ELSEVIER

Journal of Process Control 12 (2002) 857–873

JOURNAL OF PROCESS CONTROL

www.elsevier.com/locate/jprocont

A simple graphic approach for observer decomposition

Chuei-Tin Chang[a,*], Yi-Shin Lin[a], Christos Georgakis[b]

[a]*Department of Chemical Engineering, National Cheng Kung University, Tainan, Taiwan 70101, ROC*
[b]*Chemical Process Modeling and Control Center, Lehigh University, D315 Iacocca Hall, 111 Research Drive, Bethlehem, PA 18015-4732, USA*

Received 13 August 2001; received in revised form 29 November 2001; accepted 22 April 2002

Abstract

Based upon the proposition that the roles of inputs and outputs in a physical system and those in the corresponding output-injection observer do not really have to be consistent, a systematic procedure is developed in this work to properly divide a set of sparse system models and measurement models into a number of independent subsets with the help of a visual aid. Several smaller sub-observers can then be constructed accordingly to replace the original one. The size of each sub-observer may be further reduced by strategically selecting one or more appended states. These techniques are shown to be quite effective in relieving on-line computation load of the output-injection observers and also in identifying detectable sub-systems. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Observer; Appended state; Output injection; State diagram

1. Introduction

In system identification and fault monitoring applications, one is often confronted with the problem that some key states and non-stationary parameters (or disturbances) cannot be measured on-line. On the other hand, the system and measurement models of many chemical processes are actually well known. In this situation, an *observer* or *state estimator* is usually adopted for the purpose of estimating these unmeasured process variables [1–6]. In general, an observer consists of two parts: (1) a mathematical model of the physical, chemical and measurement processes under study and (2) a correction mechanism that makes use of the output errors. Since the outputs are 'injected' back to produce estimates, these observers are also referred to as the *output-injection observers* [7].

The observer model certainly should be an accurate and complete description of the real system. However, for the large-scale chemical processes, there are genuine incentives to replace the full-size observer with several smaller ones. First of all, one may not be interested in the estimation of *every* process state and parameter. Thus, it is computationally more efficient to implement only a subset of the decomposed sub-observers. Furthermore, this practice may even create opportunities for identifying detectable subsystems from an overall undetectable system. Finally, it should be noted that a realistic system model is often nonlinear and/or stochastic in nature. A matrix Riccati differential equation must be solved if the popular extended Kalman filter is used for such applications. In this case, the on-line computation load for numerical integration can be reduced significantly by using the smaller observer models.

A simple graphic procedure is proposed in this paper to decompose any output-injection observer on the basis of model structure. Notice first that the estimates of an observer are produced with on-line measurement data of the inputs and outputs which are defined implicitly by the observer model. In this study, it is claimed that the classification of these measurement signals does not have to be consistent with their roles in the real system. Consequently, it is possible to construct simpler sub-observers by strategically selecting various different combinations of measurements as inputs and outputs according to the model structure of the open-loop observer. Secondly, it should be noted that, by properly choosing appended states in an observer, the on-line computation demand can also be reduced. In the past, the appended states were used mainly for estimating the unknown disturbances [2,5]. However, if a real state is

* Corresponding author. Fax: +886-6-234-4496.
E-mail address: ctchang@mail.ncku.edu.tw (C.T. Chang).

