A MULTIOBJECTIVE PROGRAMMING APPROACH TO
WASTE MINIMIZATION IN THE UTILITY SYSTEMS OF
CHEMICAL PROCESSES

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Abstract—In the field of process synthesis, heat integration methodologies have been matured considerably
during the past two decades. Today, these techniques are widely accepted as effective tools for improving
chemical processes in terms of capital investment and energy consumption. However, as the problems of
environmental pollution have become more and more serious in recent years, the development of process
integration methods for waste reduction is now recognized as an area of urgent research. In this paper,
mathematical programming models, which take into account both economical incentives and environ-
mental penalties, are formulated for the design of “best” utility systems. The pollution problem associated
with a utility system can be mainly attributed to gas emissions (e.g. COx, NOx and SOx) caused by burning
fuels for generating power and/or heating utilities. In some cases, in order to satisfy the additional demand
for power, electricity is imported from a central power plant which may also consume fuels. This demand
for external electricity should therefore be considered as a hidden source of emission indirectly caused by
running the utility plant. To address these environmental concerns, an improved version of the traditional
MILP model for utility network design is proposed in this work. Not only the problem of cost minimization
can be handled efficiently with an elaborate heat recovery scheme embedded in the modified superstructure,
but also the concept of global emission can be incorporated in the model formulation. By making use of the
goal programming techniques, appropriate designs of the utility networks can be obtained according to the
decision maker’s priority. From the experiences we have gathered so far in solving the improved MILP
model, it can be concluded that the proposed techniques are applicable for a wide variety of processes
having extremely different utility demands and, also, it is a sensible design approach for establishing
a compromise among conflicting evaluation criteria. Copyright © 1996 Elsevier Science Ltd

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INTRODUCTION
During the past several decades, the use of integration
techniques as a design tool to minimize the operating
and capital costs of a chemical plant has matured
considerably and evolved into a common practice in
the process industries, e.g. see Gunderson (1991). Also,
as a result of serious concerns about environmental
problems in recent years, development of process syn-
thesis methods for waste reduction purpose has be-
come a research issue of growing importance. The
“wastes” resulted from chemical-producing activities
can generally be classified into two types, i.e. the
process wastes discharged from the chemical process
itself and the utility wastes, which are mainly the gases
(COx, SOx and NOx) and ashes created in the process
of generating utilities. The direct consequences of us-
ing utilities are essentially air pollution problems
caused by burning fuels (e.g. natural gas, fuel oil and
coal, etc.) in equipments such as boilers and furnaces.
However, the demand for imported electricity should
also be viewed as an indirect source of utility wastes
resulting from operating a chemical plant. Thus, the
proper design criteria for a modern utility plant
should include both economical and environmental
requirements. In other words, not only the capital
investment of a utility plant but also the correspond-
ing direct and indirect utility wastes must be mini-
mized.

 Apparently, the direct gas emissions can be lowered
by minimizing the utility consumption. This task can
be accomplished with heat integration techniques.
There are in general two approaches reported in the
literature. The first is to synthesize the process man-
ually with design heuristics developed from thermo-
dynamic principles and engineering insights, i.e. the
“pinch” analysis (Linnhoff, 1994). The second ap-
proach is to generate the system structure automati-
cally with the aid of mathematical programming soft-
ware packages, e.g. Cerda and Westerburg (1983),
Papoulias and Grossmann (1983b), Floudas et al.
(1986), Ciric and Floudas (1989), Yee et al. (1990a, b),
Yee and Grossmann (1990) and Chang et al. (1994).

The most obvious method to reduce the indirect
utility wastes is to cut down the need for imported
electricity with local cogeneration systems. There have
already been a number of publications concerning
heat and power integration in the past. On the basis of the supply-and-demand relation implied in a grand composite curve, principles for the appropriate placement of heat engines and heat pumps were developed by Townsend and Linnhoff (1983a, b) and Linnhoff (1989). Studies on the optimal design of power generation system can also be found in Nishio et al. (1980), Petroulas and Reklaitis (1984), Doldan et al. (1985) and Chou and Shih (1987). The mathematical programming approach to utility system synthesis was adopted in several other recent works. Papoulias and Grossmann (1983a) developed a superstructure which includes the options for incorporating steam and gas turbines in the utility plants. The best system configuration was obtained by solving the corresponding MILP model. Also, Colmenares and Seider (1987, 1989) combined models of heat engines, heat pumps and utility systems into a NLP formulation and generated minimum-cost designs with standard optimization methods.

It should be noted that all studies mentioned above addressed essentially only economical issues and none of them adopted waste reduction as one of their design criteria. Research in the latter area has not received much attention until recently. Smith et al. (1990) proposed an approach to the minimization of environmental emissions through improved process integration, i.e. the Pinch technology. Smith and Petela (1991a, b; 1992a–c) then presented the specific design techniques for waste reduction in a series of papers. In addition, Smith and Delaby (1991) tried to establish the minimum targets for flue gas emissions in the utility system. More importantly, they introduced the concept of global emissions which allows us to address the design problem from a broader perspective. Finally, Douglas (1992) also developed a hierarchical decision procedure for waste reduction in process design.

From a review of the current literature, one can find that the results obtained with the programming approach should in general be more comprehensive and less error-prone as long as all the essential engineering insights are formulated in the mathematical models (Linnhoff, 1994). However, this type of numerical optimization method has never been adopted to generate utility system designs which satisfy both economical and environmental constraints. Thus, the main objective of this study is to assess the feasibility and practical value of incorporating gas emission models into existing mathematical formulations and solving the resulting problem with multiobjective optimization techniques. It should also be noted that these mathematical programs are basically nonlinear in nature. Although effective algorithms have been developed for NLPs, e.g. Floudas and Visweswaran (1990); Visweswaran and Floudas (1990), their application in solving the multiobjective problems still requires an extremely large number of iterations. One of the proven remedial measures is to approximate the original model with a mixed-integer linear program (Papoulias and Grossmann, 1983a). Since equally acceptable designs can be obtained with much less computational effort, this alternative approach was thus adopted in the present work.

THE SUPERSTRUCTURE OF UTILITY SYSTEMS

To incorporate design options for waste minimization and for further reduction in capital costs, the superstructure developed by Papoulias and Grossmann (1983a) has been modified in this study. This modified version is presented in Fig. 1.

Notice first that there are several steam headers at different pressure levels in this generalized utility system. Although three are shown in Fig. 1, the actual number of headers in the superstructure is determined on a case-by-case basis. Specifically, their pressure levels are selected at the saturated steam pressures corresponding to the hot stream supply temperatures of the above-pinch temperature intervals in which heat-deficit values are positive. Since superheated steam is needed to drive the steam turbines, the degree of superheating associated with each chosen pressure is treated as a decision variable in optimization. In particular, several discrete temperature levels in each header are allowed, but only one operating state can be chosen in the final design.

