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The use of mixers in heat recovery system design

Chuei-Tin Chang* and Lien-Cheng Chen
Department of Chemical Engineering, National Cheng Kung University, Tainan, Taiwan 70101, Republic of China

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Abstract—A systematic procedure is proposed in the present paper to incorporate the options of merging and/or splitting streams from multiple origins in process synthesis. In particular, design problems associated with the heat recovery systems in chemical processes are discussed in detail. First, an input–output system structure is described to facilitate the derivation of the improved LP and MILP models for calculating the design targets corresponding to the operating and capital costs. Next, the construction procedure of a general stream structure is proposed to formulate a NLP model for generating the optimal network configurations. The feasibility and effectiveness of this approach is demonstrated with five examples. Copyright © 1996 Elsevier Science Ltd

Keywords: Heat exchanger network; mixer; two-scale temperature partition; mathematical programming.

INTRODUCTION

In many existing chemical processes, there are situations where two or more streams are allowed to be merged. For example, the overhead product streams of a multi-effect distillation unit may be combined and then cooled (or heated) to the operating temperature of a downstream equipment; the hot contaminated waste water from different areas in a food additive plant may be merged and sent to the treatment unit before disposal; naphtha and hydrogen may be merged and heated to the reaction temperature in the hydrotreating plant of a refinery. In designing the heat exchanger networks (HENs) of these systems, merging process streams originated from different units is often a viable alternative in addition to the traditional synthesis techniques.

Although there are a large number of potential applications of merging and splitting schemes in process design, very few papers have been published on methods that take advantage of these additional opportunities. In a preliminary study, Chang and Yu (1988) showed that such techniques can be used in an evolutionary synthesis procedure to reduce the number of heat exchangers in a maximum energy recovery network without energy penalty. In a later study, Chang *et al.* (1994) developed mathematical programs that solved the same problem on a more comprehensive basis. From the results obtained by implementing this approach, one can observe that the capital costs of HENs can be lowered significantly without increasing the operating costs.

One of the restrictions imposed in the previous studies is that heat exchange between hot and cold streams is allowed only in an exchanger. This constraint is *relaxed* in the present work, i.e. the exchange of heat in a HEN can also be realized with mixers. By introducing this extra degree of flexibility in design, it is our intention to show in this paper that further reduction in *both* capital and operating costs is possible.

Traditionally, the mathematical programming approach to solve HEN design problems is divided into three steps (Papoulias and Grossmann, 1983; Floudas *et al.*, 1986; Chang *et al.*, 1994):

- solve a linear programming (LP) model to determine the minimum consumption rates of utilities;
- solve a mixed integer linear programming (MILP) model to determine the minimum number of matches and the corresponding heat duties;
- solve a nonlinear programming (NLP) model to obtain a cost-optimal network.

This study essentially follows the same procedure. However, since heat exchange may be carried out either in an exchanger or a mixer in the present problem, adjustments must be made in all three models mentioned above. In particular, in order to represent correctly the heat flow pattern in a modified transshipment LP model, a two-scale temperature partition scheme has been developed in this study. On the basis of the same temperature partition, a revised

* Corresponding author.

version of the MILP model has been used to minimize the number of *exchangers* in the heat recovery system. Finally, a generalized stream structure has been adopted to incorporate all possible configurations connecting the exchangers and mixers identified from the solutions of mixed integer linear program. A NLP model can then be formulated accordingly to determine the optical flowsheet of heat recovery system.

To demonstrate the feasibility of the proposed synthesis procedure, the results of several application examples are presented in this paper. One can clearly observe that, as a result of integrating the proposed new techniques in process synthesis, better alternatives may be generated for the same design problem.

THE INPUT–OUTPUT SYSTEM STRUCTURE

To facilitate later discussions, the input–output system structure of a typical exchanger network (EN) must be defined first. In this study, it is described by the diagram presented in Fig. 1. Notice that a number $l(l = 1, 2, \ldots, l_M)$ is assigned to each group of process streams in which they are allowed to be mixed with one another. Corresponding to a specific group l, the streams entering the system may be originated from M_l^{in} different units and those leaving the EN may be required to be delivered to M_l^{out} destinations ($M_l^{\text{in}} \neq M_l^{\text{out}}$). To be specific, let us define the following stream set:

$\mathbf{M} = \{l \mid l$ is the label of the lth group of process streams in which the streams are allowed to be merged with one another$\}$.

Also, there may be additional hot (source) and cold (sink) process streams in the exchanger network which are not allowed to be merged with other streams, i.e.

$\mathbf{H}' = \{i \mid i$ is the label of a hot process stream which is not allowed to be merged with other streams$\}$.

$\mathbf{C}' = \{j \mid j$ is the label of a cold process stream which is not allowed to be merged with other streams$\}$.

Finally, there must be utility streams and they are included in two stream sets in this paper, i.e.

$\mathbf{S} = \{m \mid m$ is the label of a hot utility stream$\}$

$\mathbf{W} = \{n \mid n$ is the label of a cold utility stream$\}$.

Based on the principle of mass conservation and the assumption that the heat capacity of every process steam is independent of temperature, the sum of heat-capacity flow rates of input streams associated with each $l \in \mathbf{M}$ should be equal to that corresponding to the output streams. Specifically, let us define

$\mathbf{MI}_l = \{\imath \mid \imath$ represents the label of the $\imath$th input stream in group $l\}$

and

$\mathbf{MO}_l = \{o \mid o$ represents the label of the oth output stream in group $l\}$.

The heat-capacity flow rates of the streams in these two sets can be related by

$$\sum_{\imath \in \mathbf{MI}_l} F_{CP_\imath}^{\mathbf{MI}_l} = \sum_{o \in \mathbf{MO}_l} F_{CP_o}^{\mathbf{MO}_l}, \quad l \in \mathbf{M} \tag{1}$$

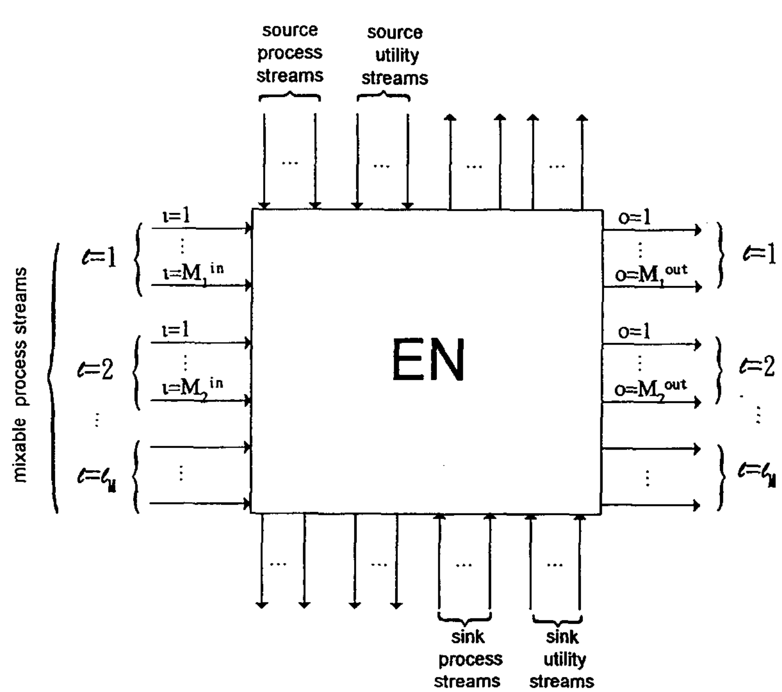

Fig. 1. The input–output system structure.