PII: S0959-1524(02)00020-3

Nomenclature

A_i	the cross-sectional area of the tank i (Example 9.1).
cl_i	the parameter that characterizes tank i leaks (Example 9.1).
d_i	the diameter of pipeline i (Example 9.1).
f_i	the friction factor for the flow in pipeline i (Example 9.1).
F_1	flow rate of phase I (Application Example).
F_0, F_2, F_3	flow rates of phase II (Application Example).
h_i	the height of liquid level in tank i (Example 9.1).
l_i	the length of pipeline i (Example 9.1).
k_1, k_2	reaction-rate constants in reactor 1 and 2 (Application Example).
K_1, K_2	adjustable constants in the level controller of evaporator (Application Example).
q_i	the volumetric flow rate in pipeline i (Example 9.1).
S	cross-sectional area of the evaporator (Application Example).
t	time.
X	concentration of B entering reactor R_1 ($X = \frac{F_0\ x_0 + F_3\ x_5}{F_2}$) (Application Example).
$\bar{X}$	reference value of X (Application Example).
x_0	entering concentration of B in phase II (Application Example).
Y	concentration of A entering reactor 1 (Application Example).
y_0	entering concentration of A in phase I (Application Example).
W	flow rate of vapor from the evaporator (Application Example).
V	total volume of reactors 1 and 2 (Application Example).
v_0	total volume of liquid in the evaporator and in the heat exchanger E_3 (Application Example).
V_3	volume of heat exchanger E_2 (Application Example).
V'	total volume of decanters 1 and 2 (Application Example).
V'_1, V'_2	volume of the individual decanter ($V'_1 = V'_2 = V'/2$) (Application Example).
ρ	the density of liquid (Example 9.1).
α	volume fraction occupied by phase II in each reactor (Application Example).
α_1, β	constants in the model of evaporator ($x_6 = \alpha_1\ x_5 + \beta$) (Application Example).
λ	variation from the reference level in the evaporator (Application Example).
ξ	concentration of B leaving exchanger E_2 (Application Example).
τ	time constant of the multifunctional controller (Application Example).

artificially treated as an appended state, the governing equations of this state and those affecting it can essentially be removed from the observer model. Thus, a smaller observer can be identified with this approach.

It should also be noted that the above sub-observers are *not* the same as the reduced-order observers [8–11]. In fact, each sub-observer may be further converted to a reduced-order observer to improve computation efficiency. The outputs of the decomposed sub-observers are essentially the estimates of a complete set of states of the overall system. On the other hand, the system states must be reconstructed from the outputs of a reduced-order observer. The number of these outputs is the difference between the number of states and that of the measurements [8,9]. Since the reduced-order observer is well known, a detailed discussion concerning its applications after observer decomposition is not provided in this paper.

In order to relieve the on-line computation effort as much as possible, the main emphasis of this study is placed upon the development a simple approach to decompose any output-injection observer into the *maximum* number of sub-observers. The proposed techniques are first developed on the basis of a thorough analysis of the roles of inputs, outputs and appended states in linear deterministic observers. These techniques can be implemented with the help of a visual aid, i.e. the modified state diagram. Since the suggested graphic manipulations involve only role-switching operations of inputs and outputs in the observer models, the same approach can be readily extended to the nonlinear stochastic systems without modifications. Although the issue of detectability is not addressed in the present paper, the feasibility and effectiveness of the decomposition procedure are demonstrated with a series of simple examples and a realistic case study.

2. The interchangeable roles of inputs and outputs in an observer

Let us consider a system model and the corresponding measurement model in the following general form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d} \tag{2.1}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{2.2}$$

where $\mathbf{x} \in R^n$ denotes the vector of state variables; $\mathbf{y} \in R^m$ denotes the vector of system outputs; $\mathbf{u} \in R^p$ represents the system inputs; $\mathbf{d} \in R^q$ is the vector of unmeasured deterministic disturbances; $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are constant matrices and they are assumed to be available. Further, we assume that $q \leqslant m \leqslant n$ and, for convenience, let $p = m$.

The most common approach to construct an output-injection observer to estimate both states and disturbances is to treat the disturbances as appended states and augment the results with system Eq. (2.1), i.e.

$$\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}\mathbf{u} \tag{2.3}$$

where $\tilde{\mathbf{x}} = \left[\mathbf{x}^T \mathbf{d}^T\right]^T$ and

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{D} \\ 0 & 0 \end{bmatrix} \tag{2.4}$$

$$\tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} \tag{2.5}$$

The measurement model (2.2) can also be transformed into the following form:

$$\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} \tag{2.6}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \tag{2.7}$$

The output-injection observer for this augmented open-loop system can then be formulated accordingly, i.e.