Steam can be generated with either fired or waste heat boilers operating at conditions consistent with those of the steam headers. The available steam in each header can be used to provide a required demand of hot utility, drive steam turbines or be transferred to the steam header at the next lower level with a pressure reducer where water is added to match the steam quality. After heating a process stream with high-pressure steam, the resulting high-pressure saturated water can be utilized in several downstream equipments. Firstly, it can be introduced directly into the deaerator for heating the fresh make-up water. Secondly, it can also be flashed at a lower pressure. The overhead steam from the flash drum then can be used to heat cold process streams at lower temperatures. The bottom stream is again sent to the deaerator or lower-pressure flash drum. Finally, in order to provide hot utility at a steady temperature, this water can be recycled and mixed with the superheated steam in a saturator to produce saturated steam at the pressure of the header.

There are basically three power-generating devices in the superstructure, i.e. steam turbines, gas turbines and electric motors. Both the condensing and back-pressure steam turbines are considered. Due to operational constraints, they are not connected to the low-pressure headers. A complete gas turbine system includes a turbine, a compressor, a combustor and a regenerator. The one adopted here is of the simple open-cycle type, with air as the working medium. The hot air exiting the turbine section can be used in a regenerator to preheat the compressed air before it enters the combustor, or it can be used as preheated air for further combustion in fired boilers or as heating medium in process heaters and waste heat boilers.
Fig. 1. The superstructure of a utility network.
In this study, all power demands are assumed to be satisfied with electric motors. Since the electric motors are operated independently in the utility system, the corresponding symbols are not presented in Fig. 1 for the sake of overall clarity. Electricity can be either imported or produced by a combination of steam and/or gas turbines connected with a common shaft on an electric generator. In other words, the power produced by steam and gas turbines in the proposed utility system is used solely for generating electricity. Finally, the options for selling surplus electricity to the neighboring plants or community are also included in the superstructure.

For high-temperature processes, furnaces may be needed to produce flue gas as a hot utility. It is assumed in this study that the heat content between the theoretical flame temperature (1800°C) and the stack temperature (160°C) is utilized exclusively for heating process streams (Linnhoff and de Luer, 1988; Smith and Delaby, 1992). Since the operation of furnaces can also be decoupled from other equipments in the utility system, they are thus not shown in Fig. 1.

Several essential auxiliary equipments are included in the superstructure. If a condensing turbine is chosen in the utility system design, then a vacuum condenser is necessary for its operation. Also, there should be a water treatrer for the make-up water and a deaerator to provide feedwater to the boilers and to meet the process water demand. Before entering the boiler, the water is raised to the operating pressure with a feedwater pump and preheated with steam. Notice that the water leaving the deaerator is assumed to be maintained at 100°C and 1 atm by appropriately flashing the high-pressure water before entering the deaerator. The flashed stream can be used to heat process streams above the pinch. If this is not possible, i.e. the pinch temperature is higher than 100°C, then it is purged to the atmosphere.

Finally, several alternative fuels should be allowed to be burned in the fired boilers, gas turbines and furnaces. The links between the energy consumption and gas emission can then be established with the corresponding combustion models.

THE CONSTRAINTS OF MILP MODEL

Having developed the superstructure for the utility system, one can then formulate a mathematical program accordingly for the synthesis of a utility system. Let us first consider the constraints. Naturally, the material and energy balance equations associated with every unit in the system should all be included as the equality constraints of the optimization problem. Since 0–1 binary variables are needed to signify the non-existence or existence of units in the superstructure and many of the balance equations are nonlinear, the resulting optimization problem is inevitably a mixed integer nonlinear program (MINLP). To reduce the effort in solving the corresponding process synthesis problem, Papoulias and Grossmann (1983) developed a general procedure to transform this MINLP model into a MILP problem. This study follows essentially the same approach. Since the discretized balance equations are not novel, their descriptions are thus omitted in this paper for the sake of brevity. Specific formulations of these equations can be found in Hwang (1994).

Other than the balance equations associated with all units, models of gas emission, electricity utilization and process heat exchange have also been integrated into the mixed integer linear program in this study. For convenience, let us define the set of all units as N = {n|n is the label of a unit in the superstructure}.

These units are fired and waste heat boilers, steam headers at different pressure levels, steam and gas turbines, furnaces, flash drums, pressure reducers and all other auxiliary equipments. In order to describe the consumption of different fuels in various units, a subset of N is defined:

\[ N_F = \{ n|n \text{ is the label of a fuel-burning unit}\} \]

(2)

Specifically, the fired boilers, the gas turbines and the furnaces are included in the set. The set of all alternative fuels that can be used in a unit n is defined as

\[ Z_n = \{ z|\text{fuel } z \text{ can be used to meet the heat demand of unit } n, n \in N_F \} \]

(3)

Thus, the demands for fuels can be represented with the following equation:

\[ Q^F_n = \eta_n \sum_{z \in Z_n} \Omega_n \bar{F}_{mn}, \quad n \in N_F \]

(4)

where \( Q^F_n \) is the heat absorbed by the utility stream, i.e. water or air, in a fuel-burning unit n, \( \Omega_n \) denotes the heat released by burning a unit mass fuel z, \( \bar{F}_{mn} \) is the mass flow rate of fuel z in unit n and \( \eta_n \) represents the overall efficiency (including combustion efficiency and heat transfer efficiency) of unit n. If only one fuel is allowed to be used in a fuel-burning unit, then binary variables \( \bar{y}_{zn} \) should be introduced to represent this restriction, i.e.

\[ \sum_{z \in Z_n} \bar{y}_{zn} = 1, \quad n \in N_F \]

(5)

\[ \bar{F}_{mn} - U^F_n \bar{y}_{zn} \leq 0, \quad z \in Z_n, \quad n \in N_F \]

(6)

where

\[ \bar{y}_{zn} = \begin{cases} 1 & \text{fuel } z \text{ is selected in unit } n \\ 0 & \text{otherwise} \end{cases} \]

and \( U^F_n \) is an arbitrarily chosen large number, usually the maximum supply rate of fuel z. The emission rates of various air pollutants, can be computed according to the mass flow rates of fuels, i.e.

\[ E_k = \sum_{n \in N_F} \sum_{z \in Z_n} \gamma_{kz} \bar{F}_{mn}, \quad k \in \mathcal{G} \]

(7)

where \( \mathcal{G} = \{ k \} \) is the set of air pollutants considered in design, \( E_k \) is the emission rate of pollutant \( k \), and \( \gamma_{kz} \) represents the mass of pollutant \( k \) produced after
burning a unit mass of fuel \( z \). The values of \( y_{\text{ke}} \) are assumed to be constants and determined according to Smith and Delaby (1991).