where $F_{CP_i}^{\mathbf{MI}_l}$ denotes the heat-capacity flow rate of the ith input stream in set $\mathbf{MI}_l$ and, similarly, $F_{CP_o}^{\mathbf{MO}_l}$ the heat-capacity flow rate of the oth output streams in set $\mathbf{MO}_l$. Throughout this paper, the same convention is used to represent other heat-capacity flow rates and thus their definitions will not be repeated later.

THE FICTITIOUS STREAMS

On the basis of the input–output system structure described in Fig. 1, one can postulate that, in every group of mixable streams, there exists a fictitious stream which is associated with each pair of input and output, i.e.

$$\mathbf{MIO}_l = \{(i,o) \mid i \in \mathbf{MI}_l,\ o \in \mathbf{MO}_l\}, \quad l \in \mathbf{M}.$$

Notice that the design objectives implied in Fig. 1 can be achieved if each input stream is split into M_l^{out} branches and then heated or cooled to the temperatures of the output streams with a combination of exchanger, mixers and additional splitters. Naturally, the heat-capacity flow rates of these fictitious streams must satisfy the following constraints:

$$\sum_{o \in \mathbf{MO}_l} F_{CP_{i,o}}^{\mathbf{MIO}_l} = F_{CP_i}^{\mathbf{MI}_l}, \quad i \in \mathbf{MI}_l,\ l \in \mathbf{M} \tag{2}$$

$$\sum_{i \in \mathbf{MI}_l} F_{CP_{i,o}}^{\mathbf{MIO}_l} = F_{CP_o}^{\mathbf{MO}_l}, \quad o \in \mathbf{MO}_l,\ l \in \mathbf{M}. \tag{3}$$

In essence, the heats added to or removed from input streams are signified by the enthalpy changes of fictitious streams in our model. Since the heat-capacity flow rates of the fictitious streams are variables, the transformation of input states to output states in any network configurations can be described with this approach. Thus, if a minimum-utility network exists, it should also be one that can be viewed from this standpoint.

Notice also that the fictitious streams can be further classified into two stream sets, $\mathbf{MH}_l$ and $\mathbf{MC}_l$, according to their initial and final temperatures, i.e.

$$\mathbf{MH}_l = \{(i,o) \mid (i,o) \in \mathbf{MIO}_l,\ T_i^{\mathbf{MI}_l} > T_o^{\mathbf{MO}_l}\}, \quad l \in \mathbf{M}$$

$$\mathbf{MC}_l = \{(i,o) \mid (i,o) \in \mathbf{MIO}_l,\ T_i^{\mathbf{MI}_l} < T_o^{\mathbf{MO}_l}\}, \quad l \in \mathbf{M}$$

where $T_i^{\mathbf{MI}_l}$ and $T_o^{\mathbf{MO}_l}$ denote, respectively, the ith input temperature and the oth output temperature in group l.

THE TWO-SCALE TEMPERATURE PARTITION PROCEDURE

Since two different types of heat exchange are allowed in the heat recovery system design, the traditional approach for computing the minimum utility costs must be modified to accomodate this added degree of freedom. In this study, a revised version of the expanded transshipment model (Papoulias and Grossmann, 1983) has been adopted for determining the energy targets. It should be noted that a transshipment model describes only the *heat flow pattern* in a heat recovery system, i.e. the heat flows within the imaginary temperature intervals and also the heat flows among hot streams, temperature intervals and cold streams. It is thus critical to select an appropriate temperature-partition scheme so that all possible channels of heat exchange can be modeled correctly.

A two-scale temperature partition procedure is proposed in this section. This procedure is developed on the basis of the assumption that a mixer can be viewed as a heat exchanger without the need for maintaining a nonzero minimum temperature approach in operation. Consequently, the entire temperature range of all streams should be partitioned into K intervals according to two different $\Delta T_{\min}$s. A value of zero is used to represent the heat transfer in mixers. On the other hand, a positive $\Delta T_{\min}$ should be adopted as the lower limit of driving force in a heat exchanger. The validity of this approach will become clear later when the heat flow pattern in a temperature interval is examined in detail.

Before a justification of the proposed assumption can be provided, it is necessary to first outline the specific steps of two-scale temperature partition procedure.

1. Compare the initial and final temperatures of all *real and fictitious* cold streams and order them sequentially from the highest down to lowest level. Transform these cold-stream temperatures T^C to hot-stream temperatures T^H with the relation $T^H = T^C + \Delta T_{\min}$, where $\Delta T_{\min}$ is nonzero.
2. Compare the initial and final temperatures of all *fictitious* streams and order them sequentially from the highest down to the lowest level. Transform these cold-stream temperatures T^C to hot-stream temperatures T^H with the relation $T^H = T^C$.
3. Compare the hot-stream temperatures obtained in the previous two steps and the initial and final temperatures of all real and fictitious hot streams. Order these temperatures sequentially from the highest to the lowest level and label them as T_k^H $(k = 0, 1, 2, \ldots, K)$.
4. Let $T_k^E = T_k^H - \Delta T_{\min}$ $(k = 0, 1, 2, \ldots, K)$, where T_k^E denotes the interval temperatures of the cold streams which exchange heat in exchangers.
5. Let $T_k^M = T_k^H$ $(k = 0, 1, 2, \ldots, K)$, where T_k^M denotes the interval temperatures of the cold streams which exchange heat in mixers.

By following the above procedure, the distributions of T_k^E and T_k^M may not be consistent over the entire temperature range of all fictitious cold streams. This is inconvenient since constraints of mass conservation corresponding to each group of mixable streams must be imposed within the same temperature intervals. Thus, the following steps must be taken to identify if there is a need to further partition a cold-stream temperature interval into subintervals.

- Compare T_k^E and T_k^M over the entire temperature range of all fictitious cold streams.

