchanger network can be obtained by solving

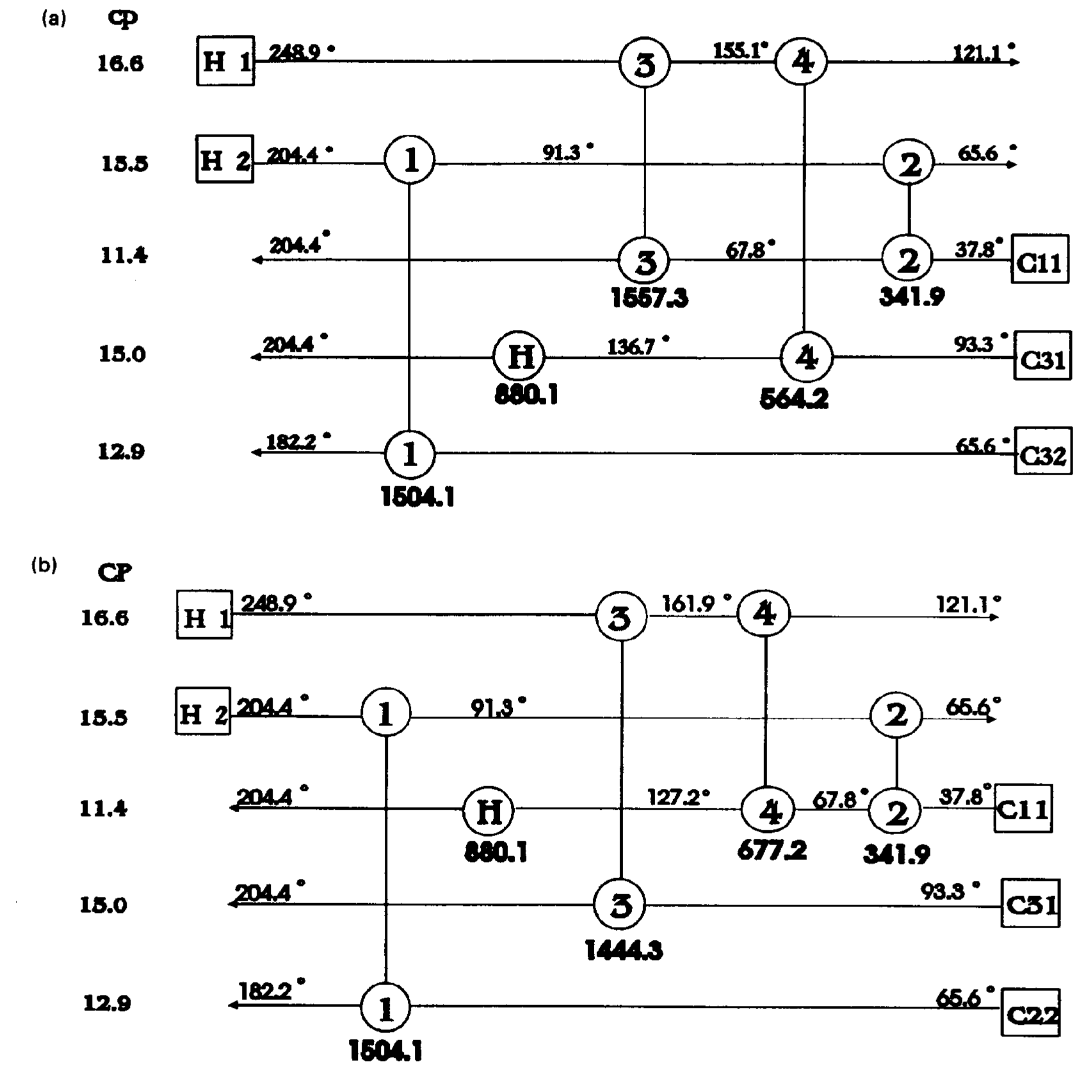

Fig. 7. (a) A candidate network for generating the feasible initial solutions. (b) Another candidate network for generating the feasible initial solutions.

the corresponding NLP problem (MP3) with GINO. For this example, the best network configuration was found to be the one presented in Fig. 10.

Example 2

This example is a realistic industrial HEN design problem. The problem is concerned with the heat recovery system of the distillation area in a food additive processing plant. Double-effect distillation columns are used to purify a mixture of iso-propyl alcohol (IPA) and water from 41.3% wt to approximately 81% wt of IPA.

A simplified process flow diagram of the distillation area is presented in Fig. 11. The feed to this separation system is drawn from an intermediate tank, in which a large inventory of 41.3% wt of IPA is stored at 43.3°C. Two feed streams, C1 and C2, are heated to 93.3 and 60.3°C, respectively and, then, fed to a high pressure column (Col. 2) and a low pressure column (Col. 1) separately. Another heat sink in the system is the process water stream C3 which is consumed in another part of the plant and is required to be heated from 10.0 to 93.3°C.