In this study, the electricity produced by the stream and/or gas turbines can be used to meet the needs of the utility plant itself, the chemical process and the neighboring community. These demands can also be satisfied with electricity imported from the central power station. Thus, an overall balance equation can be written accordingly:

\[
\sum_{m \in M} \dot{W}_m + \sum_{n \in N^P} \dot{W}_n + \dot{W}_{0}^P = \sum_{n \in N^S} \dot{W}_n^S + \dot{W}_0^S
\]

where

\[ M = \{ m | m \text{ is the label of an electricity-consuming unit in the chemical process} \} \]

\[ N^P = \{ n | n \text{ is the label of an electricity-consuming unit in the utility system} \} \]

\[ N^S = \{ n | n \text{ is the label of an electricity-producing unit in the utility system} \} \]

Notice that \( \dot{W}_0^P \) denotes the electricity demand of the neighboring community and \( \dot{W}_0^S \) represents the electricity imported from the central power station. The \( \dot{W}_m \) and \( \dot{W}_n \) in the first and second terms on the left-hand side of eq. (8) are the demands of electricity-consuming equipments in the chemical process and in the utility plant, respectively. On the other hand, \( \dot{W}_n^S \) represents electricity produced with a gas or steam turbine.

In the case of electric motors, a constant conversion efficiency, \( \eta_{\text{mc}} \), is introduced to account for the energy loss in the process of transforming electricity into work, i.e.

\[
\dot{W}_m = \eta_{\text{mc}} \dot{W}_m, \quad m \in M
\]

and

\[
\dot{W}_n = \eta_{\text{mc}} \dot{W}_n, \quad n \in N^P
\]

where \( \dot{W}_m \) and \( \dot{W}_n \) denote the power produced by unit \( m \) in the chemical process and unit \( n \) in the utility system, respectively. Notice that all other power-producing units in the utility system, i.e. the steam turbines and gas turbines, are used to generate electricity in our design. Thus, another conversion efficiency, \( \eta_{\text{gen}} \), must also be introduced to describe the energy balance in the process of transforming work \( \dot{W}_n \) into electricity \( \dot{W}_n^S \), i.e.

\[
\dot{W}_n^S = \eta_{\text{gen}} \dot{W}_n, \quad n \in N^S
\]

It is further assumed in this study that the electricity produced by the utility system cannot be exported unless the demands in itself and also the chemical process are both satisfied. Thus, the following constraints must be imposed in the MILP model:

\[
y_{\text{wt}} + y_{\text{wo}} \leq 1
\]

\[
\dot{W}_0^P - U_{\text{wo}} y_{\text{wo}} \leq 0
\]

\[
\dot{W}_0^S - U_{\text{wt}} y_{\text{wt}} \leq 0
\]

where, \( y_{\text{wo}} \) and \( y_{\text{wt}} \) are 0–1 binary variables and \( U_{\text{wo}} \) and \( U_{\text{wt}} \) are arbitrarily chosen large numbers. In other words, only one of the two quantities \( \dot{W}_0^P \) and \( \dot{W}_0^S \) can be larger than zero. If there is surplus electricity, i.e. \( \sum_{m \in M} \dot{W}_m + \sum_{n \in N^P} \dot{W}_n < \sum_{n \in N^S} \dot{W}_n^S \), then there is no need for importing electricity. Otherwise, exporting electricity is not reasonable practice. In addition, depending upon the capacity of the market demands, there should be an upper limit \( \dot{W}_0^U \) for exporting electricity, i.e.

\[
\dot{W}_0^P \leq \dot{W}_0^U
\]

In order to set the pressure levels of the steam headers appropriately, the MILP model of the utility system should be integrated with the transshipment model of HEN in the chemical process. Usually, the first step to construct such a model is to partition the entire temperature range of all process and utility streams into \( K \) temperature intervals for which any suitable partition method can be adopted, e.g. Cerda et al. (1983). These intervals are labeled from the highest level (\( k = 1 \)) down to the lowest level (\( k = K \)) of temperature. In this study, a modified version of the PI model (Papoulias and Grossmann, 1983b) is used, i.e.

\[
R_k - R_{k-1} - \sum_{i \in H_k} Q_{ki}^H = \sum_{j \in C_k} Q_{kj}^H - \sum_{j \in C_k} Q_{kj}^P,
\]

\[
k = 1, 2, \ldots, p
\]

\[
R_0 = R_p = 0
\]

\[
k = 1, 2, \ldots, p - 1
\]

Here, eq. (19) represents the energy balance around the temperature interval \( k \). Since the facilities for producing cold utilities are not included in the superstructure, \( p \) in this formulation is chosen to be the number of temperature intervals above the highest pinch temperature and \( p < K \). Notice that the pinch temperatures can be determined by solving the conventional PI model with fictitious hot utilities available at an arbitrarily high temperature.

From eq. (19), one can see that the heat inputs to interval \( k \) are coming from several sources, i.e. the residual heat from interval \( k - 1 \) (\( R_{k-1} \)) and the cold streams and heating utilities whose temperature range includes interval \( k \). Notice that \( H_k \) is the set of all hot utilities in interval \( k \). If the utility is steam, then its temperature equals \( T_k \) and

\[
Q_{ki}^H = F_{ki}^H \Delta H_{ki}
\]

where \( \Delta H_{ki} \) is the latent heat of steam and \( F_{ki}^H \) denotes the corresponding flow rate. On the other hand, if the hot utility is the flue gas coming from a furnace, then

\[
Q_{ki}^P = F_{ki}^P C_{ki}^P (T^U - T^L)
\]

and

\[
T^U = \min (T_{\text{flue}}, T_{k-1})
\]

\[
T^L = \max (T_{\text{stack}}, T_k)
\]
where \( C_f^k \) is the heat capacity of flue gas in temperature interval \( k \), \( T_{yf} \) is the theoretical flame temperature and \( T_{stack} \) is the stack temperature. Notice also that the flow rate \( F_f^k \) should remain unchanged in all temperature intervals between \( T_{yf} \) and \( T_{stack} \). Also, if the flue gas is coming from a gas turbine, then \( T_{yf} \) in eq. (24) should be replaced by the temperature of exhaust gas.

**THE DESIGN OBJECTIVES**

If only economical criteria are considered in designing the utility system, the objective function of the MILP model should be the sum of annualized capital and operating costs. The former includes the fixed and variable costs of all plant units. The latter consists of the costs of fuels, fresh water and purchased electricity. The capital cost of a unit \( n \in N \) is in general a nonlinear function of the equipment capacity \( G_n \), where \( G_n \) is usually represented by the total output flow rate of the equipment or the total work load if \( n \) denotes a turbine or motor. To establish the MILP model, the objective function must be approximated by a piecewise linear function. Specifically, \( G_n \) is first divided into \( L_n \) intervals and then, in each interval, linear approximation is introduced (see Fig. 2). Let us adopt a set of \( L_n \) variables \( G_{nm} \) to express \( G_n \) in an alternative formulation, i.e.

\[
G_n = \sum_{m \in L_n} G_{nm}. \tag{26}
\]

The set \( L_n \) is defined as follows:

\[
L_n = \{ m | m \text{ is the label of an interval of } G_n \text{ in which } G_{nm} \leq G_n \leq G_{nm+1} \}. \tag{27}
\]