- Order these temperatures sequentially from the highest to the lowest level and relabel them as T_s^S $(s = 0, 1, 2, \ldots, N)$. Let $[T_s^S, T_{s-1}^S]$ be subinterval s.
- Determine the subintervals in temperature intervals partitioned according to T_k^E and also T_k^M, i.e. identify sets

$$\mathbf{I}_k^E = \{s \mid s \text{ is the label of a subinterval and } T_k^E \leqslant T_s^S < T_{s-1}^S \leqslant T_{k-1}^E\}$$

$$\mathbf{I}_k^M = \{s \mid s \text{ is the label of a subinterval and } T_k^M \leqslant T_s^S < T_{s-1}^S \leqslant T_{k-1}^M\}$$

and

$$k = 1, 2, \ldots, K.$$

Finally, it should be noted that the constraints of mass conservation should be imposed within each subinterval, i.e.

$$F_{CP_{\iota os}}^E + F_{CP_{\iota os}}^M = F_{CP_{\iota o}}^{\mathrm{MC}l}, \quad (\iota, o) \in \mathbf{MC}_l, \quad l \in \mathbf{M}, \quad s = 1, 2, \ldots, N. \tag{4}$$

Notice that, within each subinterval, heat exchange between a cold fictitious stream (ι, o) and other hot streams can be carried out either in an exchanger or a mixer. Thus, the superscript 'E' is adopted to denote exchanger and 'M' mixer.

THE HEAT FLOW PATTERN WITHIN A TEMPERATURE INTERVAL

Let us now consider the heat flow pattern in a temperature interval k obtained by following the proposed two-scale temperature partition procedure (Fig. 2). In general, there are four types of heat inputs entering a temperature interval, i.e. the heat released by hot process and utility streams, i.e. Q_{ik}^H and Q_{mk}^S, and the corresponding residual heat flows from interval $k - 1$, i.e. $D_{i,k-1}$ and $D_{m,k-1}$. The outputs leaving interval k include the heat received by utility streams (Q_{nk}^W), by process streams with heat exchangers (Q_{jk}^C) and mixers ($Q_{jk}^{C''}$), and the corresponding heat flows to interval $k + 1$ ($D_{i,k}$ and $D_{m,k}$). Within interval k, four types of heat exchange take place and their respective heat duties are $Q_{ijk}^E, Q_{ink}^E, Q_{mjk}^E$ and Q_{ijk}^M. The superscripts of these symbols denote the equipments in which heat exchange is carried out, i.e. 'E' represents exchanger and 'M' mixer. The subscripts consist of three indices. The first is either the label of a hot process stream i or a hot utility stream m. The second one is used to represent a cold process stream j or cold utility stream n. The last index k is simply the interval number.

Notice that the heat exchanges corresponding to Q_{ijk}^E, Q_{ink}^E and Q_{mjk}^E also exist in the original transshipment model and thus there is no need to prove their validity again. The term Q_{ijk}^M represents the amount of heat exchange in mixers between cold stream j in interval k and hot stream i in the intervals $\bar{k} \leqslant k$. Notice also that, since the constraints imposed upon Q_{ijk}^M are only energy balances, it is important to ensure

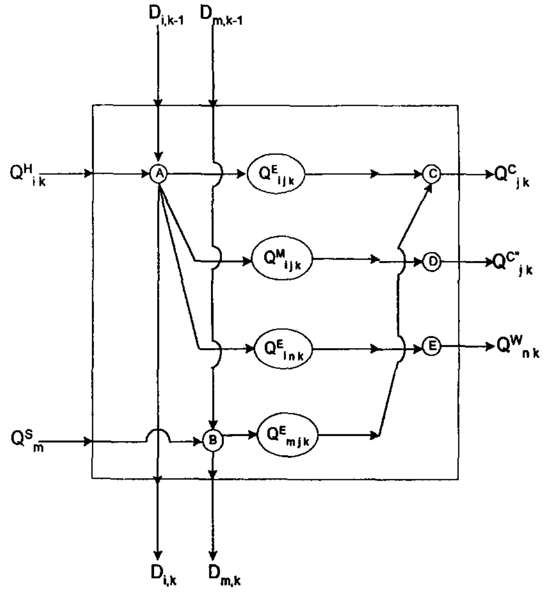

Fig. 2. Heat flows in interval k.

that all possible ways of heat exchanges via mixing can be represented by the heat flow pattern described in Fig. 2 and are feasible under the proposed temperature partition scheme. One can see that there are essentially two types of mixable hot streams, i.e. those associated with $D_{i,k-1}$ and with Q_{ik}^H. In the former case, the corresponding heat transfer can always be carried out by mixing since the temperature of hot stream is always higher than that of cold stream and their final temperatures are equal. In the latter case, heat exchanges between hot and cold streams within the *same* interval k are required. Although it may not be always possible to implement physically the heat exchange implied in the proposed model, one can always postulate a fictitious one which creates the same effects. Let us use a simple example to illustrate this point.

Example 1: Let us consider the stream data presented in Table 1(a). If mixing is the only means of heat exchange considered in this case, then the system can be partitioned into just one interval according to the proposed procedure. The maximum amount of heat that should be allowed to be exchanged in the transshipment model is

$$\min\{[3 \times (400 - 300)], [2 \times (400 - 300)]\} = 200 \text{ kW}.$$

On the surface, this is not possible since the temperature of two-stream mixture is always between 300 and 400 K. On the other hand, since streams 1 and 2 are mixable and should be identical in composition, the original problem can be viewed as an equivalent one presented in Table 1(b). In other words, stream 1 is split into streams 2′ and 3′ and stream 2 becomes stream 1′ in the new problem. The target temperatures of streams 1′ and 3′ are the same as that of stream 1. Also, the final temperature of stream 2′ is the same as

Table 1

Stream no.	F_{CP} (kW/K)	TS (K)	TT (K)
(a) *The original stream data of example* 1			
1	3	400	300
2	2	300	400
(b) *The equivalent stream data for example* 1			
1′	2	300	300
2′	2	400	400
3′	1	400	300

that of stream 2. It should be noted that, to achieve the final states specified in the original problem, only stream 3′ needs to be cooled from 400 to 300 K in the new formulation. In a sense, the heat exchange required in the transshipment model is accomplished by switching identities of the streams. The heat released from stream 3′ can be consumed with cold utilities or treated as residual heat if a temperature interval exists below 300 K.

From the above discussions, one can conclude that the heat flow pattern associated with mixers in the proposed transshipment model is really the same as that of heat exchangers which allow $\Delta T_{\min} = 0$.

THE MODIFIED TRANSSHIPMENT MODEL

The objective function ϕ of the modified transshipment model is the same as that of the original model, i.e.

$$\phi = \sum_{m \in \mathbf{S}} C_m Q_m^S + \sum_{n \in \mathbf{W}} C_n Q_n^W \tag{5}$$

where C_m and Q_m^S represent the cost coefficient (in \$/kW) and the consumption rate (in kW) of the mth hot utility, respectively, and C_n and Q_n^W denote the corresponding quantities of the nth cold utility.