The overhead stream of Col. 2 is condensed in the reboiler of the low pressure column. This condensed stream is then flashed in vessel V1. The bottom stream of V1, i.e. H4, is cooled from 80.0 to 32.2°C and sent to the storage tank for the concentrated IPA/water mixture. The flashed vapor stream from V1 is combined with the superheated overhead

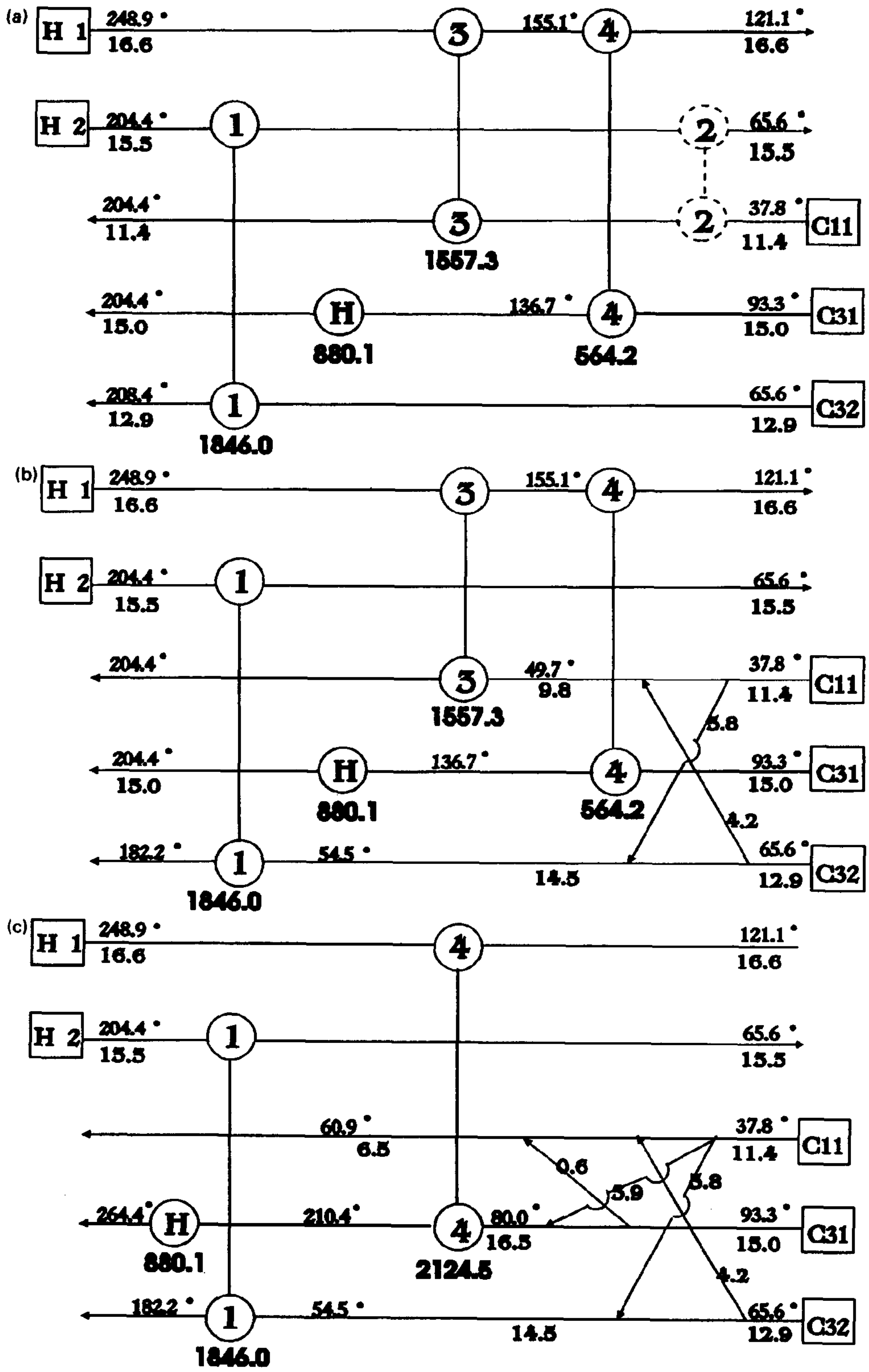

Fig. 8. (a) Generation of the initial feasible solutions—Step 4. (b) Generation of the initial feasible solutions—Step 6. (c) Generation of the initial feasible solutions—Step 8.

