Pergamon

Chemical Engineering Science, Vol. 53, No. 22, pp. 3853–3862, 1998

Printed in Great Britain
PII: S0009-2509(98)00208-5
0009-2509/98/$ See front matter

Simplification techniques for EKF computations in fault diagnosis—suboptimal gains

Chuei-Tin Chang* and Jung-Ing Hwang
Department of Chemical Engineering, National Cheng Kung University, Tainan, Taiwan 70101, Republic of China

(Received 9 February 1998; in revised form 4 June 1998; accepted 13 July 1998)

Abstract—The extended Kalman filter (EKF) is one of the most popular model-based techniques for fault detection and diagnosis. In the present study, the idea of suboptimal EKF is utilized to enhance computation efficiency without sacrificing diagnostic accuracy. In particular, a simple procedure is developed to decompose the filter model according to the precedence order of the state/parameter estimation process. As a result, a large portion of the computations needed for propagating error covariances and updating state estimates can be neglected. Extensive simulation results are also presented to demonstrate the effectiveness of these proposed techniques. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: Fault diagnosis; extended Kalman filter; suboptimal gains.

INTRODUCTION

Due to the frequency and seriousness of chemical accidents in recent years, the importance of incipient fault detection and diagnosis in complex process plants has become apparent. Among various different model-based approaches adopted in the past, the extented Kalman filter (EKF) is clearly one of the most popular methods, e.g. Watanabe and Himmelblau (1983a, b; 1984). In essence, EKFs of one form or another were employed to estimate both the states and parameters of chemical engineering systems and, then, causes of abnormal system behaviors were identified accordingly.

Although the effectiveness of EKFs has been widely recognized, its use in commercial units was in fact very limited. This is mainly due to a critical drawback of EKF, i.e. its inability to guarantee unbiased estimates (Watanabe and Himmelblau, 1984). Obviously, incorrect information about the system parameters and/or states can mislead diagnosis. To overcome this problem, we modified the traditional way in implementing the extended Kalman filters (Chang *et al.*, 1993; Chang and Chen, 1995). Instead of estimating all parameters simultaneously in a large EKF, several EKFs were used in parallel. Although this approach served the purpose of eliminating bias and misdiagnosis, the computational effort needed to carry out the parallel parameter estimation scheme can be overwhelming.

This article is one of the two companion papers in which potential methods for reducing the computation load required in implementing EKF-based fault diagnostic algorithms are discussed. In essence, the idea of suboptimal Kalman filter is utilized in our studies to enhance computation efficiency without sacrificing diagnostic performance. Generally speaking, there are two approaches which can be adopted to achieve this purpose, i.e. (1) choosing simplified system models and (2) choosing suboptimal filter gains (Gelb, 1974). For illustration convenience, the former method is discussed in detail in a separate paper (Chang and Hwang, 1998). The present article is primarily concerned with the latter. Specifically, the lower-order suboptimal filter suggested by Oh *et al.* (1996) has been improved to simplify the calculation procedure involved in estimating states and parameters simultaneously with EKFs. To facilitate implementation of this technique, a systematic method has been developed to classify the state variables and parameters on the basis of system structure and then partition the gain matrix and error covariance matrix accordingly. The effectiveness of the proposed method has been verified with extensive numerical simulation studies.

SIMPLIFICATION OF FILTER GAINS FOR A SPECIAL CLASS OF SYSTEMS

Let us consider a special class of nonlinear systems which can be described with the following

*Corresponding author. Fax: 886-6-234-4496; e-mail: ctchang@mail.ncku.edu.tw.

equations:

$$\frac{d\mathbf{x}_1(t)}{dt} = \mathbf{f}_1(\mathbf{x}_1(t), \mathbf{x}_2(t), t) \tag{1}$$

$$\frac{d\mathbf{x}_2(t)}{dt} = \mathbf{f}_2(\mathbf{x}_1(t), \mathbf{x}_2(t), t) + \omega_2(t) \tag{2}$$

where $\mathbf{f}_1$ and $\mathbf{f}_2$ both denote vectors of nonlinear functions of the states $\mathbf{x}_1 \in \mathbf{R}^{N-r}$ and $\mathbf{x}_2 \in \mathbf{R}^r$. It is assumed that the variations in initial states of this system are negligibly small and thus $\mathbf{x}_1(0)$ and $\mathbf{x}_2(0)$ can be regarded as exact quantities. Notice also that ω_2 is a zero mean Gaussian noise vector and the corresponding covariance matrix is $\mathbf{Q}_2$, i.e.

$$\omega_2 \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_2). \tag{3}$$

The measurement model adopted in this work is

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_1(t_k), \mathbf{x}_2(t_k), t_k) + v_k, \quad k = 0, 1, 2, \cdots \tag{4}$$

where $\mathbf{h}$ is also a vector of nonlinear functions, and $\mathbf{z}_k$ and v_k denote, respectively, the system output vector and measurement noise vector at time t_k. The measurement noises are again assumed to be normally distributed with zero means, i.e.

$$v_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \tag{5}$$

where $\mathbf{R}$ is the covariance matrix associated with v_k.

The continuous-discrete EKF for the above system can be written as:

- State estimate propagation

$$\frac{d\hat{\mathbf{x}}}{dt} = \mathbf{f}(\hat{\mathbf{x}}(t), t) \tag{6}$$

- Error covariance propagation

$$\dot{\mathbf{P}} = \mathbf{A}(\hat{\mathbf{x}}, t)\,\mathbf{P} + \mathbf{P}\mathbf{A}^{\mathrm{T}}(\hat{\mathbf{x}}, t) + \mathbf{Q} \tag{7}$$

- Gain matrix

$$\mathbf{K}_k = \mathbf{P}^-(t_k)\mathbf{H}_k^{\mathrm{T}}(\hat{\mathbf{x}}^-(t_k))[\mathbf{H}_k(\hat{\mathbf{x}}^-(t_k))\mathbf{P}^-(t_k) \times \mathbf{H}_k^{\mathrm{T}}(\hat{\mathbf{x}}^-(t_k)) + \mathbf{R}]^{-1} \tag{8}$$