$$\dot{\tilde{\mathbf{z}}} = \tilde{\mathbf{A}}\tilde{\mathbf{z}} + \mathbf{L}\left(\mathbf{s}_\mathbf{y} - \mathbf{H}\tilde{\mathbf{z}}\right) + \tilde{\mathbf{B}}\mathbf{s}_\mathbf{u} \tag{2.8}$$

where $\tilde{\mathbf{z}} = [\mathbf{z}_x^T \mathbf{z}_d^T]^T$ is a vector of state and disturbance estimates; $\mathbf{L}$ is the output-injection matrix of dimension $(n+q)\times m$; $\mathbf{s}_\mathbf{y}$ and $\mathbf{s}_\mathbf{u}$ denote the vectors of measurement signals used in the observer as systems outputs and inputs respectively. In the present case, $\mathbf{s}_\mathbf{y} = \mathbf{y}$ and $\mathbf{s}_\mathbf{u} = \mathbf{u}$.

Notice that the observer design problem is essentially concerned with the task of selecting the proper $\mathbf{L}$ so that the estimation errors vanish quickly. The transient behavior of the estimation errors can be described with an equation obtained by subtracting Eq. (2.3) from Eq. (2.8), i.e.

$$\dot{\tilde{\mathbf{e}}} = \dot{\tilde{\mathbf{z}}} - \dot{\tilde{\mathbf{x}}} = \left[\tilde{\mathbf{A}} - \mathbf{L}\mathbf{H}\right]\tilde{\mathbf{e}} \tag{2.9}$$

From Eq. (2.9), it is clear that, as long as we select $\mathbf{L}$ so that the system is stable, i.e. all eigenvalues of the *observer system matrix*

$$\tilde{\mathbf{A}}_o = \tilde{\mathbf{A}} - \mathbf{L}\mathbf{H} \tag{2.10}$$

are on the left-hand plane, the estimation errors will vanish with time for any initial guess of $\tilde{\mathbf{x}}$.

As claimed before, the classification of measurement signals in an observer does not really have to be consistent with the roles of inputs and outputs in the real system. The implications of this practice are analyzed in detail next. For convenience, let us consider a physical system with *no* measured inputs $\mathbf{u}$, i.e. $\mathbf{u} = \mathbf{0}$ in Eq. (2.1). This assumption will simplify the equations, but does not represent any fundamental limitation. We can then classify the outputs into two groups, i.e.

$$\mathbf{y} = \begin{pmatrix} \mathbf{s}_y \\ \mathbf{s}_u \end{pmatrix} \tag{2.11}$$

in which $\mathbf{s}_y \in R^{m-r}$ is treated as the output vector and $\mathbf{s}_u \in R^r$ as the *input* in the observer. The measurement model (2.2) can be written as

$$\begin{pmatrix} \mathbf{s}_y \\ \mathbf{s}_u \end{pmatrix} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} \tag{2.12}$$

where the state vector $\mathbf{x}$ is broken down to two components, $\mathbf{x}_1$ and $\mathbf{x}_2$, so that $\mathbf{C}_{22}$ is a nonsingular $r\times r$ matrix. The open-loop system model (2.1) can then be formulated accordingly:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} + \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix} \mathbf{d} \tag{2.13}$$

One can solve the lower half of Eq. (2.12) for $\mathbf{x}_2$ and substitute it into its upper half and also into Eq. (2.13) to yield

$$\mathbf{s}_y = \left(\mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}\right)\mathbf{x}_1 + \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{s}_u \tag{2.14}$$

$$\dot{\mathbf{x}}_1 = \left(\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}\right)\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{C}_{22}^{-1}\mathbf{s}_u + \mathbf{D}_1\mathbf{d} \tag{2.15}$$

$$\dot{\mathbf{x}}_2 = \left(\mathbf{A}_{21} - \mathbf{A}_{22}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}\right)\mathbf{x}_1 + \mathbf{A}_{22}\mathbf{C}_{22}^{-1}\mathbf{s}_u + \mathbf{D}_2\mathbf{d} \tag{2.16}$$

Since $\mathbf{s}_u$ will be considered as an input vector and $\mathbf{x}_2$ can be estimated with the lower half of Eq. (2.12), Eq.