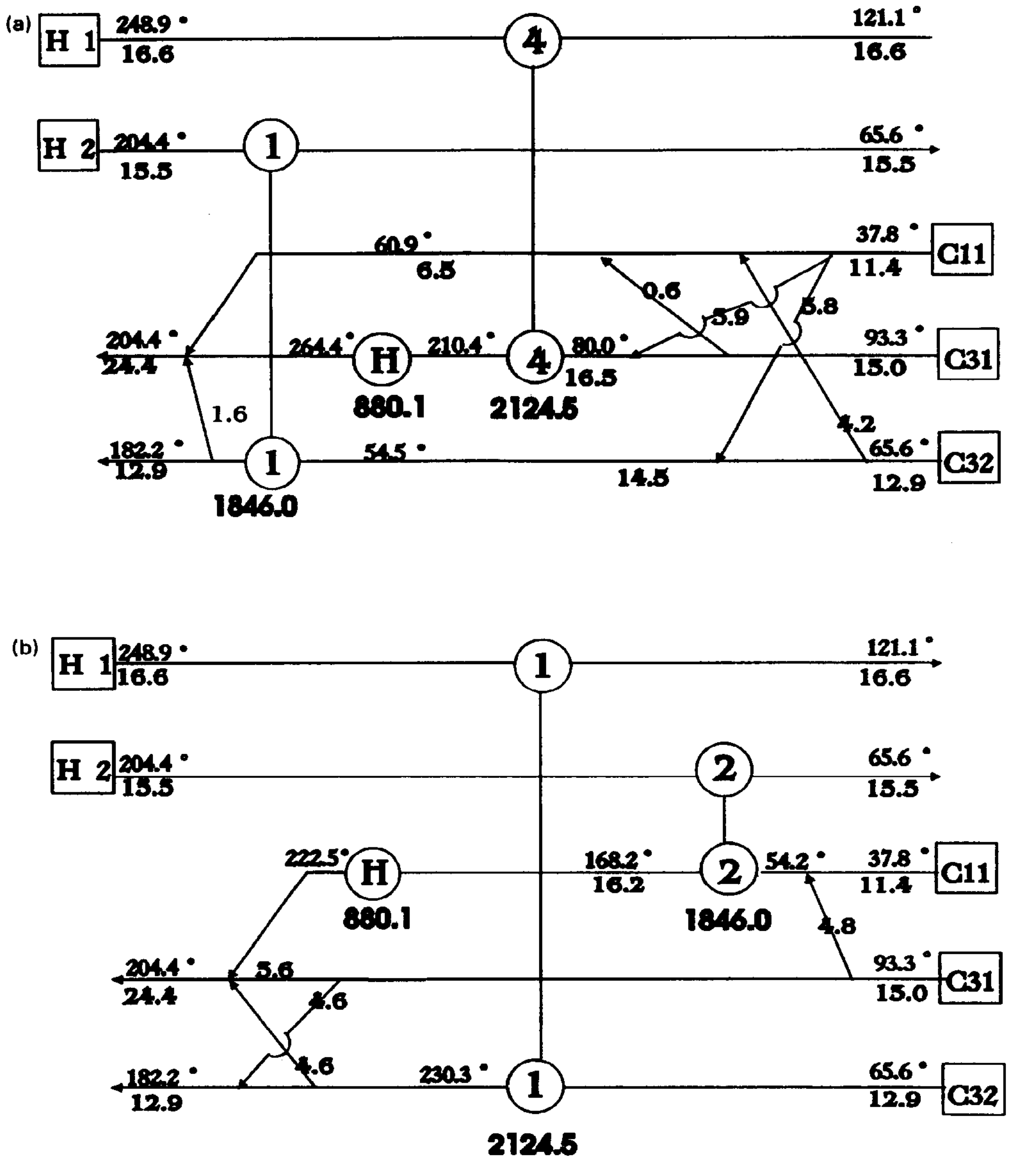

Fig. 9

vapor from Col. 1 to form a new stream H5, which is then cooled from 81.1°C to the target temperature of 32.2°C for storage in the same tank mentioned above. The bottom stream from Col. 2 is sent to a flash drum (V2) operated at a pressure slightly above 1 atm, to which the blowdown water from the reboiler is also delivered for the purpose of recovering some useful steam. The stream from the bottom of V2, i.e. H3, is contaminated by trace IPA and can be considered as a heat source since its temperature is at 108.9°C. The waste heat should be utilized before H3 being cooled to 18.3°C for treatment. Also, two other wastewater streams, H1 and H2, coming from water removal area of the plant are available at 76.7 and 54.4°C, respectively, for use as heat sources in this heat recovery system. Both streams are contaminated by IPA and should be cooled down to 18.3°C before treatment. The detailed stream data of this system are summarized in Table 3.

It should be pointed out that H5 is a vapor stream initially. Under the operating pressure, its bubble-point temperature is about 80.6°C. Also, the heat of vaporization of stream H5 has been estimated to be 9.636×10^5 J/kg and its flowrate is 11,463 kg/h. From these data, the cooling process of H5 can be