- State estimate update

$$\hat{\mathbf{x}}^+(t_k) = \hat{\mathbf{x}}^-(t_k) + \mathbf{K}_k[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}^-(t_k))] \tag{9}$$

- Error covariance update

$$\mathbf{P}^+(t_k) = [\mathbf{I} - \mathbf{K}_k\mathbf{H}_k(\hat{\mathbf{x}}^-(t_k))]\mathbf{P}^-(t_k) \tag{10}$$

where

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_1 \\ \hat{\mathbf{x}}_2 \end{bmatrix} \tag{11}$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} \tag{12}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \tag{13}$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix} \tag{14}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \tag{15}$$

$$\mathbf{H}_k = [\mathbf{H}_{1k}\mathbf{H}_{2k}] \tag{16}$$

and

$$\mathbf{A}_{ij} = \frac{\partial \mathbf{f}_i(\hat{\mathbf{x}}(t))}{\partial \hat{\mathbf{x}}_j(t)}, \quad i, j = 1, 2 \tag{17}$$

$$\mathbf{H}_{jk} = \frac{\partial \mathbf{h}(\hat{\mathbf{x}}^-(t_k))}{\partial \hat{\mathbf{x}}_j^-(t_k)}, \quad j = 1, 2. \tag{18}$$

If the members of $\mathbf{f}_1$ are only functions of $\mathbf{x}_1$, i.e. $\mathbf{f}_1 = \mathbf{f}_1(\mathbf{x}_1(t), t)$, then

$$\mathbf{A}_{12} = \mathbf{0} \tag{19}$$

Consequently, eqs (7) can be written as

$$\dot{\mathbf{P}}_{11} = \mathbf{A}_{11}\mathbf{P}_{11} + \mathbf{P}_{11}\mathbf{A}_{11}^{\mathrm{T}} \tag{20}$$

$$\dot{\mathbf{P}}_{12} = \mathbf{A}_{11}\mathbf{P}_{12} + \mathbf{P}_{11}\mathbf{A}_{21}^{\mathrm{T}} + \mathbf{P}_{12}\mathbf{A}_{22}^{\mathrm{T}} \tag{21}$$

$$\dot{\mathbf{P}}_{21} = \mathbf{A}_{21}\mathbf{P}_{11} + \mathbf{A}_{22}\mathbf{P}_{21} + \mathbf{P}_{21}\mathbf{A}_{11}^{\mathrm{T}} \tag{22}$$

$$\dot{\mathbf{P}}_{22} = \mathbf{A}_{21}\mathbf{P}_{12} + \mathbf{A}_{22}\mathbf{P}_{22} + \mathbf{P}_{21}\mathbf{A}_{21}^{\mathrm{T}} + \mathbf{P}_{22}\mathbf{A}_{22}^{\mathrm{T}} + \mathbf{Q}_2. \tag{23}$$

Let us further assume that the estimates of EKF are exact initially, i.e.

$$\mathbf{P}_{11}(t_0) = \mathbf{P}_{12}(t_0) = \mathbf{P}_{21}(t_0) = \mathbf{P}_{22}(t_0) = \mathbf{0}. \tag{24}$$

From the above assumption and eqs (20)–(23), one can see that all elements in the error covariance matrix at the next time step should still be zero except those in $\mathbf{P}_{22}$. In other words,

$$\mathbf{P}^-(t_k) = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{22}^-(t_k) \end{bmatrix}, \quad k = 1. \tag{25}$$

The gain matrix at the next time step can therefore be computed according to eq. (8):

$$\mathbf{K}_k = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{22}^-(t_k) \end{bmatrix} \begin{bmatrix} \mathbf{H}_{1k}^{\mathrm{T}} \\ \mathbf{H}_{2k}^{\mathrm{T}} \end{bmatrix} \times \left\{ [\mathbf{H}_{1k}\mathbf{H}_{2k}] \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{22}^-(t_k) \end{bmatrix} \begin{bmatrix} \mathbf{H}_{1k}^{\mathrm{T}} \\ \mathbf{H}_{2k}^{\mathrm{T}} \end{bmatrix} + \mathbf{R} \right\}^{-1} = \begin{bmatrix} \mathbf{0} \\ \mathbf{P}_{22}^-(t_k)\ \mathbf{H}_{2k}^{\mathrm{T}}[\mathbf{H}_{2k}\mathbf{P}_{22}^-(t_k)\ \mathbf{H}_{2k}^{\mathrm{T}} + \mathbf{R}]^{-1} \end{bmatrix}, \quad k = 1. \tag{26}$$

For convenience, let

$$\mathbf{K}_k = \begin{bmatrix} \mathbf{K}_{1k} \\ \mathbf{K}_{2k} \end{bmatrix} \tag{27}$$

and

$$\mathbf{K}_{1k} = \mathbf{0} \tag{28}$$

$$\mathbf{K}_{2k} = \mathbf{P}_{22}^{-}(t_k)\,\mathbf{H}_{2k}^{\mathrm{T}}[\mathbf{H}_{2k}\mathbf{P}_{22}^{-}(t_k)\,\mathbf{H}_{2k}^{\mathrm{T}} + \mathbf{R}]^{-1}. \tag{29}$$

Thus, the updated estimate of the error covariance matrix can be obtained on the basis of eq. (10):

$$\mathbf{P}^{+}(t_k) = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & [\mathbf{I}_r - \mathbf{K}_{2k}\mathbf{H}_{2k}]\mathbf{P}_{22}^{-}(t_k) \end{bmatrix}, \quad k = 1 \tag{30}$$

where $\mathbf{I}_r$ is a $r \times r$ identity matrix.