Notice that energy balance associated with *every* hot and cold stream must be established. This requirement causes a problem in formulating the modified model, i.e. the symbols used to distinguish the fictitious flows are too cumbersome. For the sake of conciseness, the previously described fictitious streams are now relabeled with single indices, i.e.

$\mathbf{H}_l'' = \{i \mid i$ is the label of a fictitious hot stream in the lth group of mixable streams$\}$

and

$\mathbf{C}_l'' = \{j \mid j$ is the label of a fictitious cold stream in the lth group of mixable streams$\}$.

Notice that there is a one-to-one correspondence between the elements in set $\mathbf{H}_l''$ and those in $\mathbf{MH}_l$ and the same relationship also exists between $\mathbf{C}_l''$ and $\mathbf{MC}_l$.

Again, to simplify notation, all hot streams (real and fictitious) are combined and grouped into a set $\mathbf{H}$, i.e.

$$\mathbf{H} = \mathbf{H}' \cup \mathbf{H}_1'' \cup \mathbf{H}_2'' \cup \cdots \cup \mathbf{H}_{IM}''.$$

Similarly, all the cold streams can be collected in a set $\mathbf{C}$, i.e.

$$\mathbf{C} = \mathbf{C}' \cup \mathbf{C}_1'' \cup \mathbf{C}_2'' \cup \cdots \cup \mathbf{C}_{IM}''.$$

Since, in the transshipment model, the energy balance equations associated with *every* temperature interval are all used as equality constraints, it is necessary to define subsets of $\mathbf{H}$ and $\mathbf{C}$:

$$\mathbf{H}_k = \{i \mid i \in \mathbf{H} \text{ and } i \text{ is present in interval } \bar{k} \leqslant k\}$$

$$\mathbf{C}_k = \{j \mid j \in \mathbf{C} \text{ and } j \text{ is present in interval } k\}.$$

In addition, the sets $\mathbf{S}_k, \mathbf{W}_k, \mathbf{H}_k', \mathbf{C}_k', \mathbf{H}_{lk}''$ and $\mathbf{C}_{lk}''$ are defined according to the same convention in this study.

To establish the constraints of the transshipment model, energy balance equations associated with nodes A, B, C, D and E in Fig. 2 must be formulated, respectively. They are presented in the sequel.

Node A:

$$D_{ik} - D_{i,k-1} + \sum_{j \in \mathbf{C}_k} Q_{ijk}^E + \sum_{l \in \mathbf{M}} \sum_{j \in \mathbf{C}_l''} Q_{ijk}^M + \sum_{n \in \mathbf{W}_k} Q_{ink}^E = Q_{ik}^H, \quad i \in \mathbf{H}_k \tag{6}$$

Node B:

$$D_{mk} - D_{m,k-1} + \sum_{j \in \mathbf{C}_k} Q_{mjk}^E = Q_{mk}^S, \quad m \in \mathbf{S}_k \tag{7}$$

Node C:

$$\sum_{i \in \mathbf{H}_k} Q_{ijk}^E + \sum_{m \in \mathbf{S}_k} Q_{mjk}^E = Q_{jk}^C, \quad j \in \mathbf{C}_k \tag{8}$$

Node D:

$$\sum_{i \in \mathbf{H}_{lk}''} Q_{ijk}^M = Q_{jk}^{C''}, \quad j \in \mathbf{C}_{lk}'' \subset \mathbf{C}_k, \ l \in \mathbf{M} \tag{9}$$

Node E:

$$\sum_{i \in \mathbf{H}_k} Q_{ink}^E = Q_{nk}^W, \quad n \in \mathbf{W}_k \tag{10}$$

where, in the above equations, $k = 1, 2, \ldots, K$. Also, notice that the fourth term on the left-hand side of eq. (6), i.e. $\sum_{l \in M} \sum_{j \in C_l''} Q_{ijk}^M$, exists only when $i \in \mathbf{H}_{lk}'' \subset \mathbf{H}_k$.

Since there should be no residual heats entering the first and leaving the last temperature interval, the following equations should be augmented in the transshipment model:

$$D_{i0} = D_{iK} = D_{m0} = D_{mK} = 0. \tag{11}$$

The values of Q_{ik}^H in eq. (6) can be computed with

$$Q_{ik}^H = F_{CPik}^H \Delta T_k \tag{12}$$

where ΔT_k denotes the temperature difference between the upper and lower limits of the kth temperature interval. Notice that F_{CPik}^H represents the heat-capacity flow rate of the ith hot stream ($i \in \mathbf{H}$) *in* *interval k*. Since the symbols representing heat-capacity flow rates of all process and utility streams *within a given temperature interval* are written according to the same convention, their definitions will not be repeated later in this paper. If $i \in \mathbf{H}'$, then

$$F^H_{ik} = \begin{cases} F^{H'}_{CP_i}, & [T^H_k, T^H_{k-1}] \subset [T^{\mathrm{OUT}}_i, T^{\mathrm{IN}}_i] \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

where T^{IN}_i and T^{OUT}_i, respectively, represent the inlet and outlet temperatures of the ith hot stream. On the other hand, if $i \in \mathbf{H}''_l$, then

$$F^H_{ik} = \begin{cases} F^{H''_l}_{CP_i}, & [T^H_k, T^H_{k-1}] \subset [T^{\mathrm{OUT}}_i, T^{\mathrm{IN}}_i] \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

Similarly, the heat flows Q^C_{jk} in eq. (8) can be expressed as

$$Q^C_{jk} = F^C_{CP_{jk}} \Delta T_k. \quad (15)$$

If $j \in \mathbf{C}'$, the heat-capacity flow rate $F^C_{CP_{jk}}$ can be determined according to

$$F^C_{CP_{jk}} = \begin{cases} F^{C'}_{CP_j}, & [T^C_k, T^C_{k-1}] \subset [T^{\mathrm{IN}}_j, T^{\mathrm{OUT}}_j] \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

where T^{IN}_j and T^{OUT}_j, respectively, denote the inlet and outlet temperatures of the jth cold stream. On the other hand, if $j \in \mathbf{C}''_l$, then

$$Q^C_{jk} = \sum_{s \in \mathbf{I}^E_k} F^E_{CP_{js}} \Delta T_s. \quad (17)$$

Finally, the heat flows $Q^{C''}_{jk}$ in eq. (9) can be expressed with

$$Q^{C''}_{jk} = \sum_{s \in \mathbf{I}^M_k} F^M_{CP_{js}} \Delta T_s. \quad (18)$$

Notice also that the heat-capacity flow rates of all fictitious streams, i.e. $F^{H''_l}_{CP_i} F^E_{CP_{js}}$ and $F^M_{CP_{js}}$ are nonnegative unknowns and they have to satisfy the constraints specified in eqs (2)–(4). The other unknowns in this model are $D_{ik}, D_{mk}, Q^W_{nk}, Q^S_{mk}, Q^E_{ijk}, Q^M_{ijk}, Q^E_{mjk}$ and Q^E_{ink}. Naturally, their values also have to be greater than or equal to zero.