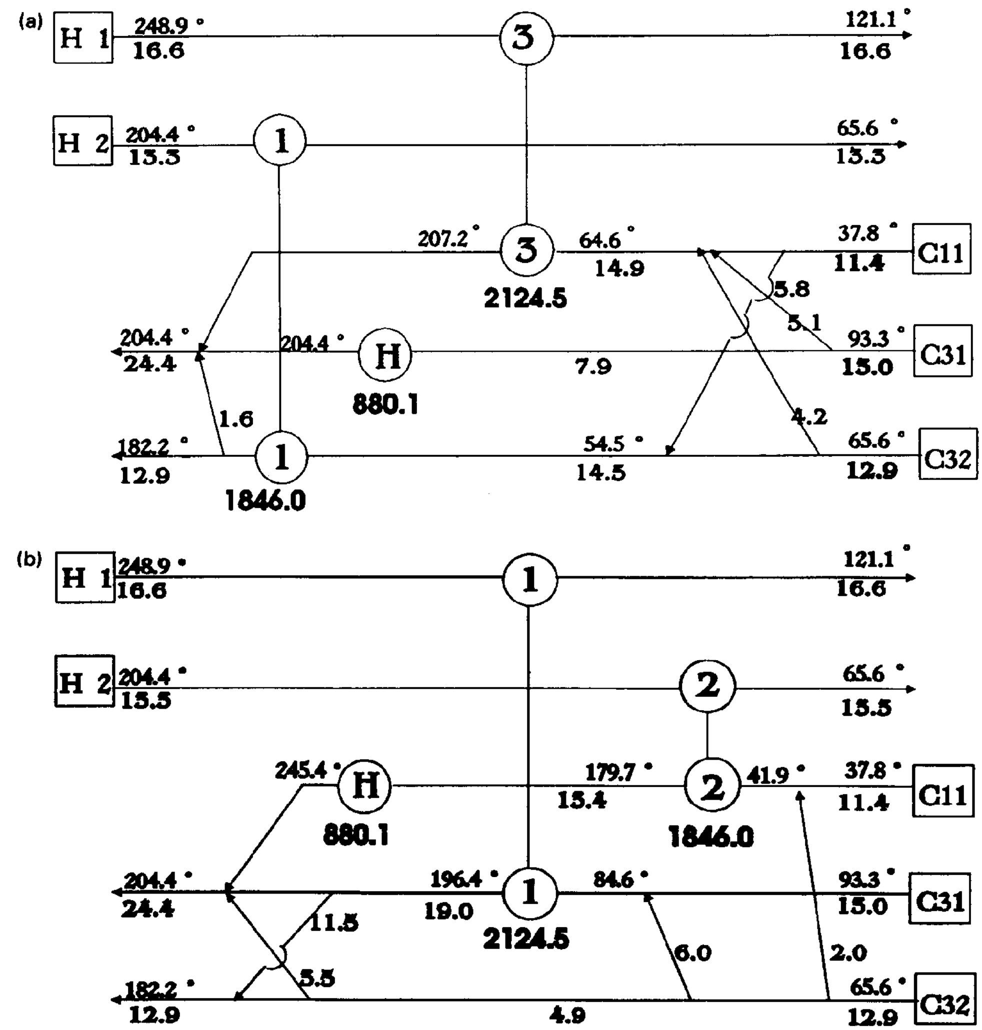

Fig. 9. (a) The initial feasible solution for Example 1—alternative one. (b) The initial feasible solution for example 1—alternative two. (c) The initial feasible solution for Example 1—alternative three. (d) The initial feasible solution for Example 1—alternative four.

divided into two steps. The stream is condensed initially from 81.1 to 80.6°C. In this example, the enthalpy change occurred in this temperature interval is described by the heat released from a "pseudo" liquid stream with a heat-capacity flowrate of 6137.1 kW/°C. The next step is to cool the condensed H5 from 80.6°C to the storage temperature (32.2°C). The corresponding enthalpy change can be calculated with the estimated heat-capacity flowrate of the liquid mixture in H5 (10.4 kW/°C).

Since H1, H2 and H3 are all wastewater streams contaminated by trace of IPA, the designer is allowed to merge the three streams in any possible configuration. Also, H4 and H5 can be mixed in liquid form due to the facts that they are both concentrated IPA/water mixtures and bound to be sent to the same storage tank eventually. Notice that, since the considerations in designing a condenser are different from those for liquid–liquid heat exchangers, it is necessary to treat the "pseudo" liquid stream in H5 as a separate and individual stream in the HEN synthesis. Finally, from the observation that streams C1 and C2 are originated from the same intermediate storage tank, one can conclude that they can be combined and, then, split and heated to their respective target temperatures. On the basis of the above discussions, the stream data presented in Table 3 can be modified for formulating the LP, MILP and NLP models in this example (see Table 4).

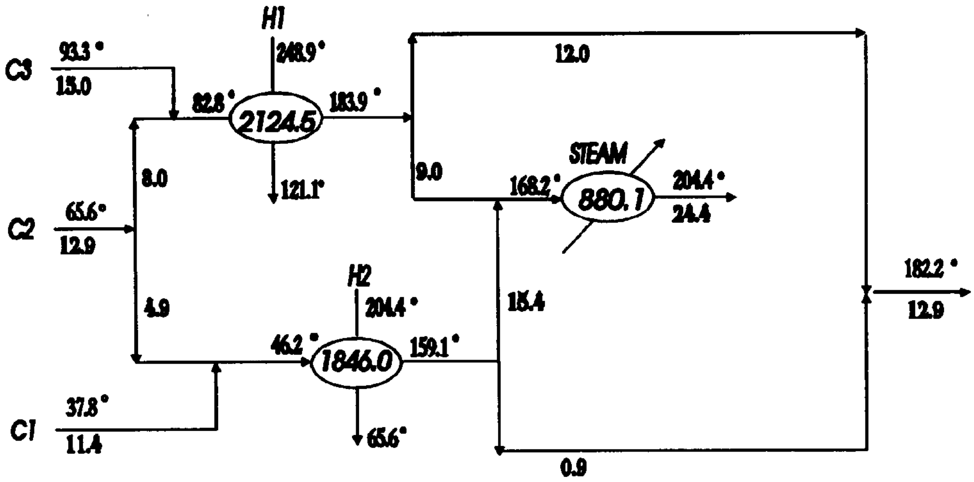

Fig. 10. The optimal heat exchanger network for Example 1.