(2.16) can be discarded as the dynamics of $\mathbf{x}_2$ are not needed to determine the dynamics of $\mathbf{x}_1$. A smaller observer can therefore be formulated according to Eqs. (2.14) and (2.15). Since this observer can be used only to produce the estimates of $\mathbf{x}_1$, it is referred to as a *partial observer*. Let us now consider the observer system matrix for the partial observer:

$$\tilde{\mathbf{A}}' - \mathbf{L}'\mathbf{H}' = \begin{bmatrix} \mathbf{A}_{11} - \mathrm{L}'_x\mathbf{C}_{11} - (\mathbf{A}_{12} - \mathbf{L}'_x\mathbf{C}_{12})\mathbf{C}_{22}^{-1}\mathbf{C}_{21} & \mathrm{D}_1 \\ -\mathbf{L}'_d(\mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}) & \mathbf{0} \end{bmatrix} \tag{2.17}$$

where

$$\tilde{\mathbf{A}}' = \begin{bmatrix} \mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{C}_{22}^{-1}\mathrm{C}_{21} & \mathbf{D}_1 \\ 0 & \mathbf{0} \end{bmatrix} \tag{2.18}$$

$$\mathbf{H}' = \begin{bmatrix} \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21} & \mathbf{0} \end{bmatrix} \tag{2.19}$$

and the output-injection matrix is written as

$$\mathbf{L}' = \begin{bmatrix} \mathrm{L}'_x \\ \mathbf{L}'_d \end{bmatrix} \tag{2.20}$$

If we had not performed the elimination of $\mathbf{x}_2$ stated above, the observer system matrix for the original system, i.e. Eqs. (2.12) and (2.13) with the disturbances appended, can be written as

$$\breve{\mathbf{A}}' - \breve{\mathbf{L}}'\breve{\mathbf{H}}' = \begin{bmatrix} \mathbf{A}_{11}-\mathbf{L}'_{11}\mathbf{C}_{11}-\mathbf{L}'_{12}\mathbf{C}_{21} & \mathbf{D}_1 & \mathbf{A}_{12}-\mathbf{L}'_{11}\mathbf{C}_{12}-\mathbf{L}'_{12}\mathbf{C}_{22} \\ -\mathbf{L}'_{21}\mathbf{C}_{11}-\mathbf{L}'_{22}\mathbf{C}_{21} & \mathbf{0} & -\mathbf{L}'_{21}\mathbf{C}_{12}-\mathbf{L}'_{22}\mathbf{C}_{22} \\ \mathbf{A}_{21}-\mathbf{L}'_{31}\mathbf{C}_{11}-\mathbf{L}'_{32}\mathbf{C}_{21} & \mathbf{D}_2 & \mathbf{A}_{22}-\mathbf{L}'_{31}\mathbf{C}_{12}-\mathbf{L}'_{32}\mathbf{C}_{22} \end{bmatrix} \tag{2.21}$$

where

$$\breve{\mathbf{A}}' = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{D}_1 & \mathbf{A}_{12} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{21} & \mathbf{D}_2 & \mathbf{A}_{22} \end{bmatrix} \tag{2.22}$$

$$\breve{\mathbf{H}}' = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{0} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{0} & \mathbf{C}_{22} \end{bmatrix} \tag{2.23}$$

$$\breve{\mathbf{L}}' = \begin{bmatrix} \mathbf{L}'_{11} & \mathbf{L}'_{12} \\ \mathbf{L}'_{21} & \mathbf{L}'_{22} \\ \mathbf{L}'_{31} & \mathbf{L}'_{32} \end{bmatrix} \tag{2.24}$$