Example 2: This example is designed to illustrate the procedure for computing the minimum utility cost and demonstrate the benefit of incorporating mixing as a means of heat exchange in HEN. Let us consider the problem presented in Table 2. Notice that 'M' represents a group of mixable process streams in which the numbers of input and output streams are both two. On the other hand, the labels 'H' and 'C' denote, respectively, a hot and a cold process stream which is not allowed to be mixed with other streams.

On the basis of Fig. 1, four fictitious streams can be constructed. Among them, two are hot streams, i.e. $\mathbf{MH} = \{(1,1'),(1,2')\}$, and the rest are cold streams, i.e. $\mathbf{MC} = \{(2,1'),(2,2')\}$. For convenience, let us relabel all process streams as

$$\{(1,1'),(1,2'),H\} \Rightarrow \mathbf{H} = \{H1, H2, H3\}$$

and

$$\{(2,1'),(2,2'),C\} \Rightarrow \mathbf{C} = \{C1, C2, C3\}.$$

Then the entire system can be partitioned according to the proposed procedure (see Fig. 3). One can clearly see that, as a result of this partitioning procedure, two different cold-stream temperature ranges are included in the same interval k. Notice that the two sets of the interval temperatures, T^M_k and T^C_k, may be inconsistent over the whole temperature range of the fictitious cold streams. In the present example, an additional T^M_k, i.e. 60°C, must be inserted to create two subintervals in the 9th interval. This is necessary since the fictitious cold streams appear in both types of cold-stream temperature intervals and the constraint of mass conservation must be imposed in the LP model over the same temperature ranges.

Finally, a transshipment model can be formulated according to Fig. 2. This LP model has been solved with the default solver of GAMS (Brooke *et al.*, 1992). The minimum consumption rates of the hot and cold utilities are 1150 and 80 kW, respectively. If the option of mixing is not considered in design, then the streams $(1,1')$ and $(2,2')$ can be regarded as two separate streams which are not allowed to be merged. In such case, the corresponding utility targets should be 1500 (hot) and 430 (cold) kW, respectively.

THE OPTIMAL MATCHES IN NETWORK SYNTHESIS

To synthesize a minimum-cost HEN with mathematical programming techniques, it is necessary to determine first the optimal matches for use in the NLP model, i.e. the minimum number of matches, the hot and cold streams involved in every match and the corresponding heat duty in each exchanger or mixer. If mixing is not allowed between streams coming from different units, a MILP model has already been developed, i.e. Papoulias and Grossmann (1983), for the above purpose. If, however, stream-merging can be considered in HEN design as a means of heat exchange, then additional heat exchangers can be eliminated and thus the capital investment can be lowered significantly.

In order to illustrate the modifications introduced in this study precisely, the following stream sets must be defined:

$$\hat{\mathbf{H}}' = \{i \mid i \in \mathbf{H}' \text{ or } i \in \mathbf{S}\}$$

$$\hat{\mathbf{C}}' = \{j \mid j \in \mathbf{C}' \text{ or } j \in \mathbf{W}\}$$

$$\hat{\mathbf{H}} = \{\mu \mid \mu \in \hat{\mathbf{H}}' \text{ or } \mu \in \mathbf{M}\}$$

$$\hat{\mathbf{C}} = \{\nu \mid \nu \in \hat{\mathbf{C}}' \text{ or } \nu \in \mathbf{M}\}.$$

The modified version of the MILP model is very similar to the original one. The objective function Φ in this case is the total number of *exchangers*, i.e.

$$\Phi = \sum_{\mu \in \hat{\mathbf{H}}} \sum_{\nu \in \hat{\mathbf{C}}} y_{\mu\nu} \quad (19)$$

Table 2. The stream data of example 2 ($\Delta T_{min} = 60°C$)

Group label	Inlet no.	F_{CP}^{in} (kW/°C)	T_{in} (°C)	Outlet no.	F_{CP}^{out} (kW/°C)	T_{out} (°C)
M	1	7	200	1′	7	50
	2	40	40	2′	40	80
H		8	250		8	90
C		15	60		15	180

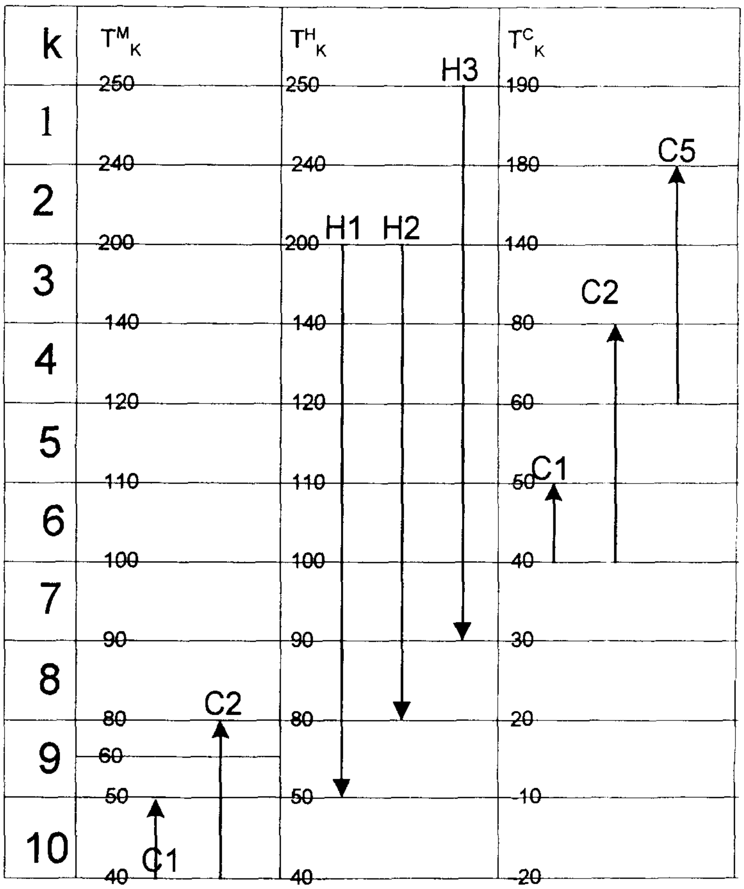

Fig. 3. Two-scale temperature partition in example 2.

where

$$y_{\mu\nu} = \begin{cases} 1 & \text{there exists a match between hot stream } \mu \in \hat{\mathbf{H}} \text{ and cold stream } \nu \in \hat{\mathbf{C}} \\ 0 & \text{otherwise.} \end{cases}$$