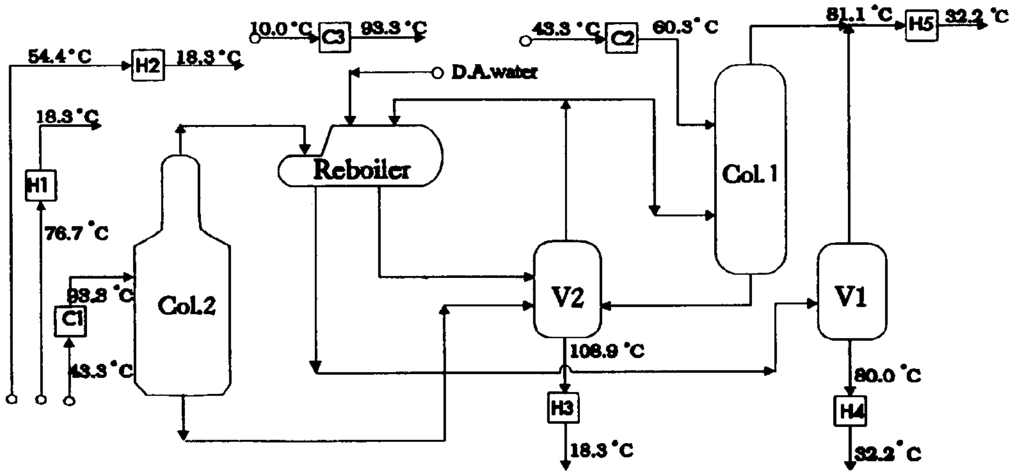

Fig. 11. The simplified flow diagram of IPA/water distillation unit in a food additive processing plant.

Based on the solution procedure proposed in this paper, the problem presented in Table 4 can be solved in three steps, i.e.

- The minimum utility consumption rates:

  The modified LP model for this example can be formulated according to MP1. Its solution can be obtained easily with LINDO. The minimum consumption rates of the hot and cold utilities were determined to be 802.7 and 2157.6 kW, respectively. The hot and cold pinch temperatures are 81.1 and 72.8°C, respectively.

- The design targets for network synthesis:

  The modified MILP model can be formulated according to MP2 and the solution of MP1. LINDO was again used for solving the optimization problem. The results are summarized in Table 5.

Although cost-efficient HENs can be obtained with the design targets in Table 5, a somewhat different approach was taken in this example.

Table 3. Stream data for the heat recovery system in an IPA/water distillation unit

Stream type and No.	Heat-capacity flowrate (kW/°C)	Supply temperature (°C)	Target temperature (°C)
H1	15.8	76.7	18.3
H2	2.6	54.4	18.3
H3	35.1	108.9	18.3
H4	7.7	80.0	32.2
H5	6137.1	81.1	80.6
	10.4	80.6	32.2
C1	22.8	43.3	93.3
C2	18.6	43.3	60.3
C3	64.0	10.0	93.3

$\Delta T_{min} = 8.3°C$.

Table 4. Data needed in formulating the LP, MILP and NLP models for Example 2

Stream type	Group No.	Input stream No.	Heat-capacity flowrate (kW/°C)	Temperature (°C)	Output stream No.	Heat-capacity flowrate (kW/°C)	Temperature (°C)
Hot	1	1	15.8	76.7	1	53.5	18.3
		2	2.6	54.4			
		3	35.1	108.9			
Hot	2	1	7.7	80.0	1	18.1	32.2
		2	10.4	80.6			
Hot	3		6137.1	81.1		6137.1	80.6
Cold	1′	1′	41.4	43.3	1′	22.8	93.3
					2′	18.6	60.3
Cold	2′		64.0	10.0		64.0	93.3

Table 5. The optimal matches for Example 2

Sub-network No.	Match: Hot stream No.	Match: Cold stream No.	Heat duty (kW)
1	utility	1′	467.2
1	utility	2′	335.5
1	1	2′	976.7
2	1	2′	1940.5
2	3	1′	989.0
2	3	2′	2079.5
2	1	utility	1285.4
2	2	utility	872.2

Notice that, due to the extra degree of flexibility introduced by the generalized stream structure, it is often possible to remove some of the exchangers from these "optimal" networks without increasing the utility demands. From Table 5, one can observe that a loop is formed between hot stream 1 and cold stream 2′ if the two subnetworks in this example are considered as one combined system in synthesizing the HEN. If the heat duties of the matches in