If we assign

$$\mathbf{L}'_{11} = \mathbf{L}'_x \tag{2.25}$$

$$\mathbf{L}'_{12} = (\mathbf{A}_{12} + \mathbf{L}'_x\mathbf{C}_{12})\mathbf{C}_{22}^{-1} \tag{2.26}$$

$$\mathbf{L}'_{21} = \mathbf{L}'_d \tag{2.27}$$

$$\mathbf{L}'_{22} = -\mathbf{L}'_d\mathbf{C}_{12}\mathbf{C}_{22}^{-1} \tag{2.28}$$

then the following results can be obtained:

$$\tilde{\mathbf{A}}' - \mathrm{L}'\mathrm{H}' = \begin{bmatrix} \mathbf{A}_{11} - \mathbf{L}'_{11}\mathbf{C}_{11} - \mathbf{L}'_{12}\mathbf{C}_{21} & \mathbf{D}_1 \\ -\mathbf{L}'_{21}\mathbf{C}_{11} - \mathbf{L}'_{22}\mathbf{C}_{21} & \mathbf{0} \end{bmatrix} \tag{2.29}$$

$$\mathbf{A}_{12} - \mathbf{L}'_{11}\mathbf{C}_{12} - \mathbf{L}'_{12}\mathbf{C}_{22} = \mathbf{0} \tag{2.30}$$

$$-\mathbf{L}'_{21}\mathbf{C}_{12} - \mathbf{L}'_{22}\mathbf{C}_{22} = \mathbf{0} \tag{2.31}$$

Based on the above results, it can be concluded that the error dynamics associated with $\mathbf{x}_1$ and $\mathbf{d}$ in the original full-size observer based on Eqs. (2.12) and (2.13) can always be made to be identical to that in the partial observer based on Eqs. (2.14) and (2.15) and, at the same time, independent of the estimation errors of $\mathbf{x}_2$. Also, it is clear that various different partial observers can be constructed according to different selections of $\mathbf{s}_y$ and $\mathbf{s}_u$.

3. Decomposition of sparse systems

In many large-scale engineering applications matrices **A**, **B** and **D** are sparse and the rows and columns of **C** can be rearranged to yield the following form:

$$\mathbf{C} = \begin{bmatrix} \mathbf{0} & \Sigma \end{bmatrix} \tag{3.1}$$

where Σ is a diagonal matrix. As a result, it is often possible to identify a set of *sub-observers* to replace the original one by strategically classifying the measurement signals. Let us use a simple example to demonstrate such a possibility.

Example 3.1. Consider an open-loop system defined by the following matrices

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0.004 & -0.3652 & -0.004 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0.004 & -0.3652 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} -1.174 & 0 \\ 0 & 0 \\ 0 & -0.8301 \\ 0 & 0 \end{bmatrix}$$

and also assume that there is no measured inputs **u**. The measurement matrix used here is

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

In this example, the symbols

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$$

$$\mathbf{y} = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$$

and

$$\mathbf{d} = \begin{bmatrix} d_1 & d_2 \end{bmatrix}^T$$

represent states, measurements and disturbances respectively.

First, let us adopt the following classification

$$s_y = y_1 \quad s_u = y_2$$

The submatrices of the measurement matrix $\mathbf{C}$ can be partitioned by selecting

$$\mathbf{x}_1 = \begin{bmatrix} x_1 & x_2 & x_4 \end{bmatrix}^T \quad \mathbf{x}_2 = [x_3]^T$$

That is

$$\mathbf{C}_{11} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{C}_{12} = [0] \\ \mathbf{C}_{21} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \mathbf{C}_{22} = [1]$$

On the basis of above classification, we can find

$$\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21} = \begin{bmatrix} 0 & -1 & 0 \\ 0.004 & -0.3652 & 0 \\ 0 & 0 & -0.3652 \end{bmatrix}$$

$$\mathbf{A}_{12}\mathbf{C}_{22}^{-1} = \begin{bmatrix} 0 \\ -0.004 \\ 0.004 \end{bmatrix}$$

$$\mathbf{D}_1 = \begin{bmatrix} -1.174 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{C}_{12}\mathbf{C}_{22}^{-1} = [0]$$