From the above discussions, it is clear that eqs (25), (26) and (30) should also be valid when $k = 2, 3, \ldots$. Thus, the optimal estimates of $\mathbf{x}_1$ can be obtained with an alternative set of equations:

$$\hat{\mathbf{x}}_1^{+}(t_k) = \hat{\mathbf{x}}_1^{-}(t_k) \tag{31}$$

$$\frac{\mathrm{d}\hat{\mathbf{x}}_1^{-}}{\mathrm{d}t} = \mathbf{f}_1(\hat{\mathbf{x}}_1^{-}(t), t). \tag{32}$$

It should be noted that the system model under consideration, i.e. eq. (1), is not always consistent with the assumption used in the above derivation. However, even when $\mathbf{A}_{12} \neq 0$, it is still possible to select $\mathbf{x}_1$ and $\mathbf{x}_2$ and artificially assign

$$\mathbf{P}_{11}^{-}(t_k) = \mathbf{P}_{12}^{-}(t_k) = \mathbf{P}_{21}^{-}(t_k) = \mathbf{0}, \quad k = 1, 2, 3, \ldots . \tag{33}$$

Consequently, $\mathbf{K}_{1k} = \mathbf{0}$ and the corresponding computations can be neglected. For completeness, the resulting suboptimal EKF is summarized in Appendix A. Oh *et al.* (1996) has demostrated on a *trial-and-error* basis that this approach is feasible if the system model can be decomposed *properly* into two parts, i.e. one is associated with the states in $\mathbf{x}_1$ and the other with $\mathbf{x}_2$. However, such a decomposition procedure has never been specifically outlined in the past.

THE SUBOPTIMAL EKFS FOR FAULT DIAGNOSIS

If an extended Kalman filter is adopted for the purpose of fault diagnosis, it is usually used to estimate both states and *parameters* of the system model. This is because parameter estimates are in general more sensitive to faults than those of the state variables and thus they are better indications of the degradation of system performance. Since it is usually possible to associate the assumed malfunctions with changes in the corresponding model parameters, these parameters can be treated as augmented states in the corresponding EKF (Himmelblau, 1978). Specifically, let us consider a system model with the following general form:

$$\frac{\mathrm{d}\bar{\mathbf{x}}}{\mathrm{d}t} = \bar{\mathbf{f}}(\bar{\mathbf{x}}, \theta, t) + \omega_{\bar{x}}, \quad \omega_{\bar{x}} \sim \mathcal{N}(\mathbf{0}, \bar{\mathbf{Q}}) \tag{34}$$

where $\bar{\mathbf{x}}$ is an n-dimensional vector of the state variables and θ is an m-dimensional vector of the parameters and inputs of the system, $\bar{\mathbf{f}}$ is a vector of nonlinear functions of $\bar{\mathbf{x}}$ and θ, $\omega_{\bar{x}}$ represents a vector of normally distributed random system noises and $\bar{\mathbf{Q}}$ is the corresponding covariance matrix. Note that $n + m = N$ in this study.

In order to estimate the time-variant parameters and/or inputs in an EKF, one can treat them as state variables and augment the corresponding equations with eq. (34), i.e.

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \bar{x} \\ \theta \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{f}} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \omega_{\bar{x}} \\ \omega_{\theta} \end{bmatrix} = \mathbf{f} + \omega, \quad \omega \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \tag{35}$$

where ω_θ is a random vector with mean zero. For the sake of convenience, the components in ω are assumed to be independent and thus $\mathbf{Q}$ is a diagonal matrix. Also, without loss of generality, it is assumed in this study that the first s state variables can be measured directly. In other words, the measurement model can be written as

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k = [\mathbf{I}_s \mid \mathbf{0}]\mathbf{x}_k + \mathbf{v}_k, \ \mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \tag{36}$$

where $\mathbf{z}_k$, $\mathbf{x}_k$ and $\mathbf{v}_k$ are the system output vector, augmented state vector and measurement noise vector, respectively, at time t_k, $\mathbf{I}_s$ is an $s \times s$ identity matrix. Also, $\mathbf{R}$ is assumed to be a diagonal matrix in this study.

As mentioned before, it is generally believed that the model parameters are more sensitive (than the state variables) to incipient faults (Watanabe and Himmelblau, 1984). If this is the case, it is clear that the parameters θ in eq. (35) should be treated as members of $\mathbf{x}_2$ in a suboptimal EKF (see Appendix A). In addition, since the requirement of $\mathbf{A}_{12} = \mathbf{0}$ implies that $\mathbf{f}_1$ is not a function of $\mathbf{x}_2$, the state estimate propagation equations in the suboptimal EKF, i.e. eqs (A1) and (A2) in Appendix A, can actually be replaced with a reduced model in the following form:

$$\frac{\mathrm{d}\hat{\mathbf{x}}_1}{\mathrm{d}t} = \mathbf{f}_2(\hat{\mathbf{x}}_1(t), \mathbf{u}(t), t) \tag{37}$$

$$\frac{\mathrm{d}\hat{\mathbf{x}}_2}{\mathrm{d}t} = \mathbf{f}_2(\hat{\mathbf{x}}_1(t), \hat{\mathbf{x}}_2(t), t) \tag{38}$$

where $\mathbf{u}$ denotes a vector of known inputs. Therefore, if correct estimates of $\mathbf{x}_2$ can somehow be obtained and substituted into eq. (37), then the results of implementing the suboptimal EKF should also be correct.

From the above analysis, one can see that, should a suboptimal EKF be used for fault diagnosis, correct estimates of the parameters and also other members in $\mathbf{x}_2$ must be obtained with the reduced model described in eqs (37) and (38). In other words, the reduced system must be *fault observable*. The definition of fault observability and a procedure for identifying observable systems have already been detailed in an earlier paper (Chang and Chen, 1995). For the sake of

brevity, these materials are not included here. Following is a systematic method for selecting $\mathbf{x}_1$ and $\mathbf{x}_2$ from the parameters and state variables of the system model:

1. Obtain the precedence diagram of the original system, i.e. eqs (A1) and (A2), with *Algorithm B*. This algorithm can be found in Appendix B.
2. Identify a path which is initiated at a parameter and terminated at a measurement variable.
3. Repeat step 2 until all parameters are exhausted. Note that the terminating node of these path must be different.
4. The parameters and state variables on the above paths should be selected as members of $\mathbf{x}_2$. The other state variables should be treated as members of $\mathbf{x}_1$.