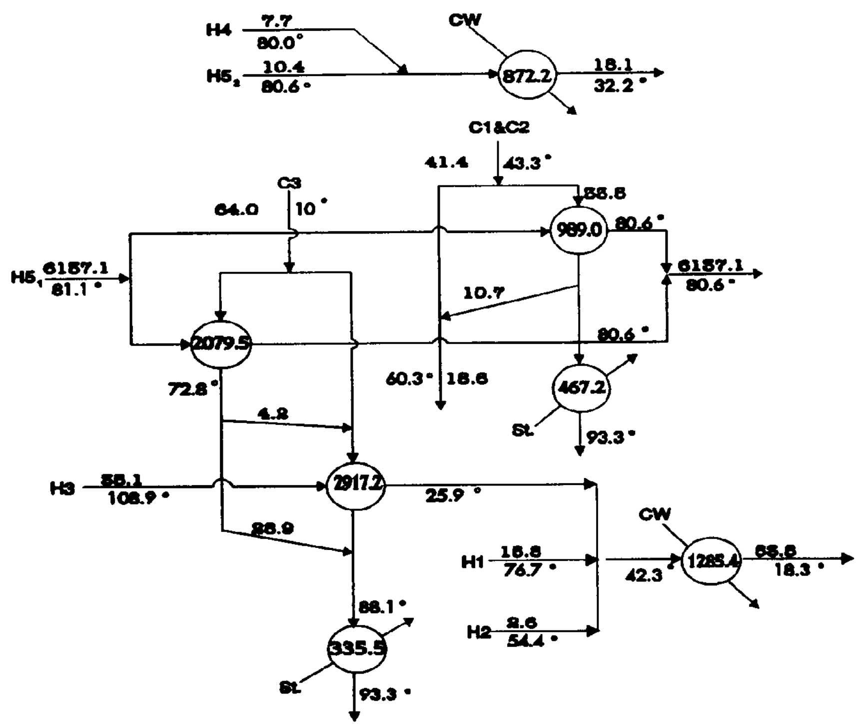

Fig. 12. The optimal heat exchanger network for Example 2.

this loop are shifted, one exchanger can be eliminated. The informations concerning the matches of this modified system, i.e. their heat duties and the streams involved in each match, can be used as the basis for formulating the NLP model of the combined system.

- The optimal network configuration

  Assuming that the temperature of the hot utility is constant at 127°C and the temperature of the cold utility varies between 10 and 20°C in every cooler, this model was again solved by GINO. The resulting network is presented in Fig. 12. Notice that the number of exchanger units in this design is 8, which is one less than that determined with the modified MILP model (see Table 5).

CONCLUSIONS

A systematic procedure has been proposed in this study to incorporate various options of merging/splitting process streams in the design of HENs. In particular, an input–output system structure was introduced to derive the modified LP and MILP models for calculating the design targets of operating and capital costs and, also, a generalized stream structure was developed to formulate the NLP Model for generating the optimal network configurations. Also, the effectiveness of this approach was greatly enhanced by the development of an evolutionary procedure for producing feasible solutions to be used as the initial guesses of the NLP problems.

The practical value of the suggested synthesis techniques has been clearly demonstrated in the application examples presented in this paper. It can be observed that the generalized stream structure is indeed useful in the synthesis of process configurations. Without increasing the operating costs, the capital costs of HENs may be reduced significantly by adopting such options in design. Also, due to the extra degree of flexibility caused by considering the proposed methods in solving the synthesis problem, it is sometimes even possible to obtain networks with fewer exchanger units than those predicted by the modified MILP Model.

Acknowledgement—This work is supported by the national Science Council of the ROC under Grant NSC81-0402-E006-545.

NOMENCLATURE

A_{ij} = Area of the exchanger corresponding to match $(i, j) \in \mathbf{MA}$

C = Union of **C′** and **C″**

C′ = Set of labels for all groups of mixable cold streams

C″ = Set of labels for all regular cold streams

$\hat{\mathbf{C}}$ = Union of **C** and **CU**

$\check{\mathbf{C}}$ = Union of **CF**, **CP** and a combined cold stream formed from the rest of the streams in $\hat{\mathbf{C}}$

$\hat{\mathbf{C}}_s$ = Subset of $\hat{\mathbf{C}}$ in which the cold streams are required to be present in subnetwork $\mathbf{SN}_s$ also

CF = Set of labels for the cold streams in all forbidden matches

CP = Set of labels for the cold streams in all compulsory matches

CU = Set of labels for all cold utility streams

$\mathbf{CI}_{l'}$ = Set of labels for all input streams in the l'th group of mixable cold streams

$\mathbf{CO}_{l'}$ = Set of labels for all output streams in the l'-th group of mixable cold streams

$\mathbf{CIO}_{l'}$ = Set of double indices representing all possible fictitious streams in the l'th group of mixable cold streams

Ct_{ij}, Cb_{ij} = Cost coefficients associated with exchanger (i, j)

$\mathbf{E}_{ij}^{\mathrm{HI}}$ = Set of labels for the branches of a hot stream entering the exchanger $(i, j) \in \mathbf{MA}$

$\mathbf{E}_{ij}^{\mathrm{HO}}$ = Set of labels for the branches of a hot stream leaving the exchanger $(i, j) \in \mathbf{MA}$

$\mathbf{E}_{ij}^{\mathrm{CI}}$ = Set of labels for the branches of a cold stream entering the exchanger $(i, j) \in \mathbf{MA}$

$\mathbf{E}_{ij}^{\mathrm{CO}}$ = Set of labels for the branches of a cold stream leaving the exchanger $(i, j) \in \mathbf{MA}$

F = Set of double indices representing all forbidden matches

$F_{Cp_\iota}^{\mathrm{HI}_l}$ = Heat-capacity flowrate of hot input stream ι in set $\mathbf{HI}_l$

$F_{Cp_o}^{\mathrm{HO}_l}$ = Heat-capacity flowrate of hot output stream o in set $\mathbf{HO}_l$

$F_{Cp_{\iota'}}^{\mathrm{CI}_{l'}}$ = Heat-capacity flowrate of cold input stream ι' in set $\mathbf{CI}_{l'}$

$F_{Cp_{o'}}^{\mathrm{CO}_{l'}}$ = Heat-capacity flowrate of cold output stream o' in set $\mathbf{CO}_{l'}$

$F_{Cp_{\iota o}}^{\mathrm{HIO}_l}$ = Heat-capacity flowrate of hot fictitious stream (ι, o) in set $\mathbf{HIO}_l$