From the above results, it is clear that the transformed system is decoupled into two smaller ones, i.e. $\{x_1, x_2\}$ and $\{x_4\}$. Notice that only the first subsystem is detectable. Its system model can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 0.004 & -0.3652 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 0 \\ -0.004 \end{bmatrix} s_u + \begin{bmatrix} -1.174 \\ 0 \end{bmatrix} d_1$$

and the measurement model is:

$$s_y = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Thus, this sub-observer can be used to estimate x_1, x_2 and d_1 only. On the other hand, if we adopt the alternative classification, i.e.

$$s_y = y_2 \quad s_u = y_1$$

then the following models can be used as the basis for estimating x_3, x_4 and d_2:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 0.004 & -0.3652 \end{bmatrix}\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} s_u + \begin{bmatrix} -0.8301 \\ 0 \end{bmatrix} d_2$$

$$s_y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{pmatrix} x_3 \\ x_4 \end{pmatrix}$$

From the above example, one can see that each sub-observer can be used to estimate only a subset of the original states and disturbances. However, if all the sub-observers are implemented on-line, every variable can still be traced satisfactorily, There are several incentives to decompose the system. For example, if one is only interested in estimating part of the states and disturbances, it may be possible to ignore some of the sub-observers. Also, the proposed technique can be utilized to identify detectable subsystems in an undetectable overall system. Let us again use an example to illustrate this point.

Example 3.2. Consider the same system described in the previous example except that there are three possible disturbances d_1, d_2 and d_3. The corresponding coefficient matrix is

$$\mathbf{D} = \begin{bmatrix} -1.174 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -0.8301 & 0 \\ 0 & 0 & -1.0926 \end{bmatrix}$$

Although the entire system is undetectable, a detectable subsystem can be identified by making the selection that $s_y = y_1$ and $s_u = y_2$. This subsystem is the same as that described in the previous example for estimating x_1, x_2 and d_1.

4. Decomposition strategy

Having demonstrated that it is indeed possible to replace an output-injection observer with a set of sub-observers, the obvious challenge next is to develop a systematic strategy to classify the measurement signals and then construct these observers accordingly. From the standpoint of creating opportunities for reducing computation load and/or identifying detectable subsystems, it is desirable to identify the *largest* number of subsystems. It can be observed from Eq. (2.15) that a smallest possible partial observer can be obtained by choosing all measurement signals as inputs. Further, notice that this practice also nullifies the interaction among states in the resulting sub-system as much as possible. Despite these advantages, the corresponding estimation scheme becomes an *open-loop observer* since there are no outputs to correct the estimates. Although this observer is not always reliable due to the presence of unknown disturbances, useful insights can be gained from its model structure. Following is an example:

Example 4.1. Consider again the system in Example 3.1. An open-loop observer can be constructed according to Eq. (2.15), i.e.

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_4\end{pmatrix}=\begin{bmatrix}0 & 0\\ 0 & -0.3652\end{bmatrix}\begin{pmatrix}x_1\\ x_4\end{pmatrix}+\begin{bmatrix}-1 & 0\\ 0 & 0.004\end{bmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}+\begin{bmatrix}-1.174 & 0\\ 0 & 0\end{bmatrix}\begin{pmatrix}d_1\\ d_2\end{pmatrix}$$

The discarded equations, i.e. those corresponding to Eq. (2.16), can be written as

$$\begin{pmatrix}\dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{bmatrix}0.004 & 0\\ 0 & -1\end{bmatrix}\begin{pmatrix}x_1\\ x_4\end{pmatrix}+\begin{bmatrix}-0.3652 & -0.004\\ 1 & 0\end{bmatrix}\times\begin{pmatrix}y_1\\ y_2\end{pmatrix}+\begin{bmatrix}0 & 0\\ 0 & -0.8301\end{bmatrix}\begin{pmatrix}d_1\\ d_2\end{pmatrix}$$

From these results, one can see that there are actually two independent open-loop observers, i.e. one associated with the state x_1 and another with x_4. However, it should also be noted that these open-loop observers are useless unless the values of all disturbances can be obtained in advance. Thus, a mechanism for output injection should be established and, specifically, at least one measurement must be treated as s_y in each of these observers.