It should also be noted that only one of the variables \( G_{nm} \) in eq. (26) is not zero. The variable adopted to replace \( G_n \) in the objective function should be the one corresponding to the interval containing the actual value of \( G_n \). Thus, additional constraints must be imposed, i.e.

\[
\sum_{m \in L_n} y_{nm} = \sum_{m \in L_n} y_{nm} \leq G_{nm} \leq \sum_{m \in L_n} y_{nm} \tag{28}
\]

\[
\sum_{m \in L_n} y_{nm} \leq 1 \tag{29}
\]

where \( K_n \) is the set of operating conditions of unit \( n \) and \( y_{nm} \) is a 0–1 binary variable signifying the nonexistence or existence of \( G_n \) in the interval \([G_{nm}, G_{n+1}]\) under operating condition \( \xi \). In the case that unit \( n \) must exist in the final design the equality in eq. (29) should be used, whereas in the case when unit \( n \) may not exist the inequality should be adopted in the MILP model. The piecewise linear objective function \( C \) can thus be written as

\[
C = \sum_{n \in N} \sum_{m \in L_n} \left( \beta_{nm} G_{nm} + \sum_{z \in K_n} (\alpha_{nm} - \beta_{nm} G_{nm}) y_{nmz} \right) + c_f \tilde{W}_f + c_w F_w + \sum_{n \in N} \sum_{z \in K_n} c_z \tilde{F}_z \tag{30}
\]

where \( F_w \) is the flow rate of make-up water entering the water treater and \( \tilde{W}_f, \tilde{F}_z \) are cost coefficients.

If, on the other hand, environmental concerns were to be addressed in the design of utility systems, emission targets of various waste gases must be lowered as much as possible. Thus, objective functions other than the annualized cost should be adopted in the MILP model. In this study, the concept of *global emission* (Smith and Delaby, 1991), i.e. the emissions corresponding to both the utility generation within the utility plant and outside the plant boundary, is used to establish these objective functions. The global emissions can be calculated with the following formula:

\[
\{ \text{Global emissions} \} = \{ \text{Emissions from on-site power station corresponding to the amount of electricity imported} \} + \{ \text{Emissions saved at the central power station corresponding to the amount of electricity exported from the site} \}.
\]

\[
E^g = E_e + \frac{\gamma_e \omega_e}{\Omega_e \eta_{e_{ip}}} \tilde{W}_f - \frac{\gamma_e \omega_e}{\Omega_e \eta_{e_{ip}}} \left( \tilde{W}_o + \sum_{n \in N} \tilde{W}_n \right) \tag{32}
\]

Since the scope of this study is limited to the design of utility plants, the chemical processes are thus considered to be outside the system boundary. The corresponding global emission rates can be formulated on this basis, i.e.
where $\kappa \in \mathcal{F}$, $z^*$ is associated with the fuel adopted in the central power station and $\eta_{cps}$ represents the overall efficiency of generating and delivering electricity from the power station to the end users, i.e. the amount of electricity delivered per unit of energy released by burning fuels. In this study, it is assumed that coal is used in the power station to produce the steam needed for driving turbines. A constant overall efficiency $\eta_{cps}$ of 0.28 is adopted in our model, which is also a value suggested by Smith and Delaby (1991).

Notice that a better understanding of the emission reduction problem can be obtained by studying eq. (32). Specifically, from its first term $E_\kappa$, one can see that one of the dominant factors contributing to a certain emission $\kappa$ is fuel selection. This is due to the fact that the value of $\gamma_{\kappa z}$ associated with a given fuel $z(z \in Z_n$ and $n \in N_F)$ may be significantly different from that of another. On the other hand, from the second, third and fourth terms in eq. (32), one may conclude that another general guideline to reduce emission is to generate all the electricity needed in the chemical process and the neighboring community within the utility plant. The effectiveness of this approach, however, hinges upon the overall efficiency of energy utilization $\eta_{eu}$ and this efficiency is defined as

$$
\eta_{eu} = \frac{\text{total amount of useful energies produced}}{\text{per unit time by the utility system}} - \frac{\text{amount of the energy released}}{\text{per unit time by burning fuels}}
$$

where the useful energies include work for driving the generators and heat for heating the process streams. If the value of $\eta_{eu}$ is too low, the amount of emissions saved at the central power station may be offset by a large increase in local emissions.

**THE SINGLE-OBJECTIVE SYSTEM DESIGN**

From the previous discussions, it is clear that a realistic utility system design should be evaluated with multiple criteria. A reasonable solution to the multi-objective programming problem can usually be established on the basis of several single-objective solutions (Ignizio and Cavaliar, 1994). In order to illustrate the solution steps of the single-objective MILP problem and to show the insights gained from this exercise, an example is presented as follows.

**Example 1**: Let us consider the HEN and utility system design problems associated with three cold streams and two hot streams in a chemical process. The corresponding stream data are presented in Table 1. The electricity and water demands of the chemical process are specified in Table 2. Notice that definitions of the abbreviations used in the above tables can be found in the Notation section. The options of fuels for the boilers and furnaces include coal, natural gas and low-sulfur diesel fuel. Due to operability considerations, only the latter two can be used in gas turbines.

Cost coefficients in this study were obtained by approximating the cost data given in Ulrich (1984) with piecewise-linear functions. The installation costs of the equipments were updated to 1994 price levels on the basis of the CE plant cost index. The purchase costs of fuels, water and electricity were adopted from government reports such as *Monthly Statistics of Imports* and *Commodity-Price Statistics Monthly* for the Taiwan area.

The standard Problem Table Algorithm (Linnhoff and Hindemarsh, 1983) has been performed according to Table 1. The hot and cold pinch temperatures were found to be 120 and 100°C, respectively. The heat deficit of each temperature interval has also been determined (see Table 4). On the basis of these results, two levels of steam-header pressures can be selected, i.e. 10 and 40 bar. They are essentially the saturated steam pressures at 180 and 250°C, respectively. In this example, the operating temperatures are chosen to be 240, 280 and 300°C for the former header and 350, 375 and 400°C for the latter.

Having determined the operating conditions of the steam headers, the modified MILP can be solved with any of the previously formulated objective functions. In this example, both the emission rate of CO$_2$ and the annualized cost have been considered. A summary of the solutions can be found in Table 5 and in Figs 3(a) and 3(b). Notice that the SO$_2$ emission rate of case I is negative. This is due to the fact that the emission rate of SO$_2$ caused by importing electricity from central power station to the chemical process is much larger than that by cogeneration. From Table 5, one can also see that the fuels chosen for the two cases are different.

In case I, natural gas is the best choice for reducing pollution. On the other hand, the logical choice for case II must be the cheaper coal.