To illustrate this decomposition procedure and demostrate the effectiveness of the proposed approach, two simple examples are presented below:

Example 1. Let us consider the system of two storage tanks connected in series (see Fig. 1). The corresponding system model can be written as:

$$\phi_1: A_1 \frac{dh_1}{dt} = q_i - q_1 \tag{39}$$

$$\phi_2: \frac{dq_1}{dt} = \frac{\pi d_1^2}{4\rho l_1}\left[\rho g h_1 - \frac{8(f_1 + \Delta f_1) l_1 \rho q_1 |q_1|}{\pi^2 d_1^5}\right] \tag{40}$$

$$\phi_3: A_2 \frac{dh_2}{dt} = q_1 - q_2 \tag{41}$$

$$\phi_4: \frac{dq_2}{dt} = \frac{\pi d_2^2}{4\rho l_2}\left[\rho g h_2 - \frac{8 f_2 l_2 \rho q_2 |q_2|}{\pi^2 d_2^5}\right] \tag{42}$$

where ρ is the density of liquid, h_ℓ and A_ℓ denote, respectively, the height of liquid level in and the cross-sectional area of tank ℓ ($\ell = 1, 2$), q_ℓ, d_ℓ, l_ℓ and f_ℓ represent, respectively, the volumetric flow rate in and the diameter, length and friction factor of the outlet pipeline from tank ℓ. Also note that the parameter Δf_1 is associated with the assumed failure, i.e. partial blockage in the pipeline between tank 1 and tank 2. According to eq. (35), the following equation must be augmented with the system model in EKF:

$$\phi_5: \frac{d\Delta f_1}{dt} = 0 \tag{43}$$

In this example, h_1 and q_2 are selected as the measurement variables. The corresponding precedence order (Fig. 2) can be determined with *algorithm B*. From Fig. 2, it is clear that there are two paths initiated at Δf_1. One path is terminated at h_1 and the other at q_2. In order to reduce computation load to the lowest level, the shorter path is chosen in this case.

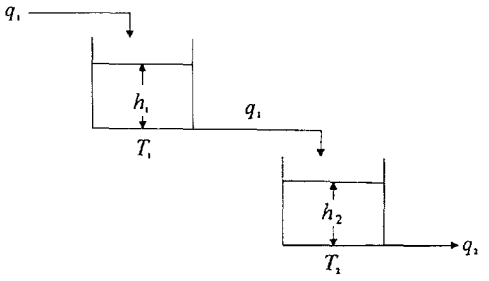

Fig. 1. The simplified process flow diagram of a two-tank system.

In other words,

$$\mathbf{x}_1 = \begin{bmatrix} h_2 \\ q_2 \end{bmatrix} \tag{44}$$

$$\mathbf{x}_2 = \begin{bmatrix} h_1 \\ q_1 \\ \Delta f_1 \end{bmatrix}. \tag{45}$$

Numerical simulation studies were carried out to verify the correctness of EKF estimates. It was assumed that the system was operated at its normal steady state initially and the above-mentioned fault occurred at $t = 50$. More specifically, the change in Δf_1 was described as

$$\Delta f_1 = C_f\{1 - \exp - \alpha(t - 50)\}u(t - 50) \tag{46}$$

where C_f and α are constants and u is a unit step function.

Equations (39)–(43) and (46) were integrated together to produce the transient behavior of the state variables. The measurement noises of h_1 and q_2 were produced with a random number generator and then added to the simulated values of these two variables to obtain the simulated on-line measurements.

The corresponding suboptimal EKF was then applied to the simulated measurement data. From the results of extensive simulation studies, we found that it is always possible to obtain correct estimates of both the parameter and states. A sample of the EKF estimates is presented in Fig. 3.

In order to carry out the computations indicated in the state estimate and error covariance propagation equations of an EKF, a set of nonlinear ordinary differential equations must be integrated numerically. The benefit of adopting the suboptimal EKF is demonstrated here by counting the number of differential equations that are required to be integrated numerically. This number N_{eq} can be computed by

$$N_{eq} = \frac{r(r+1)}{2} + n \tag{47}$$

where n is the number of the state variables in the system model and r is the dimension of $\mathbf{x}_2$ in the suboptimal EKF. Notice that

$$r = n + m = N \tag{48}$$

in a standard EKF.

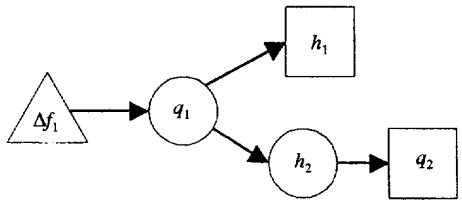

Fig. 2. The precedence diagram of the two-tank system in Fig. 1—result of implementing algorithm B with h_1 and q_2 as the measurement variables (fault parameter: Δf_1).

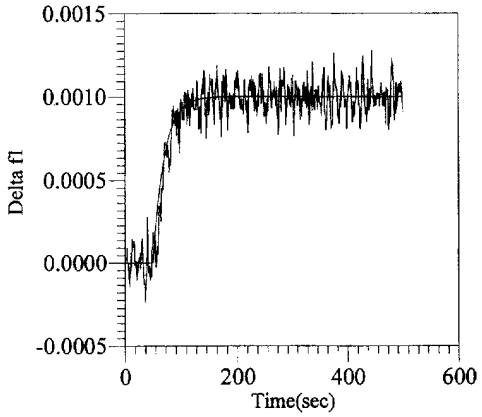

Fig. 3. Estimates of Δf_1 using a suboptimal EKF corresponding to the path ($\Delta f_1 \to q_1 \to h_1$) identified in Fig. 2.

On the basis of eqs (47) and (48), one can determine that $N_{eq} = 19$ in a standard EKF. On the other hand, this number is reduced to 10 if a suboptimal EKF is used.

Example 2. Let us now consider another two-tank system (Fig. 4). Assuming that the possible fault origin in this system is a leak in tank T_1, one can formulate the corresponding system model as eqs (41), (42) and

$$A_1\frac{dh_1}{dt} = q_i - q_1 - \Delta cl_1\sqrt{h_1} \tag{49}$$

$$\frac{dq_1}{dt} = \frac{\pi d_1^2}{4\rho l_1}\left[\rho g(h_1 - h_2) - \frac{8f_1 l_1 \rho q_1|q_1|}{\pi^2 d_1^5}\right]. \tag{50}$$

Let us again use h_1 and q_2 as the measurement variables and apply *Algorithm B*. The resulting precedence diagram is presented in Fig. 5. In this case, there is only one candidate path $\Delta cl_1 \to h_1$. Consequently,

$$\mathbf{x}_1 = \begin{bmatrix} q_1 \\ h_2 \\ q_2 \end{bmatrix} \tag{51}$$

$$\mathbf{x}_2 = \begin{bmatrix} h_1 \\ \Delta cl_1 \end{bmatrix}. \tag{52}$$

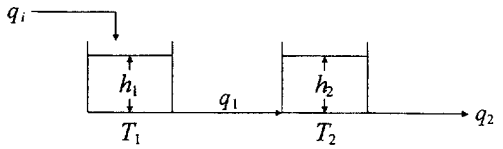

Fig. 4. The simplified process flow diagram of another two-tank system.