It should be noted that, although both sets $\hat{\mathbf{H}}$ and $\hat{\mathbf{C}}$ contain mixable streams, a match between the streams in these two sets is counted only when the corresponding heat exchange is carried out in an exchanger. The values of binary variables $y_{\mu\nu}$ are thus controlled with the following inequality constraints:

$$E_{\mu\nu} - y_{\mu\nu} U \leqslant 0, \quad \mu \in \hat{\mathbf{H}}, \ \nu \in \hat{\mathbf{C}} \tag{20}$$

where U is a large number and $E_{\mu\nu}$ denotes the amount of heat exchanged between hot stream μ and cold stream ν in an exchanger. The values of $E_{\mu\nu}$ can be determined by

$$E_{\mu\nu} = \sum_{k=1}^{K} Q^E_{\mu\nu k}, \quad \mu \in \hat{\mathbf{H}}', \ \nu \in \hat{\mathbf{C}}' \tag{21a}$$

$$E_{\mu\nu} = \sum_{k=1}^{K} \sum_{j \in C_\nu'} Q^E_{\mu jk}, \quad \mu \in \hat{\mathbf{H}}', \ \nu \in \mathbf{M} \tag{21b}$$

$$E_{\mu\nu} = \sum_{k=1}^{K} \sum_{i \in H_\mu'} Q^E_{i\nu k}, \quad \mu \in \mathbf{M}, \ \nu \in \hat{\mathbf{C}}' \tag{21c}$$

$$E_{\mu\nu} = \sum_{k=1}^{K} \sum_{i \in H_\mu'} \sum_{j \in C_\nu'} Q^E_{ijk}, \quad \mu \in \mathbf{M}, \ \nu \in \mathbf{M}. \tag{21d}$$

Table 3. The optimal matches of example 3

Match	Duty (kW)
(H, M)	550
(H, C)	650
(S, C)	1150
(H, W)	80
(M, M)	1050

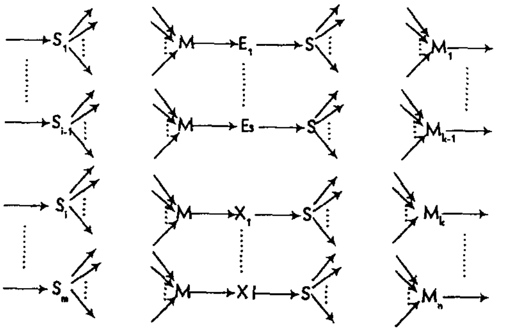

Fig. 4. The generalized stream configuration in a heat recovery system.

Notice that, on the basis of eqs (19)–(21), it is clear that the number of mixers in the HEN are *not included* in Φ.

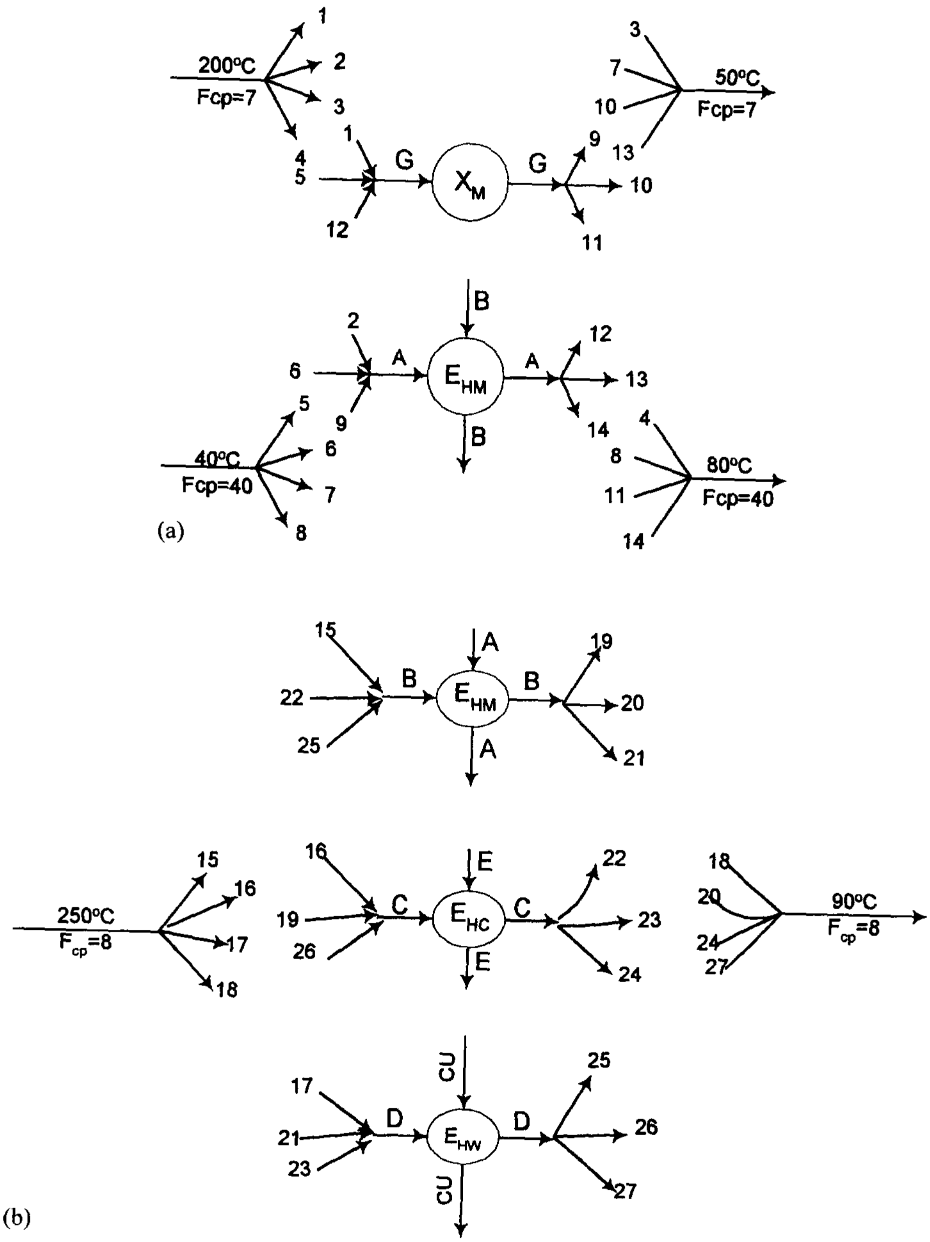

Fig. 5. The general stream structure for (a) mixable stream M in example 4, (b) hot stream H in example 4.