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### Table 1. The stream data of example 1 ($\Delta T_{Min} = 20$ C)

<table>
<thead>
<tr>
<th>Stream type and number</th>
<th>FC (kW/°C)</th>
<th>$T_{IN}$ (°C)</th>
<th>$T_{OUT}$ (°C)</th>
<th>Heat load (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>600</td>
<td>100</td>
<td>230</td>
<td>78,000</td>
</tr>
<tr>
<td>C2</td>
<td>1200</td>
<td>80</td>
<td>160</td>
<td>96,000</td>
</tr>
<tr>
<td>C3</td>
<td>150</td>
<td>40</td>
<td>200</td>
<td>24,000</td>
</tr>
<tr>
<td>H1</td>
<td>450</td>
<td>180</td>
<td>60</td>
<td>54,000</td>
</tr>
<tr>
<td>H2</td>
<td>1050</td>
<td>220</td>
<td>100</td>
<td>126,000</td>
</tr>
</tbody>
</table>

### Table 2. The electricity and water demands of the chemical process in example 1

<table>
<thead>
<tr>
<th>Electricity</th>
<th>General use 6800 kW</th>
<th>Motor no. 1 375 kW</th>
<th>Motor no. 2 820 kW</th>
<th>Motor no. 3 540 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>Deaerated water 35 kg/s</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3. The operating conditions/parameters for equipments in the utility system of example 1

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Condition/Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deaerator</td>
<td>$P = 1.1013$ bar, $T = 100\degree C$</td>
</tr>
<tr>
<td>Feedwater heater</td>
<td>BFW heated by MP steam</td>
</tr>
<tr>
<td>Boiler</td>
<td>Blow-down rate $= 5%$</td>
</tr>
<tr>
<td></td>
<td>Exhaust gas temperature $= 160\degree C$</td>
</tr>
<tr>
<td></td>
<td>Overall efficiency $= 90.5%$</td>
</tr>
<tr>
<td>Furnace</td>
<td>Exhaust gas temperature $= 160\degree C$</td>
</tr>
<tr>
<td></td>
<td>Overall efficiency $= 90.5%$</td>
</tr>
<tr>
<td>Steam turbine</td>
<td>Efficiency $= 85%$</td>
</tr>
<tr>
<td></td>
<td>Compression ratio $= 10$</td>
</tr>
<tr>
<td></td>
<td>Expansion ratio $= 9.8$</td>
</tr>
<tr>
<td></td>
<td>Turbine efficiency $= 87%$</td>
</tr>
<tr>
<td></td>
<td>Overall heat recovery efficiency $= 80%$</td>
</tr>
<tr>
<td></td>
<td>Turbine inlet temperature $= 1027\degree C$</td>
</tr>
<tr>
<td>Gas turbine</td>
<td>Compression ratio $= 10$</td>
</tr>
<tr>
<td></td>
<td>Expansion ratio $= 9.8$</td>
</tr>
<tr>
<td></td>
<td>Compressor efficiency $= 85%$</td>
</tr>
<tr>
<td></td>
<td>Turbine efficiency $= 87%$</td>
</tr>
<tr>
<td></td>
<td>Overall heat recovery efficiency $= 80%$</td>
</tr>
<tr>
<td></td>
<td>Exhaust steam $P = 0.16$ bar and $T = 55\degree C$</td>
</tr>
<tr>
<td></td>
<td>Cooling water inlet $T = 25\degree C$</td>
</tr>
<tr>
<td></td>
<td>Cooling water outlet $T = 45\degree C$</td>
</tr>
<tr>
<td>Generator</td>
<td></td>
</tr>
<tr>
<td></td>
<td>98%</td>
</tr>
<tr>
<td>Electric motor</td>
<td>95%</td>
</tr>
<tr>
<td>Vacuum condenser</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exhaust steam $P = 0.16$ bar and $T = 55\degree C$</td>
</tr>
<tr>
<td>Cooling water pump</td>
<td>Inlet pressure $= 1.013$ bar</td>
</tr>
<tr>
<td></td>
<td>Outlet pressure $= 7.94$ bar</td>
</tr>
</tbody>
</table>

Table 4. The heat deficits of the temperature intervals in example 1

<table>
<thead>
<tr>
<th>$k$</th>
<th>$T_{\text{h},k}$ ($\degree C$)</th>
<th>$T_{\text{c},k}$ ($\degree C$)</th>
<th>$\Delta T_{\text{h}}$ ($\degree C$)</th>
<th>$\Delta H_{\text{h}}$ (kW)</th>
<th>$\Delta H_{\text{c}}$ (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>250</td>
<td>230</td>
<td>20</td>
<td>18,000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>220</td>
<td>200</td>
<td>20</td>
<td>12,000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>160</td>
<td>20</td>
<td>6,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>100</td>
<td>20</td>
<td>2,000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>80</td>
<td>20</td>
<td>300</td>
<td>-150</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>40</td>
<td>20</td>
<td>300</td>
<td>-12,000</td>
</tr>
</tbody>
</table>

Table 5. The solutions of single-objective MILP model in example 1

<table>
<thead>
<tr>
<th>Case no.</th>
<th>I</th>
<th>II</th>
<th>Objective function</th>
<th>Cost ($10^6$ USD/yr)</th>
<th>CO$_2$ rate (kg/h)</th>
<th>SO$_2$ rate (kg/h)</th>
<th>NO$_x$ rate (kg/h)</th>
<th>GTW (kW)</th>
<th>GTF</th>
<th>BOF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CO$_2$ rate</td>
<td>Cost</td>
<td>CO$_2$ rate</td>
<td>12.25</td>
<td>1489.30</td>
<td>-139.70</td>
<td>GTW (kW)</td>
<td>5386.00</td>
<td>Natural gas</td>
<td>Natural gas</td>
</tr>
</tbody>
</table>

The flow diagrams presented in Figs 3(a) and 3(b) represent the optimal designs of cases I and II, respectively. Notice that the design in Fig. 3(a) uses a combination of gas and steam turbines to satisfy the demand for power, but only steam turbines are required in case II. The overall efficiency in energy utilization of the former design is obviously higher than that of the latter. In the latter case, a large excess of steam and high-temperature saturated water must be purged after supplying the specified demands for power and heating utilities. [see Fig. 3(b)]. This is due to the facts that the deaerator is required to be operated at 100\degree C and the hot pinch point is 120\degree C. Thus, although the second design costs less, the emission rates of CO$_2$, SO$_2$ and NO$_x$ are much higher due to the requirements for burning more fuel and using a fuel that creates more serious pollution problems.

Finally, it should be noted that the cost of purchasing electricity is assumed to be 0.067 USD/kW hr in this example. Since this cost is higher than that produced within the utility plant, none of the designs adopts the option of importing electricity. It is also assumed that there is no electricity demand from the neighboring community. Thus, electricity export is not necessary in both cases I and II.

THE MULTIOBJECTIVE SYSTEM DESIGN

From the results obtained with single-objective MILP model, it is clear that it is not possible to optimize multiple performance measures simultaneously. One can observe from Table 5 that, when one of the objectives is minimized, the other aspects of design may be far from satisfactory. Thus, instead of a scalar, the performance of our design problem should be characterized with a vector, i.e.

$$\min z_r = g_r(x), \quad r = 1, 2, \ldots, R$$  \hspace{1cm} (34)

subject to

$$f_m(x) \begin{cases} \geq & b_m, \quad m = 1, 2, \ldots, M \end{cases}$$  \hspace{1cm} (35)
where $z_r$ denotes the rth objective function, $f_m$ represents the mth constraint and the $b_{nm}$s are constants. Notice that the "correct" solution to this problem is now a matter of *philosophy*. The proper trade-off among competing objectives is really dependent upon the preference of the decision maker(s).