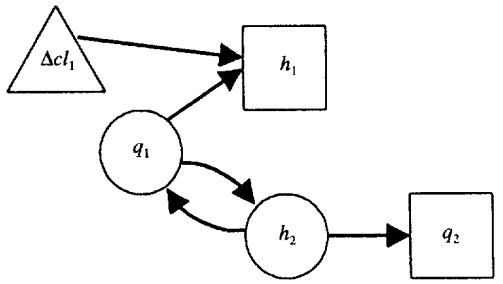

Fig. 5. The precedence diagram of the two-tank system in Fig. 4—result of implementing algorithm B with h_1 and q_2 as the measurement variables (fault parameter: Δcl_1).

A sample of the results obtained by applying the corresponding suboptimal EKF is presented in Fig. 6. One can see that these estimates are quite acceptable for the purpose of fault diagnosis. Note also that it is only necessary to numerically integrate 7 equations in this case and 19 equations are needed in a standard EKF. The reduction in computation load is apparent.

On the other hand, if the assumed fault origin is a leak in tank T_2, the corresponding system model can be described with eqs (39), (42), (50) and

$$A_2\frac{dh_2}{dt} = q_1 - q_2 - \Delta cl_2\sqrt{h_2}. \tag{53}$$

The corresponding precedence diagram is presented in Fig. 7. Although two paths can be identified, the shorter one, i.e. $\Delta cl_2 \to h_2 \to q_2$, is adopted to construct the suboptimal EKF. In particular,

$$\mathbf{x}_1 = \begin{bmatrix} h_1 \\ q_1 \end{bmatrix} \tag{54}$$

$$\mathbf{x}_2 = \begin{bmatrix} h_2 \\ q_2 \\ \Delta cl_2 \end{bmatrix}. \tag{55}$$

The effectiveness of the suboptimal EKF is again verified with simulation studies. A sample of the results is presented in Fig. 8. In this case, ten equations must be integrated numerically.

AN APPLICATION EXAMPLE

The previous examples used in this paper are concerned with two simple two-tank systems. They are designed to illustrate the decomposition procedure

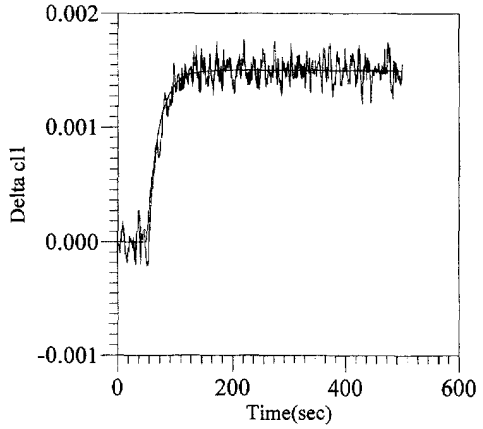

Fig. 6. Estimates of Δcl_1 using a suboptimal EKF corresponding to the path ($\Delta cl_1 \rightarrow h_1$) identified in Fig. 5.

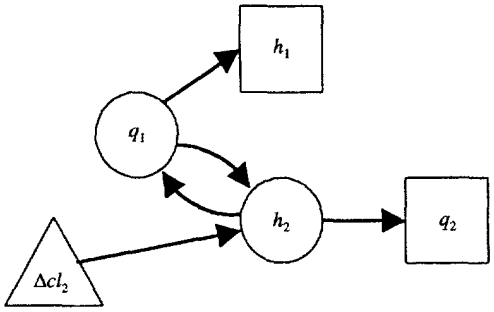

Fig. 7. The precedence diagram of the two-tank system in Fig. 4—result of implementing algorithm B with h_1 and q_2 as the measurement variables (fault parameter: Δcl_2).

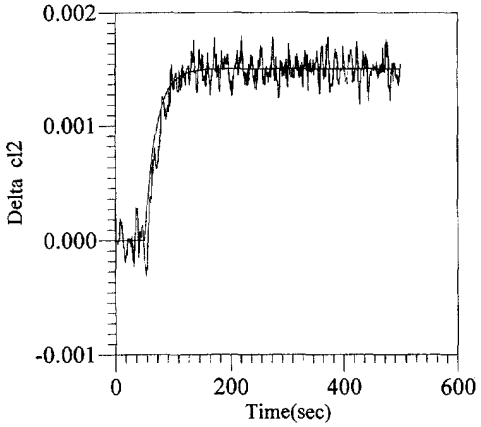

Fig. 8. Estimates of Δcl_2 using a suboptimal EKF corresponding to the path ($\Delta cl_2 \rightarrow h_2 \rightarrow q_2$) identified in Fig. 7.

and to show the effectiveness of the proposed approach. A complex two-phase reaction process is discussed here. This example is meant to demonstrate the practical value of our techniques and the potential for other realistic applications.