Let us redefine the measurement signals in the open-loop observers, i.e. $s_y=y_1$ and $s_u=y_2$. The system equation in the first open-loop observer becomes:

$$\frac{\mathrm{d}x_1}{\mathrm{d}t}=-x_2-1.174d_1$$

Since the observer model now contains the state x_2, it is necessary to recover the corresponding equation which is originally discarded, i.e.

$$\frac{\mathrm{d}x_2}{\mathrm{d}t}=0.004x_1-0.3652x_2-0.004y_2$$

One can see that the above two equations are exactly the same as those of the first sub-observer in Example 3.1.

It should also be noted that, with the present choice of s_u and s_y, it is not possible to construct an output-injection observer on the basis of the second open-loop observer. If, however, the roles of y_1 and y_2 are switched, i.e. $s_y=y_2$ and $s_u=y_1$, then the other sub-observer in Example 3.1 can be identified accordingly.

The decomposition procedure described in the above example can be summarized with two specific steps.

- *Treat all measurement signals as inputs in the observer model. Identify an open-loop observer corresponding to a distinct component in the resulting system.*

Let us consider the general formulation of an open-loop observer, i.e. Eq. (2.15), together with the rest of the system model, i.e. Eq. (2.16), obtained by treating all measurement signals as inputs in s_u. In a sparse system, it is often possible to reorganize these equations in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\hat{\mathbf{x}}_1\\ \hat{\mathbf{x}}_2\end{pmatrix}=\begin{bmatrix}\hat{\mathbf{A}}_{11} & \mathbf{0}\\ \hat{\mathbf{A}}_{21} & \hat{\mathbf{A}}_{22}\end{bmatrix}\begin{pmatrix}\hat{\mathbf{x}}_1\\ \hat{\mathbf{x}}_2\end{pmatrix}+\begin{bmatrix}\hat{\mathbf{D}}_1\\ \hat{\mathbf{D}}_2\end{bmatrix}\mathbf{d}+\hat{\mathbf{B}}\mathbf{s}_u \tag{4.1}$$

where $\hat{\mathbf{x}}_1$ contains the states in the observer under consideration and $\hat{\mathbf{A}}_{11}$ and $\hat{\mathbf{A}}_{22}$ are square matrices.

- *Treat the measurements of the states in the above open-loop observer*, i.e. $\hat{\mathbf{x}}_1$, *as outputs to recover the output-injection mechanism.*

Now let's re-classify the signals in $\mathbf{s}_u$ in such a way that

$$\mathbf{s}_u=\begin{pmatrix}\hat{\mathbf{s}}_y\\ \hat{\mathbf{s}}_u\end{pmatrix}=\begin{bmatrix}\hat{\mathbf{C}}_{11} & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{C}}_{22}\end{bmatrix}\begin{pmatrix}\hat{\mathbf{x}}_1\\ \hat{\mathbf{x}}_2\end{pmatrix} \tag{4.2}$$

where $\hat{\mathbf{C}}_{11}$ and $\hat{\mathbf{C}}_{22}$ are two non-zero matrices. Consequently, the $\hat{\mathbf{B}}$ matrix can be partitioned accordingly:

$$\hat{\mathbf{B}}=\begin{bmatrix}\hat{\mathbf{B}}_{11} & \hat{\mathbf{B}}_{12}\\ \hat{\mathbf{B}}_{21} & \hat{\mathbf{B}}_{22}\end{bmatrix} \tag{4.3}$$

Thus, the output-injection observer for the states in $\hat{\mathbf{x}}_1$ can be built with

$$\dot{\hat{\mathbf{x}}}_1 = \left(\hat{\mathbf{A}}_{11} + \hat{\mathbf{B}}_{11}\hat{\mathbf{C}}_{11}\right)\hat{\mathbf{x}}_1 + \hat{\mathbf{D}}_1\mathbf{d} + \hat{\mathbf{B}}_{12}\mathbf{s}_u \tag{4.4}$$

as the system model and the upper half of Eq. (4.2) as the measurement model.