If one can identify a common measure of effectiveness by means of which each of the objectives can be expressed, it should be possible to aggregate all of them into an *equivalent* function and solve the resulting problem with a conventional optimization approach. In fact, the well-known utility theory has been devoted, especially for establishing the utility functions that represent the preference of the decision maker, e.g. Keeney and Raiffa (1976). However, in our present case, it is difficult to justify the implied assumption that all objectives are commensurable. First of all, to convert the environmental impacts into equivalent financial implications is by far a trivial task. Further, this approach itself, i.e. to express pollution problems solely in terms of economical losses, is still highly controversial.

A more sensible tool for our problem seems to be the method of goal programming (GP) (Ignizio, 1982), in which the performance measures are ranked and then minimized *lexicographically*. Notice that, for the decision maker, it is a relatively straightforward task to just set the precedence order among objectives. Since these objectives are handled separately in the solution process, the need for weighting them is thus avoided. Further, the lexicographic notion has been demonstrated to be consistent with the multiobjective decision-making process of human being. Thus, the goal programming model is a better representation of the multicriteria design problem at hand.

There are actually a number of types of goal programs. The one adopted here is the non-Archimedean GP. A brief outline of its fundamental ideas is given below:

The first step in constructing such a GP model is to establish the aspiration levels $s_r$ of all objectives and convert them into goals. Specifically, eq. (34) can be
transformed into

\[ g_i(x) \leq s_r, \quad r = 1, 2, \ldots, R. \quad (36) \]

The aspiration levels are values that the designer hopes to achieve in the final solution. Thus, rather than attempting to achieve solution optimality (which is meaningful only in single-objective problems), the approach of GP is to find a solution that comes as close as possible to satisfying the design goals. Guidelines for determining the values of \( s_r \) have been reported in Ignizio and Cavalier (1994).

Notice that eqs (35) and (36) can be written in a generalized form, i.e.

\[ F_i(x) = b_i, \quad i = 1, 2, \ldots, M \quad (37) \]

where

\[ F_i(x) = f_i(x), \quad i = 1, 2, \ldots, M \quad (38a) \]

and

\[ F_i(x) = g_{i-M}(x), \quad b_i = s_{i-M} \quad i = M + 1, M + 2, \ldots, M + R. \quad (38b) \]

Thus, the GP model can be viewed as the one consisting only of goals. These goals can be transformed into equations via negative and positive variables, \( n_i \) and \( p_i \), i.e.

\[ F_i(x) + n_i - p_i = b_i, \quad n_i \geq 0, \quad p_i \geq 0. \quad (39) \]

Next, let us define an achievement function \( a_i \) for each goal to reflect the degree of violation in the solution, i.e.

\[ \begin{align*}
    a_i &= n_i \quad \text{when } F_i(x) \geq b_i; \\
    a_i &= p_i \quad \text{when } F_i(x) \leq b_i; \\
    a_i &= n_i + p_i \quad \text{when } F_i(x) = b_i.
\end{align*} \]

If the value of every \( a_i \) is reduced to zero, then all the goals defined in the GP model must also be achieved.
However, since this ideal outcome may not be possible, the lexicographic minimum must be identified on the basis of an achievement vector \( \mathbf{u} \), which is defined as follows:

\[
\mathbf{u} = [c^{(1)}a^{(1)}, c^{(2)}a^{(2)}, \ldots, c^{(k)}a^{(k)}]
\]  

(40)

where \( a^{(k)} \) is the vector of achievement functions at priority level \( k \) and \( c^{(k)} \) denotes the vector of corresponding positive-valued weights. The achievement functions associated with hard goals are always included in \( a^{(1)} \) and their weights are all set to unity, i.e.

\[
c^{(1)} = [1, 1, \ldots, 1].
\]  

(41)

On the other hand, the achievement functions at higher priority levels (\( k \geq 2 \)) are usually corresponding to the soft goals and their precedence order is arranged according to the decision-maker’s preference.

In this study, the hard goals are obtained from the rigid constraints of the MILP model formulated previously and the rest of the elements in the achievement vector are corresponding to the design objectives, i.e.

\[
c^{(1)}a^{(1)} = \sum_{i=1}^{M} a_{i}
\]  

(42a)

\[
c^{(k)}a^{(k)} = a_{j}
\]  

(42b)

\[ k = 2, 3, \ldots, R + 1 \] and \( j \in \{M + 1, M + 2, \ldots, M + R\} \).

Therefore, the standard form of a GP model can be written as

\[
\text{lexmin} \mathbf{u}
\]  

(43a)

s.t.

\[
F_{j}(x) + n_{i} - p_{i} = b_{i}, \quad i = 1, 2, \ldots, M + R
\]  

(43b)

\[
x \geq 0, \quad n \geq 0, \quad p \geq 0
\]  

(43c)

where \( n \) and \( p \) denote the vectors of negative and positive deviation variables, respectively.

Finally, the concept of lexicographic minimum (lexmin) needs to be further clarified. Let us consider the achievement vector \( \mathbf{u} = [u_{1}, u_{2}, \ldots, u_{R+1}] \). In searching for the solution to a GP problem, the one corresponding to a particular \( \mathbf{u}^{*} \) is preferred to another \( \mathbf{u}^{\prime} \) if \( u_{k}^{*} < u_{k}^{\prime} \) and all higher-ordered terms (i.e. \( u_{1}, u_{2}, \ldots, u_{k-1} \)) are equal. Thus, if no other solution is preferred to the one associated with \( \mathbf{u}^{*} \), then \( \mathbf{u}^{*} \) is considered as a lexicographic minimum. Under the basis of this concept, the following procedure can be adopted for solving eqs (43a)–(43c).

1. Let \( k = 1 \). Solve a single-objective optimization problem using \( a_{k} \) as the objective function and eqs (43b) and (43c) as the constraints.
2. Let \( a_{k}^{*} = \min a_{k} \).
3. If \( k \geq R + 1 \), then a lexicographic minimum has been identified. Otherwise go to the next step.
4. Let \( k = k + 1 \). Solve the single-objective optimization problem using \( a_{k} \) as the objective func-

### Table 6. The solutions of multiobjective MILP model in example 2

<table>
<thead>
<tr>
<th>Design no.</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (10^6 USD/yr)</td>
<td>10.49</td>
<td>9.93</td>
</tr>
<tr>
<td>CO₂ rate (kg/h)</td>
<td>10,000.00</td>
<td>11,294.70</td>
</tr>
<tr>
<td>SO₂ rate (kg/h)</td>
<td>99.60</td>
<td>95.30</td>
</tr>
<tr>
<td>NOₓ rate (kg/h)</td>
<td>23.80</td>
<td>25.50</td>
</tr>
<tr>
<td>GTW (kW)</td>
<td>1015.60</td>
<td>—</td>
</tr>
<tr>
<td>GTF</td>
<td>Natural gas</td>
<td>Oil</td>
</tr>
<tr>
<td>BOF</td>
<td>Oil</td>
<td>Oil</td>
</tr>
</tbody>
</table>

as the constraints. Go to step 2.