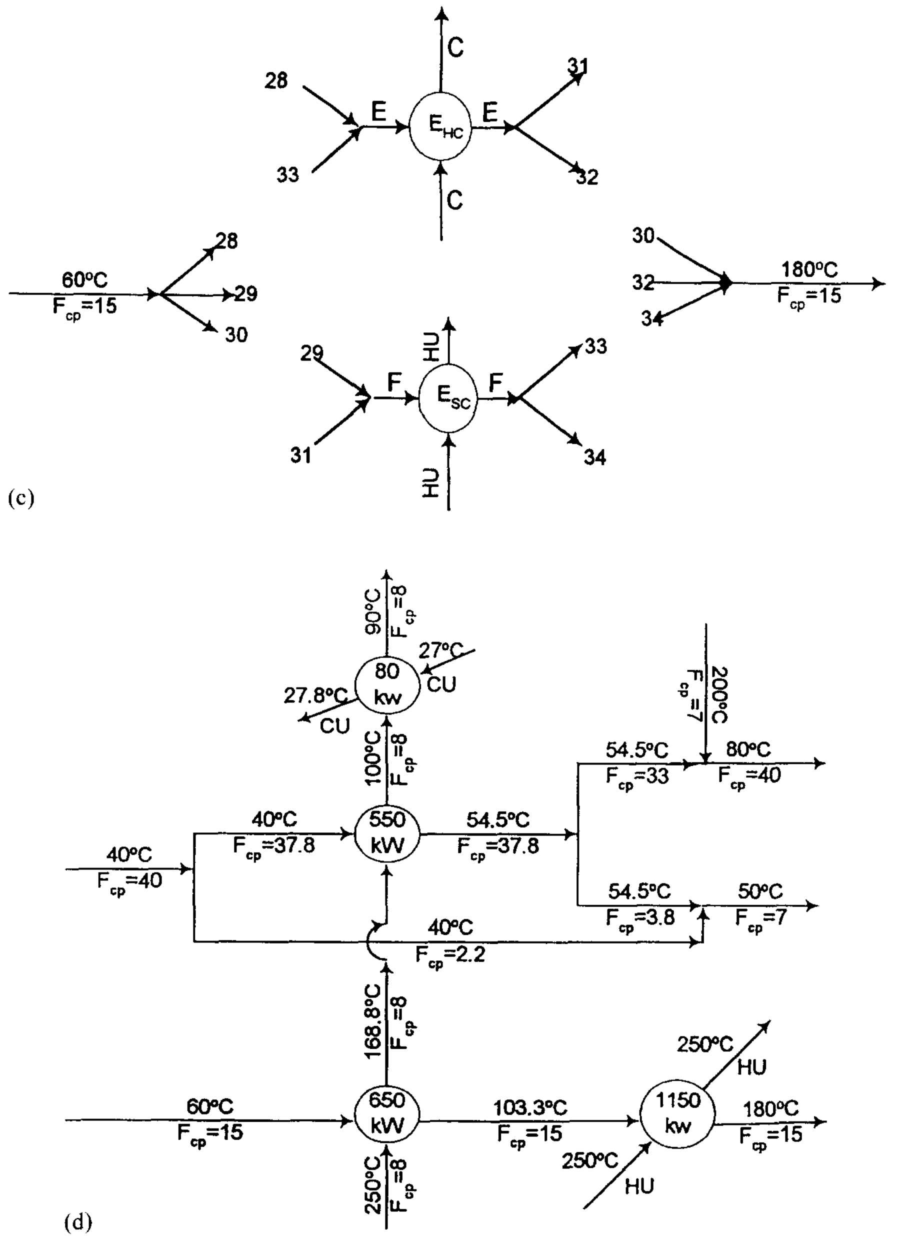

Fig. 5. (c) Cold stream C in example 4, and (d) the optimal network structure in example 4.

Finally, the energy balance equations associated with all temperature intervals should be adopted as the model constraints. These constraints are essentially the same as those used in the modified transshipment model, i.e. eqs (6)–(10).

Example 3: Let us consider the problem presented in example 2. The results obtained by solving the modified MILP model are listed in Table 3. Notice that the symbols 'S' and 'W' denote hot and cold utility, respectively. Notice also that, since the match between mixable streams, i.e. (M, M), is a mixer in the solution, the number of exchangers should therefore be 4. On the other hand, if stream merging is not allowed in design, the same problem can be solved with a traditional MILP model. It has been found that the number of heat exchangers in this latter case should be at least 6.

THE OPTIMAL NETWORK STRUCTURE

To integrate all options of merging and splitting process streams into the HEN design strategy, a general stream structure has been developed in this study. It is essentially a modified version of the superstructure derived by Floudas *et al.* (1986) and Chang *et al.* (1994). The difference between the present structure and the one developed by Chang *et al.* (1994) is that mixers can be included as an alternative means of heat

Table 4. The stream data of the additional example ($\Delta T_{min} = 20$ K)

Group label	Inlet no.	F_{CP}^{in} (kW/K)	T_{in} (K)	Outlet no.	F_{CP}^{out} (kW/K)	T_{out} (K)
M1	1	19	405.0	1′	19	333
	2	23	283.0	2′	23	366
M2	3	10	353.6	3′	18	305
	4	8	353.0			
H1	5	35	382.0	5	35	291
H2	6	6137	354.1	6	6137	353.6
C1	7	100	283.0	7	100	366

exchange. Specifically, corresponding to each group $l \in \mathbf{M}$ of mixable streams, it can be constructed according to the following procedure.

1. List all the exchangers and mixers between this group of streams and other streams according to the matches determined with the modified MILP model.
2. Place a mixing point on each output stream in the group, also one at the inlet of every exchanger and one before every mixer listed in step 1.
3. Place a splitting point on each input stream, also one at the exit of every exchanger and one after every mixer listed in Step 1. The split branches of every splitting point are connected to all previously mentioned mixing points.

This scheme can be represented by Fig. 4, in which the symbol S_i denotes the splitting point on the ith input stream, M_k denotes the mixing point on the kth output stream, $M - E_e - S$ represents the eth exchanger and the mixing and splitting points attached before and after this exchanger and, similarly, $M - X_m - S$ represents the mth mixer and the attached mixing and splitting points. All possible configurations in connecting multiple process streams are imbedded in this scheme, e.g. stream split, bypass, matches in series, matches in parallel, matches in series-parallel, etc. Notice that this structure is also

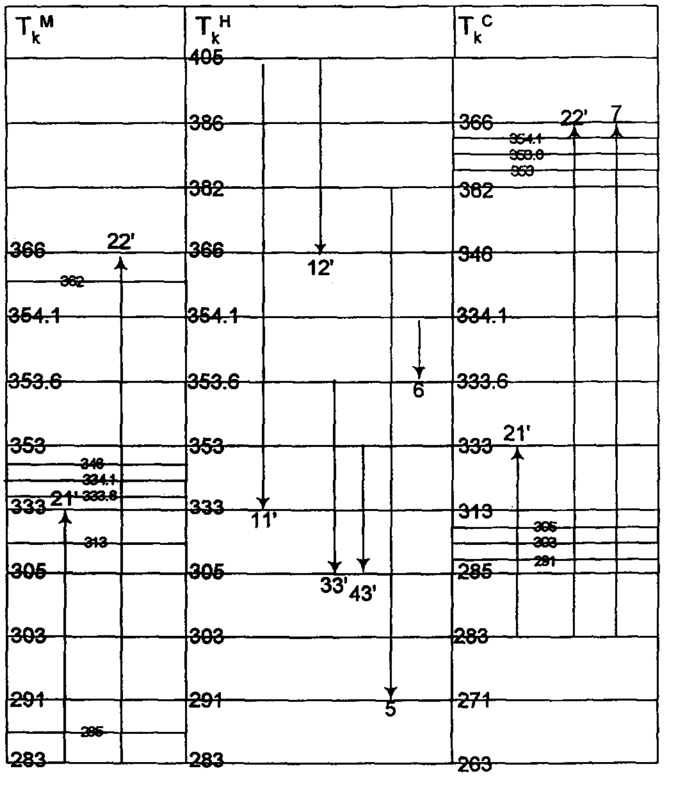

Fig. 6. Two-scale temperature partition in the additional example.