$F_{Cp_{\iota' o'}}^{\mathrm{CIO}_{l'}}$ = Heat-capacity flowrate of cold fictitious stream (ι', o') in set $\mathbf{CIO}_{l'}$

F_m^{HU}, F_n^{CU} = Mass flowrates of the mth hot and the nth cold utility, respectively

f_j^κ = Heat-capacity flowrate of branch j on stream $\kappa \in \mathbf{HCT}$

H = Union of **H′** and **H″**

H′ = Set of labels for all groups of mixable hot streams

H″ = Set of labels for all regular hot streams

$\hat{\mathbf{H}}$ = Union of **H** and **HU**

$\check{\mathbf{H}}$ = Union of **HF**, **HP** and a combined hot stream formed from the rest of the streams in $\hat{\mathbf{H}}$

$\hat{\mathbf{H}}_s$ = Subset of $\hat{\mathbf{H}}$ in which the hot streams are required to be present in subnetwork $\mathbf{SN}_s$ also

HCT = Union of $\hat{\mathbf{H}}$ and $\hat{\mathbf{C}}$

HF = Set of labels for the hot streams in all forbidden matches

HP = Set of labels for the hot streams in all compulsory matches

HU = Set of labels for all hot utility streams

$\mathbf{HI}_l$ = Set of labels for all input streams in the lth group of mixable hot streams

$\mathbf{HO}_l$ = Set of labels for all output streams in the lth group of mixable hot streams

$\mathbf{HIO}_l$ = set of double indices representing all possible fictitious streams in the lth group of mixable hot streams

K = Total number of temperature intervals

M = Number of groups of mixable hot streams

M' = Number of groups of mixable cold streams

M_{E}^{l} = Number of exchangers on the lth group of mixable hot streams

M_{I}^{l} = Number of input streams in the lth group of mixable hot streams

M_{O}^{l} = Number of output streams in the lth group of mixable hot streams

$M_{\mathrm{E}}^{\prime l'}$ = Number of exchangers on the lth group of mixable cold streams

$M_{\mathrm{I}}^{\prime l'}$ = Number of input streams in the l'th group of mixable cold streams

$M_{\mathrm{O}}^{\prime l'}$ = Number of output streams in the l'th group of mixable cold streams

MA = Set of double indices representing the matches predicted by model MP2

$\mathcal{M}_{\kappa}$ = Set of labels for all mixers on stream $\kappa \in \mathbf{HCT}$

$\mathcal{M}_{\kappa}^{\mathrm{I}}(\rho)$ = Set of labels for all branches entering the mixer $\rho \in \mathcal{M}_{\kappa}$

$\mathcal{M}_{\kappa}^{\mathrm{O}}(\rho)$ = Set of labels for all branches leaving the mixer $\rho \in \mathcal{M}_{\kappa}$

N = Sum of the number of groups of mixable hot streams and the number of regular hot streams

N' = Sum of the number of groups of mixable cold streams and the number of regular cold streams

$\mathcal{N}_{\kappa}$ = Set of labels for all the branches on stream $\kappa \in \mathbf{HCT}$

P = Set of double indices representing all compulsory matches

PH_m, PC_n = Unit costs of the mth hot and the nth cold utility, respectively

Q_{ij} = Heat exchanged between hot stream i and cold stream j

$Q_{\xi\eta k}$ = Heat exchanged between hot stream ξ and cold stream η in interval k

$Q_{l'k}^{\mathrm{C}''}$ = Heat output from interval k to cold stream $l' \in \mathbf{C}''$

$Q_{\eta k}^{\tilde{\mathrm{C}}}$ = heat output from interval k to cold stream $\eta \in \tilde{\mathbf{C}}$

$Q_{\nu k}^{\hat{\mathrm{C}}_s}$ = heat output from interval k to cold stream $\nu \in \hat{\mathbf{C}}_s$

$Q_{\iota' o' k}^{\mathrm{CIO}_{l'}}$ = Heat output from interval k to fictitious cold stream $(\iota', o') \in \mathbf{CIO}_{l'}$

$Q_{lk}^{\mathrm{H}''}$ = Heat input to interval k from hot stream $l \in \mathbf{H}''$

$Q_{\xi k}^{\tilde{\mathrm{H}}}$ = Heat input to interval k from hot stream $\xi \in \tilde{\mathbf{H}}$

$Q_{\mu k}^{\hat{\mathrm{H}}_s}$ = Heat input to interval k from hot stream $\mu \in \hat{\mathbf{H}}_s$

$Q_{\iota o k}^{\mathrm{HIO}_l}$ = Heat input to interval k from fictitious hot stream $(\iota, o) \in \mathbf{HIO}_l$

R_k = Residual heat flow of interval k at temperature T_k

$R_{\xi k}$ = Residual heat flow of hot stream ξ at temperature T_k

S = Total number of pinch points

$\mathbf{SN}_s$ = Set of labels for all intervals in the subnetwork s

$\mathcal{S}_{\kappa}$ = Set of labels for all splitters on stream $\kappa \in \mathbf{HCT}$

$\mathcal{S}_{\kappa}^{\mathrm{I}}(\sigma)$ = Set of labels for all branches entering the splitter $\sigma \in \mathcal{S}_{\kappa}$

$\mathcal{S}_{\kappa}^{\mathrm{O}}(\sigma)$ = Set of labels for all branches leaving the splitter $\sigma \in \mathcal{S}_{\kappa}$