Let us consider the process shown in Fig. 9 in which reactants A and B are converted to product C by the reaction

$$A + B \rightarrow C.$$

The reactants are fed to the process in two different immiscible phases (A is in phase I and B is in phase II), and the reaction takes place in the mixer of the mixer-settler pairs. The objective of the process is to convert a fixed percentage of A into product C. The mathematical model of this system can be found in Himmelblau and Bischoff (1968):

$$(1-\alpha)V\frac{dy_1}{dt} = F_1(Y - y_1) - k_1\alpha V x_1 y_1 \quad (56)$$

$$\alpha V\frac{dx_1}{dt} = F_0 x_0 + F_3 x_5 - F_2 x_1 - 2k_1\alpha V x_1 y_1 \quad (57)$$

$$V_1'\frac{dy_2}{dt} = F_1(y_1 - y_2) \quad (58)$$

$$V_2'\frac{dx_2}{dt} = F_2(x_1 - x_2) \quad (59)$$

$$V_3\frac{d\xi}{dt} = F_2(x_2 - \xi) \quad (60)$$

$$(1-\alpha)V\frac{dy_3}{dt} = F_1(y_0 - y_3) - k_2\alpha V x_3 y_3 \quad (61)$$

$$\alpha V\frac{dx_3}{dt} = F_2(\xi - x_3) - 2k_2\alpha V x_3 y_3 \quad (62)$$

$$V_1'\frac{dY}{dt} = F_1(y_3 - Y) \quad (63)$$

$$V_2'\frac{dx_4}{dt} = F_2(x_3 - x_4) \quad (64)$$

$$(v_0 + S\lambda)\frac{dx_5}{dt} = F_2(x_4 - x_5) - W[(\alpha_1 - 1)x_5 + \beta] \quad (65)$$

$$S\frac{d\lambda}{dt} = F_2 - W - F_3 \quad (66)$$

$$\frac{dW}{dt} = \frac{K_1}{S}(F_2 - W - F_3) + K_2\dot{\lambda} \quad (67)$$

$$\frac{dF_0}{dt} = \frac{F_2 - F_0}{x_0 - X}\frac{\bar{X} - X}{\tau} \quad (68)$$

$$X = \frac{F_0 x_0 + F_3 x_5}{F_2} \quad (69)$$

$$\frac{dF_3}{dt} = -\frac{dF_0}{dt}. \quad (70)$$

Most of the notation used in this model is presented in the block diagram of the process shown in Fig. 10. Additional explanations can also be found in the Notation section at the end of this paper.

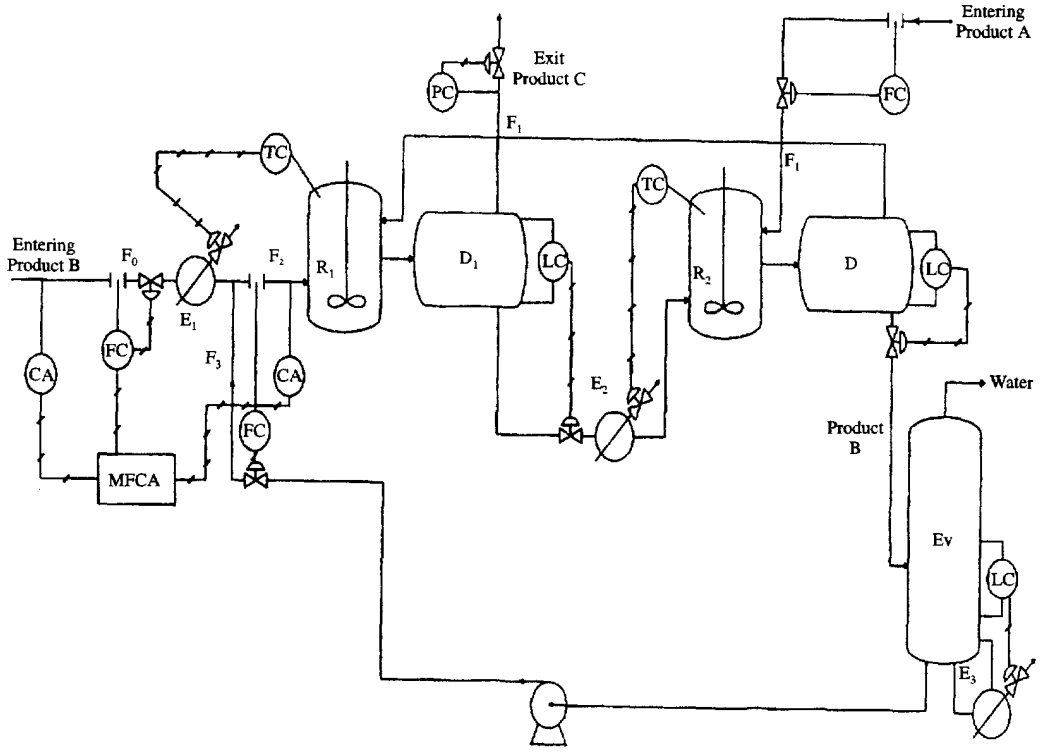

Fig. 9. The process flow diagram of a two-phase reaction process.

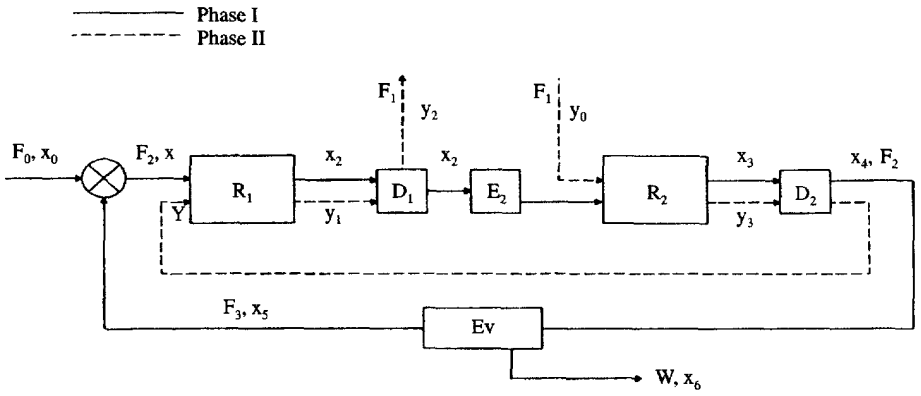

Fig. 10. The process block diagram of a two-phase reaction process.