Let us use a simple example to demonstrate the feasibility of the proposed multicriteria design technique.

**Example 2:** The problem stated in example 1 is solved again using the GP model. Let us assume that, after a careful study of the results presented in Table 5, the designer has decided that the appropriate aspiration levels of CO₂ emission rate and annualized cost are 10,000 kg/h and 10 million U.S. dollars, respectively. Two designs have been obtained in this example. The first (design A) places more emphasis on environmental protection, i.e. the CO₂ emission rate is used as the first soft goal at the second level and the annualized cost is next at the third level. In design B, this priority order is reversed. A summary of these two designs is presented in Table 6. One can clearly see that, in each case, the first soft goal can be achieved satisfactorily and, although a slight deviation exists, the extent of violation in the second soft goal is still quite acceptable. One can also observe that, if economic performance is the main concern in designing a utility plant, then it is in general advantageous to produce power using a combination of steam turbines and boilers driven by the low sulfur diesel fuel. On the other hand, if the first priority in design is to reduce air pollution, natural gas is certainly the best choice for fuel. Also, the use of gas turbine should be considered to cut down the total consumption rate of fuel and, consequently, the gas emission rate can be reduced as well.

### AN ADDITIONAL EXAMPLE

In this section, an additional example is presented to show the effects of varying process requirements on the utility system design. Let us consider a chemical process in which three hot streams and three cold streams can be identified (see Table 7). Notice that cold stream C2 is originally a liquid which is vaporized at 119°C. The total electricity demand of this process is assumed to be 14,000 kW. The options for fuels are essentially the same as those adopted in example 1. The operating conditions and parameters of equipments in the utility system can be found in Table 3.
After carrying out the Problem Table Algorithm, the hot and cold pinch temperatures of this process have been found to be 139 and 119°C, respectively. From the heat deficits listed in Table 8, one can see that the first four temperature intervals require heat inputs. However, if the demand for heating utility in the first interval is to be satisfied with steam, then the saturated steam pressure must be higher than 500 bar. On the basis of safety and cost considerations, the steam header corresponding to the first interval is thus excluded in the superstructure. Three steam headers are used in this example. Their operating pressures are set to be the saturated steam pressures at the upper bounds of the second, third and fourth temperature intervals, i.e. 55, 17.5 and 3.5 bar, respectively. The discretized operating temperatures corresponding to 55 bar are 375, 400 and 425°C and those corresponding to 17.5 bar are 325, 350 and 375°C. Since the steam pressure of the third header is too low to drive a turbine, the operating temperature is fixed at 139°C.

As indicated before, the first step in design process is to solve several single-objective MILP problems. Four performance measures have been considered, i.e. the emission rates of CO\textsubscript{x}, NO\textsubscript{x} and SO\textsubscript{x} and the annualized cost. A summary of the solutions is shown in Table 9. Contrary to the findings in example 1, a gas turbine is used in the minimum-cost design. The only difference between this design and those aimed at reducing gas emission is that the gas turbine in the former case produces less power. This result is obviously due to the fact the steam cannot be produced at very high temperatures. Also, one can see that appropriate fuel selection is critical in reducing the emission rate of a particular pollutant. Notice that the best fuel for minimizing NO\textsubscript{x} emission is low-sulfur diesel oil but the choice for the other pollutants is natural gas. These selections are made apparently on the basis of fuel compositions. Accordingly to Smith and Delaby (1991), oil has the lowest nitrogen content and, on the other hand, the carbon and sulfur contents are lowest in natural gas.

After a review of the data given in Table 9, the priority order of the design objectives and the corresponding aspiration levels can be determined according to the designer's preference. Let us assume that these decisions have been made and the GP model has been solved according to Table 10. The final design of the utility plant is shown in Fig. 4. The corresponding grand composite curve can be found in Fig. 5. A summary of the design is listed in Table 11. One can see

### Table 7. The stream data of the additional example (\(\Delta T\text{MIN} = 20^\circ\text{C}\))

<table>
<thead>
<tr>
<th>Stream type and number</th>
<th>FC (kW/°C)</th>
<th>(T_{IN}) (°C)</th>
<th>(T_{OUT}) (°C)</th>
<th>Heat load (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>40</td>
<td>185</td>
<td>414</td>
<td>9160</td>
</tr>
<tr>
<td>C2</td>
<td>294</td>
<td>119</td>
<td>250</td>
<td>51,352</td>
</tr>
<tr>
<td>C3</td>
<td>126</td>
<td>205</td>
<td>60</td>
<td>24,360</td>
</tr>
<tr>
<td>H1</td>
<td>52</td>
<td>270</td>
<td>100</td>
<td>8840</td>
</tr>
<tr>
<td>H2</td>
<td>252</td>
<td>90</td>
<td>45</td>
<td>11,340</td>
</tr>
</tbody>
</table>

### Table 8. The heat deficits of the temperature intervals in the additional example (\(T_{P,CH} = 139^\circ\text{C}, T_{P,CH} = 119^\circ\text{C}\))

<table>
<thead>
<tr>
<th>(k)</th>
<th>(T_{A,k}) (°C)</th>
<th>(T_{A,k}) (°C)</th>
<th>(\sum (F_i C_i - F_h C_h)_k) (kW/°C)</th>
<th>(\Delta H_k) (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>434</td>
<td>414</td>
<td>40</td>
<td>6560</td>
</tr>
<tr>
<td>1</td>
<td>270</td>
<td>250</td>
<td>164</td>
<td>24,700</td>
</tr>
<tr>
<td>2</td>
<td>205</td>
<td>185</td>
<td>65</td>
<td>172</td>
</tr>
<tr>
<td>3</td>
<td>139</td>
<td>119</td>
<td>66</td>
<td>11,352</td>
</tr>
<tr>
<td>4</td>
<td>139</td>
<td>119</td>
<td>—</td>
<td>25,200</td>
</tr>
<tr>
<td>5</td>
<td>125</td>
<td>105</td>
<td>14</td>
<td>222</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>80</td>
<td>25</td>
<td>222</td>
</tr>
<tr>
<td>7</td>
<td>90</td>
<td>70</td>
<td>10</td>
<td>168</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>40</td>
<td>30</td>
<td>12,600</td>
</tr>
<tr>
<td>9</td>
<td>45</td>
<td>25</td>
<td>15</td>
<td>3780</td>
</tr>
</tbody>
</table>