T_l^{I}, T_l^{O} = Input and output temperatures of stream l and $l \in \mathbf{H}''$

$T_{l'}^{\mathrm{I}}$, $T_{l'}^{\mathrm{O}}$ = Input and output temperatures of stream l' and $l' \in \mathbf{C}''$

$T_{\iota}^{\mathrm{HI}_l}$ = Temperature of hot input stream ι in set $\mathbf{HI}_l$

$T_o^{\mathrm{HO}_l}$ = Temperature of hot output stream o in set $\mathbf{HO}_l$

$T_{\iota'}^{\mathrm{CI}_{l'}}$ = Temperature of cold input stream ι' in set $\mathbf{CI}_{l'}$

$T_{o'}^{\mathrm{CO}_{l'}}$ = Temperature of cold output stream o' in set $\mathbf{CO}_{l'}$

T_j^{κ} = Temperature of branch j on stream $\kappa \in \mathbf{HCT}$

$Y_{\mu\nu s}$ = Binary variable representing the status of heat exchange between hot stream μ and cold stream ν in subnetwork s

Δh_{mk}, Δh_{nk} = Enthalpy changes of hot utility m and cold utility n, respectively, in interval k

ΔT_k = Temperature change in interval k

$\Delta T_{\min}$ = Minimum temperature approach in a heat exchanger

REFERENCES

Cerda J. and A. W. Westerberg, Synthesizing heat exchanger networks having restricted stream/stream match using transportation problem formulations. *Chem. Engng Sci.* **38,** 1723 (1983).

Cerda J., A. W. Westerberg, D. Mason and B. Linnhoff, Minimum utility usage in heat exchanger network synthesis—a transportation problem. *Chem. Engng Sci.* **38,** 373 (1983).

Ciric A. R. and C. A. Floudas, A retrofit approach for heat exchanger networks, *Computers chem. Engng* **13,** 703 (1989).

Ciric A. R. and C. A. Floudas, Heat exchanger network synthesis without decomposition. *Computers chem. Engng* **15,** 385 (1991).

Chang C. T. and T. P. Yu, Development of an evolutionary stream merging method in heat exchanger network design. *IChemE Symp. Ser.* **109,** 159 (1988).

Duran M. A. and I. E. Grossmann, Simultaneous optimization and heat integration of chemical processes. *AIChE Jl* **32,** 123 (1986).

El-Halwagi M. M. and V. Manousiouthakis, Synthesis of mass exchange networks. *AIChE Jl* **35,** 1233 (1989).

El-Halwagi M. M. and V. Manousiouthakis, Automatic synthesis of mass-exchange networks with single-component targets. *Chem. Engng Sci.* **45,** 2813 (1990).

Floudas C. A. and I. E. Grossmann, Automatic generation of multiperiod heat exchanger network configurations. *Computers chem. Engng* **11,** 123 (1987).

Floudas C. A., A. R. Ciric and I. E. Grossmann, Automatic synthesis of heat exchanger network configurations. *AIChE Jl* **32,** 276 (1986).

Grossmann I. E., Mixed-integer programming approach for the synthesis of integrated process flowsheets. *Computers chem. Engng* **9,** 463 (1985).

Gundersen T. and L. Naess, The synthesis of cost optimal heat exchanger networks—an industrial review of the state of the art. *Computers chem. Engng* **12,** 503 (1988).

Liebman J., L. Lasdon, L. Schrage and A. Waren, *Modeling and Optimization with GINO.* The Scientific Press, South San Francisco (1986).

Linnhoff B. and E. Hindmarsh, The pinch design for heat exchanger networks. *Chem. Engng Sci.* **38,** 745 (1983).

Murtagh B. A. and M. A. Saunders, *MINOS 5.1 User's Guide.* Tech. Rpt SOL 83-20R, System Optimization Laboratory, Stanford University (1983).

Papoulias S. A. and I. E. Grossmann, A structural optimization approach in process synthesis—II. Heat recovery networks. *Computers chem. Engng* **7,** 707 (1983).

Paterson W. R., A replacement for the logarithmic mean. *Chem. Engng Sci.* **39,** 1635 (1984).

Ponton J. W. and R. A. Donalson, A fast method for the synthesis of heat exchanger networks. *Chem. Engng Sci.* **29,** 2375 (1974).

Schrage L., *User's Manual for Linear, Integer and Quadratic Programming with LINDO,* Release 5. The Scientific Press, South San Francisco (1991).

Yee T. F., I. E. Grossmann and Z. Kravanja, Simultaneous optimization models for heat integration—I. Area and energy targeting and modeling of multi-stream exchangers. *Computers chem. Engng* **14**, 1151 (1990a).

Yee T. F. and I. E. Grossmann, Simultaneous optimization models for heat integration—II. Heat exchanger network synthesis. *Computers chem. Engng* **14**, 1165 (1990b).

Yee T. F., I. E. Grossmann and Z. Kravanja, Simultaneous optimization models for heat integration—III. Process and heat exchanger network synthesis. *Computers chem. Engng* **14**, 1185 (1990c).

Yee T. F. and I. E. Grossmann, A screening and optimization approach for the retrofit of heat-exchanger networks, *Ind. Engng Chem. Res.* **30**, 146 (1991).