Let us assume that there are two possible fault origins in this process, i.e. a change in the inlet concentration of reactant A and/or reactant B. The parameters used to describe their effects are Δy_0 and Δx_0, respectively. The measurement variables selected for this system are: x_1, x_4, Y, F_0 and F_3. The corresponding precedence diagram (Fig. 11) can be obtained by applying *Algorithm B*. Two shortest candidate paths can be identified from this diagram, i.e. (1) $\Delta x_0 \rightarrow F_3$ and (2) $\Delta y_0 \rightarrow y_3 \rightarrow Y$. The system can be decomposed accordingly, i.e.

$$\mathbf{x}_1 = [y_1 \ y_2 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ \xi \ \lambda \ W \ F_0]^T \quad (71)$$

$$\mathbf{x}_2 = [F_3 \quad \Delta x_0 \quad Y \quad y_3 \quad \Delta y_0]^T. \quad (72)$$

The results of implementing the suboptimal EKF can be found in Figs 12 and 13. One can see that these EKF estimates are perfectly acceptable for the purpose of diagnosis. Notice further that the number N_{eq} needed for applying the suboptimal EKF is only 29. When compared with a standard EKF ($N_{eq} = 150$), a tremendous amount of computations can be saved.

CONCLUSIONS

A new approach has been proposed in this study to simplify EKF computations in fault identification without sacrificing diagnostic performance. In essence, this improvement is achieved with a suboptimal

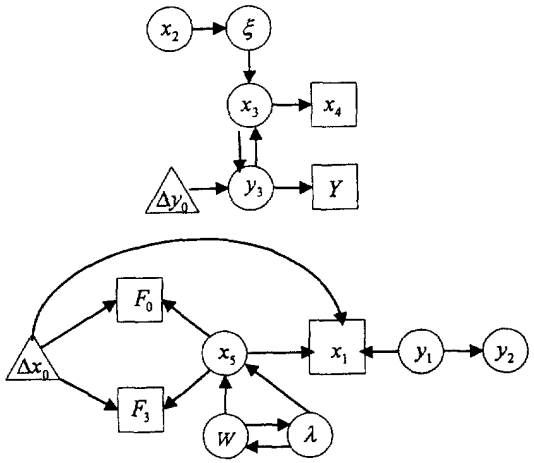

Fig. 11. The precedence diagram of the two-phase reaction process in Fig. 9—result of implementing algorithm B with x_1, x_4, Y, F_0 and F_3 as the measurement variables (fault parameter: Δx_0 and Δy_0).

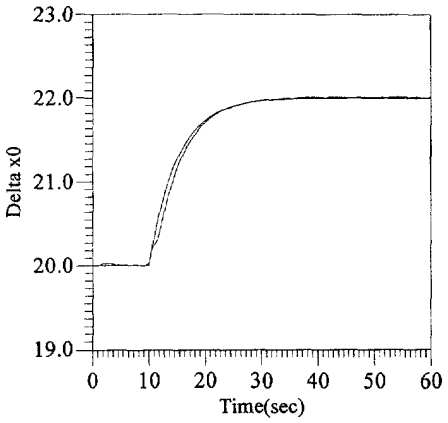

Fig. 12. Estimates of Δx_0 using a suboptimal EKF corresponding to the two paths ($\Delta x_0 \to F_3$ and $\Delta y_0 \to y_3 \to Y$) identified in Fig. 11.

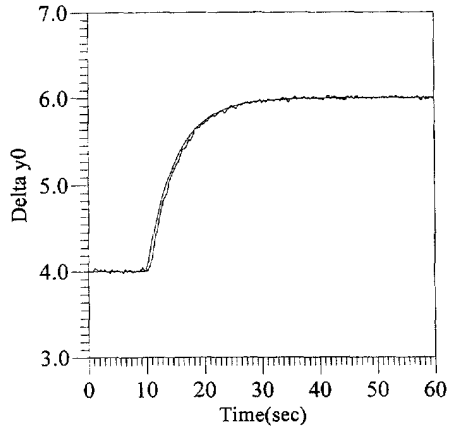

Fig. 13. Estimates of Δy_0 using a suboptimal EKF corresponding to the two paths ($\Delta x_0 \to F_3$ and $\Delta y_0 \to y_3 \to Y$) identified in Fig. 11.

Kalman filter in which only part of the state estimates are updated. This filter can be constructed with an effective system decomposition method which is developed on the basis of the precedence order of the state/parameter estimation process. From the results of extensive numerical simulation studies, one can see that the estimates of the simpified EKFs are indeed correct and, furthermore, the computation load may be reduced to less than 20% of the original level.

Acknowledgement

This work is supported by the national science council of the ROC under grant NSC85-2214-E006-015.

NOTATION

A_ℓ	the cross-sectional area of the tank ℓ (example 1 and 2)
cl_ℓ	the parameter that characterizes tank ℓ leaks (example 1 and 2)
d_ℓ	the diameter of pipeline ℓ (example 1 and 2)
f_ℓ	the friction factor for the flow in pipeline ℓ (example 1 and 2)
F_1	flow rate of phase I (application example)
F_0, F_2, F_3	flow rates of phase II (application example)
h_ℓ	the height of liquid level in tank ℓ (example 1 and 2)
l_ℓ	the length of pipeline ℓ (example 1 and 2)
k_1, k_2	reaction-rate constants in reactor 1 and 2 (application example)
K_1, K_2	adjustable constants in the level controller of the evaporator (application example)
q_ℓ	the volumetric flowrate in pipeline ℓ (example 1 and 2)
S	cross-sectional area of the evaporator (application example)
t	time
X	concentration of B entering reactor R_1 (application example)
$\bar{X}$	reference value of X (application example)
x_0	entering concentration of B in phase II (application example)
Y	concentration of A entering reactor 1 (application example)
y_0	entering concentration of A in phase I (application example)
W	flow rate of vapor from the evaporator (application example)
V	total volume of reactors 1 and 2 (application example)
v_0	total volume of liquid in the evaporator and in the heat exchanger E_3 (application example)
V_3	volume of heat exchanger E_2 (application example)
V'	total volume of decanters 1 and 2 (application example)
V'_1, V'_2	volume of the individual decanter ($V'_1 = V'_2 = V'/2$) (application example)

Greek letters

α volume fraction occupied by phase II in each reactor (application example)

α_1, β constants in the model of the evaporator ($x_6 = \alpha_1 x_5 + \beta$) (application example)

λ variation from the reference level in the evaporator (application example)

ξ concentration of B leaving exchanger E_2 (application example)

ρ the density of liquid (example 1 and 2)