### Table 9. The solutions of single-objective MILP model in the additional example

<table>
<thead>
<tr>
<th>Case no.</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function</td>
<td>CO\textsubscript{x} rate</td>
<td>NO\textsubscript{x} rate</td>
<td>SO\textsubscript{x} rate</td>
<td>Cost (10\textsuperscript{6} USD/yr)</td>
</tr>
<tr>
<td>Cost (10\textsuperscript{6} USD/yr)</td>
<td>20.115</td>
<td>17.796</td>
<td>20.115</td>
<td>16.325</td>
</tr>
<tr>
<td>CO\textsubscript{x} rate (kg/h)</td>
<td>2350.2</td>
<td>9903.4</td>
<td>2350.2</td>
<td>17581.8</td>
</tr>
<tr>
<td>SO\textsubscript{x} rate (kg/h)</td>
<td>—226.7</td>
<td>—175.4</td>
<td>—226.7</td>
<td>291</td>
</tr>
<tr>
<td>NO\textsubscript{x} rate (kg/h)</td>
<td>17.2</td>
<td>46.9</td>
<td>17.2</td>
<td>53.8</td>
</tr>
<tr>
<td>GTW (kW)</td>
<td>8286.8</td>
<td>8296.8</td>
<td>8286.8</td>
<td>5007</td>
</tr>
<tr>
<td>GTF</td>
<td>Natural gas</td>
<td>Oil</td>
<td>Natural gas</td>
<td>Natural gas</td>
</tr>
<tr>
<td>BOF</td>
<td>Natural gas</td>
<td>Oil</td>
<td>Natural gas</td>
<td>Coal</td>
</tr>
</tbody>
</table>
Table 10. The priority order and aspiration levels used in the multi-objective MILP model of the additional example

<table>
<thead>
<tr>
<th>Priority order</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function</td>
<td>CO\textsubscript{x} rate</td>
<td>SO\textsubscript{x} rate</td>
<td>Cost</td>
<td>NO\textsubscript{x} rate</td>
</tr>
<tr>
<td>kg/h</td>
<td>kg/h</td>
<td>(10\textsuperscript{6} USD/yr)</td>
<td>kg/h</td>
<td></td>
</tr>
<tr>
<td>Aspiration level</td>
<td>8500</td>
<td>- 120</td>
<td>17.5</td>
<td>- 20</td>
</tr>
</tbody>
</table>

![Diagram of the multiobjective design for the additional example](image)

**Fig. 4.** The multiobjective design for the additional example.

that, other than the annualized cost, all design goals are achieved. Since the cost overrun is not significant, the system in Fig. 4 should be considered as a suitable solution when multiple conflicting criteria have to be considered in a single design.

**CONCLUSIONS**

A practical utility system design must be evaluated with both economic and environmental criteria. To address this need, a multiobjective MILP model has been developed in our study. Not only the problem of cost minimization can be solved efficiently with the proposed superstructure, but also the concept of global emission can be incorporated in the model formulation. On the basis of goal programming (GP) philosophy, a satisfactory design can be generated according to the priority order given by the designer. From the results we have gathered so far in implementing...
the proposed approach, one can conclude that the MILP model is suitable for the design of a wide variety of utility systems and, furthermore, the GP method is a natural and sensible design tool for establishing a compromise among conflicting objectives.

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**Table 11. The solution of multiobjective MILP model in the additional example**

<table>
<thead>
<tr>
<th>Cost (10^6 USD/yr)</th>
<th>18.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂ rate (kg/h)</td>
<td>8500</td>
</tr>
<tr>
<td>SO₂ rate (kg/h)</td>
<td>-187.3</td>
</tr>
<tr>
<td>NOₓ rate (kg/h)</td>
<td>-30.1</td>
</tr>
<tr>
<td>GTW (kW)</td>
<td>7309.1</td>
</tr>
<tr>
<td>BOF Natural gas</td>
<td></td>
</tr>
<tr>
<td>Oil</td>
<td></td>
</tr>
</tbody>
</table>

**NOTATION**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_{hk}</td>
<td>heat capacity of flue gas h in interval k</td>
<td>kW/°C</td>
</tr>
<tr>
<td>E_{x}</td>
<td>local emission rate of pollutant x</td>
<td>kg/h</td>
</tr>
<tr>
<td>E_{G}</td>
<td>global emission rate of pollutant x</td>
<td>kg/h</td>
</tr>
<tr>
<td>F_{W}</td>
<td>mass flow rate of make-up water</td>
<td>kW</td>
</tr>
<tr>
<td>F_{HU}</td>
<td>mass flow rate of utility h in interval k</td>
<td>kW</td>
</tr>
<tr>
<td>F_{z}</td>
<td>mass flow rate of fuel z in unit n</td>
<td>kW</td>
</tr>
<tr>
<td>G_{n}</td>
<td>capacity of equipment n</td>
<td>kW</td>
</tr>
<tr>
<td>G_{nm}</td>
<td>discretized capacity of equipment n in range m</td>
<td>kW</td>
</tr>
<tr>
<td>Q_{nf}</td>
<td>heat absorbed by the utility stream in a fuel-burning unit n</td>
<td>kW</td>
</tr>
<tr>
<td>Q_{h}</td>
<td>heat input from utility h to interval k</td>
<td>kW</td>
</tr>
<tr>
<td>Q_{i}</td>
<td>heat input from hot stream i to interval k</td>
<td>kW</td>
</tr>
<tr>
<td>Q_{k}</td>
<td>heat output from interval k to cold stream j</td>
<td>kW</td>
</tr>
<tr>
<td>R_{k}</td>
<td>residual heat from temperature interval k</td>
<td>kW</td>
</tr>
<tr>
<td>T_{f}</td>
<td>theoretical flame temperature</td>
<td>°C</td>
</tr>
<tr>
<td>T_{stack}</td>
<td>stack temperature</td>
<td>°C</td>
</tr>
<tr>
<td>W_{m}, W_{n}</td>
<td>power produced by unit m in the chemical process and unit n in the utility system, respectively</td>
<td>kW</td>
</tr>
<tr>
<td>\dot{W}<em>{m}, \dot{W}</em>{n}</td>
<td>electricity consumed by motor m in the chemical process and motor n in the utility system, respectively</td>
<td>kW</td>
</tr>
</tbody>
</table>
\[ \tilde{W}_s \] the electricity generated by unit \( n \) in the utility system
\[ \tilde{W}_d \] the electricity demand of the neighboring community
\[ \tilde{W}_u \] the upper limit of electricity demand of the neighboring community
\[ \tilde{W}_i \] the electricity imported from central power station

Greek letters
\[ \eta \] the overall efficiency of a fuel-burning unit \( n \)
\[ \eta_{mc} \] the conversion efficiency of a motor
\[ \eta_{cp} \] the overall efficiency of generating and delivering electricity from power station to the end users
\[ \eta_{en} \] the conversion efficiency of a generator
\[ \Omega_2 \] the heat released by burning a unit mass of fuel \( z \)

Abbreviations
BFW boiler-feed water
BOF boiler fuel
GTF gas-turbine fuel
GTW power generated by the gas turbine
MP medium pressure

REFERENCES