Table 5. The optimal matches of the additional example

Match	Duty (kW)
(M1, M1)	540.3
(M1, C1)	329.0
(M2, M1)	870.0
(H1, C1)	2765.0
(H1, W)	420.0
(H2, C1)	3158.5
(S, C1)	2047.5

suitable for streams that are not allowed to be mixed with other streams. In such case, there are no mixers and only one input and one output stream and it reduces to the superstructure suggested by Floudas *et al.* (1986).

Having constructed the general stream structure for a given problem, a NLP model can then be formulated to determine a cost-optimal HEN. The constraints of such a model are simply the material and energy balance equations associated with all exchangers, mixers and mixing and splitting points in this structure. For the sake of brevity, the detailed model descriptions are omitted in this paper.

Example 4: This example is a continuation of the previous two. By following the proposed procedure, three general stream structures can be constructed for the streams listed in Table 2, i.e. M, H and C, according to the matches determined in example 3. The corresponding general stream structures are presented in Figs 5(a), (b) and (c), respectively. Notice that, for clarity, the connections among the inlet splitting points, exit mixing points, exchangers and mixers are not shown explicitly. However, the actual configurations of these general stream structures can be easily identified from the stream numbers assigned to the outputs of the splitting points and the inputs of the mixing points. Also, the process streams involved a match are labeled with letters A–G and the hot and

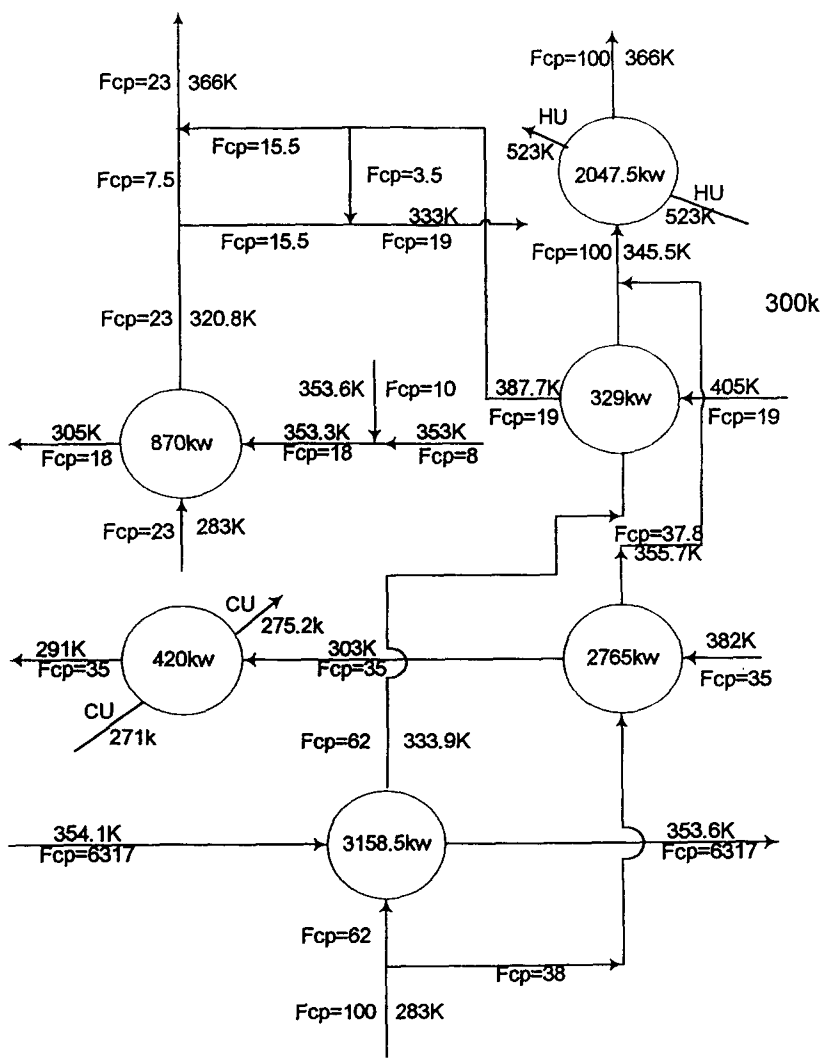

Fig. 7. The optimal network structure in the additional example.

cold utilities are denoted by HU and CU, respectively. On the basis of these structures, a NLP can be formulated and solved with the default solver of GAMS. The resulting network is presented in Fig. 5(d).

AN ADDITIONAL EXAMPLE

To demonstrate the effectiveness of the proposed approach, a more complicated example problem is presented here. Let us consider the stream data listed in Table 4. Based on the proposed solution procedure, this problem can be solved in three steps:

- The minimum utility consumption rates: the system should be first divided into several temperature intervals with the two-scale temperature partition procedure (see Fig. 6). Then, the modified transshipment model can be formulated accordingly. The solutions were obtained with GAMS and the minimum consumption rates of hot and cold utilities were found to be 2047.5 and 420 kW, respectively.
- The optimal matches in network synthesis: the modified MILP model was also solved with GAMS. The results are presented in Table 5.
- The optimal network structure: assuming that the temperature of the hot utility is constant at 523 K and the temperature of the cold utility varies between 271 and 272.2 K in every cooler, a NLP model can be formulated to minimize the annualized capital cost of the network. This model was again solved with GAMS. The resulting optimal network structure is presented in Fig. 7.

CONCLUSIONS

In order to incorporate mixing as a means of heat or mass exchange in heat recovery system design, modified LP, MILP and NLP models for network synthesis have been developed in this study. These models were formulated on the basis of a series of new concepts, i.e. the input–output system structure, the fictitious stream, the two-scale temperature partition scheme, and the generalized stream structure. From the results we have obtained in applications, it can be concluded that the proposed approach can be used to cut down not only capital investments but also utility costs of heat recovery systems.

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