τ time constant of the multifunctional controller (application example)

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APPENDIX A: THE SUBOPTIMAL EKF

- State estimate propagation

$$\frac{d\hat{\mathbf{x}}_1}{dt} = \mathbf{f}_1(\hat{\mathbf{x}}_1(t), \hat{\mathbf{x}}_2(t), t) \tag{A.1}$$

$$\frac{d\hat{\mathbf{x}}_2}{dt} = \mathbf{f}_2(\hat{\mathbf{x}}_1(t), \hat{\mathbf{x}}_2(t), t) \tag{A.2}$$

- Error covariance propagation

$$\dot{\mathbf{P}}_{22} = \mathbf{A}_{22}\mathbf{P}_{22} + \mathbf{P}_{22}\mathbf{A}_{22}^{\mathrm{T}} + \mathbf{Q}_2 \tag{A.3}$$

- Gain matrix

$$\mathbf{K}_{2k} = \mathbf{P}_{22}^{-}(t_k)\,\mathbf{H}_{2k}^{\mathrm{T}}[\mathbf{H}_{2k}\mathbf{P}_{22}^{-}(t_k)\mathbf{H}_{2k}^{\mathrm{T}} + \mathbf{R}]^{-1} \tag{A.4}$$

- State estimate update

$$\hat{\mathbf{x}}_1^{+}(t_k) = \hat{\mathbf{x}}_1^{-}(t_k) \tag{A.5}$$

$$\hat{\mathbf{x}}_2^{+}(t_k) = \hat{\mathbf{x}}_2^{-}(t_k) + \mathbf{K}_{2k}[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}^{-}(t_k))] \tag{A.6}$$

- Error covariance update

$$\mathbf{P}_{22}^{+}(t_k) = [\mathbf{I}_r - \mathbf{K}_{2k}\mathbf{H}_{2k}]\mathbf{P}_{22}^{-}(t_k). \tag{A.7}$$

APPENDIX B: THE MODIFIED PARTITIONING AND TEARING ALGORITHMS

For the completeness of this paper, modified versions of the partitioning algorithm (Steward, 1965) and the tearing technique (Christensen and Rudd, 1969; Christensen, 1970; Stadtherr *et al.*, 1974) are presented here. These algorithms can be best explained with the so-called *structural matrix*. Specifically, the (i, j) th entry in this array is filled with an "$\times$" if the ith equation in eqs (35) involves the jth variable. Otherwise, it is blank. Note that the diagonal positions are reserved for the output variables. The output of each equation should always be the one which appears in the time derivative.

After constructing the structural matrix, the following procedure can be followed to obtain a partition of the system:

1. We look for a row with no off-diagonal element and eliminate that row and the column corresponding to it. We repeat this process until there are no further rows without off-diagonal elements.
2. We begin tracing a path through the structural matrix by following the off-diagonal elements in search of a loop as follows:
   (a) Select the first row remaining in the matrix as the 'row to be examined' and enter its row number on a list.
   (b) Locate the first off-diagonal element in the row being examined.
   (c) Select the row corresponding to the column in which the off-diagonal element was found as the next row to be examined and add the row number to the list of rows examined.
   (d) If the new row number has not been previously examined (i.e. is not already on the list), return to step b and continue tracing.
   (e) If the new row number is already on the list, then we have found a loop containing all the rows whose numbers appear on the list between the two occurrences of the last row number on the list.
3. When we find a loop, we replace the set of rows in the loop by one row which is the *union* of the rows replaced. The union of two rows is a row which contains an element in each column in which *either* row originally contained an element. This we call *collapsing* the rows in the loop. Similarly, we collapse the columns corresponding to these rows.
4. We proceed to step 1 and look for a row with no off-diagonal element. When a row is eliminated in step 1, that row and the rows which collapsed to form it represent the equations in a block. The order in which rows without off-diagonal elements are eliminated gives an order in which the changes in the variables of these blocks may occur.

Table B.1. The structural matrix of the two-tank system in example 1

	h_1	q_1	h_2	q_2	Δf_1
ϕ_1	×	×			
ϕ_2	×	×			×
ϕ_3		×	×	×	
ϕ_4			×	×	
ϕ_5					×

Table B.2. The structural matrix of two-tank system after applying algorithm A

	Δf_1	h_1	q_1	h_2	q_2
ϕ_5	×				
ϕ_1		×	×		
ϕ_2	×	×	×		
ϕ_3			×	×	×
ϕ_4				×	×

Table B.3. The structural matrix of two-tank system after applying algorithm B

	Δf_1	q_1	h_1	h_2	q_2
ϕ_5	×				
ϕ_2	×	×			
ϕ_1		×	×		
ϕ_3		×		×	
ϕ_4				×	×

Originally, the above procedure was used to promote the computation efficiency in solving nonlinear algebraic equations. A *partition* is the division of the set of equations into subsets or *blocks* so that each block in the partition is the smallest set of equations that must be solved simultaneously. After such a partition is established, the blocks can be solved one at a time in series.

In this work, this partitioning algorithm is extended to determine the *precedence order of influences* among state variables due to changes in the parameters (Chang and Chen, 1995). For convenience, it is referred to as *Algorithm A* in this paper. As an example, let us first construct the structural matrix corresponding to eqs (39)–(43). This matrix is presented in Table B.1. After implementing algorithm A, the resulting precedence order, i.e. Table B.2, can be obtained.

On the other hand, the so-called *tearing* technique was originally used to determine an appropriate numerical iteration procedure for a given set of nonlinear algebraic equations. In this research, this method is adopted as an aid for clarifying the cause-and-effect relations between the model parameters and the measurement variables in the state estimate propagation equations of EKF. Specifically, all s measurement variables should be treated as the 'tear variables,' i.e. the variables which are guessed, and the following procedure can be applied accordingly:

1. *Tear* the measurement variables from the structural matrix, i.e. remove all off-diagonal elements in each of the corresponding columns.
2. Apply algorithm A to the post-tearing structural matrix.

This procedure is referred to as algorithm B in the present paper. Note that the precedence order in Fig. 2 can be easily converted to the structural matrix presented in Table B.3 and *vice versa*. The latter is actually the result of implementing algorithm B.