Finally, note that the above two steps should be repeated until all components and the corresponding sub-observers are identified.

5. A visual aid

Although only simple role-switching operations of inputs and outputs in the observer model are needed in the above procedure, the required mathematical manipulations are still rather cumbersome. A visual aid, i.e. the modified *state diagram*, has been developed in this study to facilitate implementation. A state diagram is essentially a special form of digraph representing the cause-and-effect relation of inputs, outputs, states and their derivatives. A well-established method can be used for converting a set of mathematical models to the state diagram, e.g. see Kuo [12]. In this study, the standard state diagram has been modified to simplify analysis. Specifically,

- the nodes representing inputs and initial states are removed, and
- the nodes representing each state and its derivatives are combined.

The usefulness of such a graphic tool is demonstrated in the following example:

Example 5.1. Once again let us consider the system in Example 3.1. The system model and measurement model can be converted to the state diagram presented in Fig. 1.

In step 1 of the observer decomposition procedure, the measurement signals y_1 and y_2 should be treated as inputs. The corresponding state diagram can be obtained by replacing the outward arcs of x_2 and x_3 with ones that connect y_1 and y_2 respectively (see Fig. 2). Furthermore, since the inputs are not included in the modified version of state diagram, y_1 and y_2 should also be removed (see Fig. 3). Notice that two independent components can be identified from the above result and two open-loop observers can be constructed accordingly.

In step 2, the output-injection mechanism should be established individually for each of the two open-loop observers. Specifically, the measurement signals of the state variables within each component should be used as outputs of the corresponding observer. The resulting state diagrams are presented in Figs. 4 and 5.

Notice that it is always possible to establish output injection mechanism for each identified component in the open-loop observer. This is due to our assumption concerning the measurement model of sparse systems. In other words, the partition of measurement matrix described in Eq. (4.2) is always feasible if **C** is defined by Eq. (3.1). Consequently, it is sufficient to construct only the state diagram of open-loop observer, i.e. Fig. 3, and there is no need to carry out additional steps to obtain the diagrams for sub-observers, i.e. Figs. 4 and 5. Further, since the nodes representing inputs are not included in analysis, a short-cut procedure can be utilized to build the state diagram associated with the open-loop observer. In particular, one can start with the state diagram representing only system model and then remove the outward arcs of the measured states.

Finally it should be noted that the minimum requirement for observer decomposition is the availability of *model structure* only. The exact mathematical system

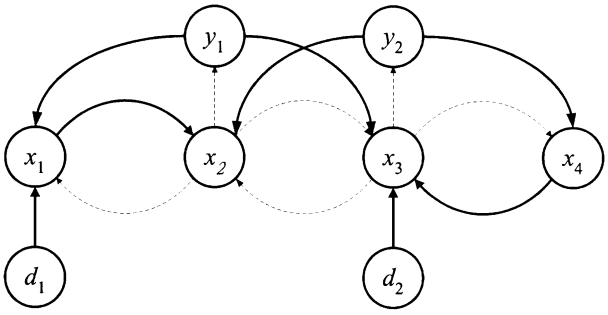

Fig. 2. State diagram of the open-loop observer obtained from Fig. 1.

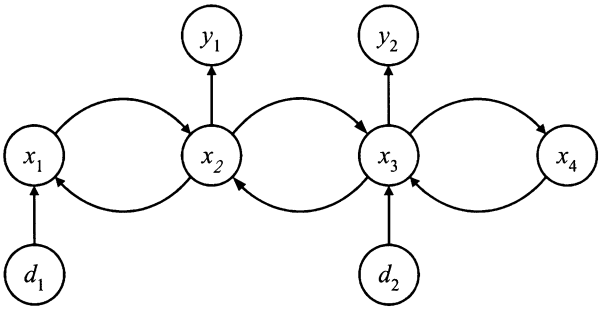

Fig. 1. State diagram of the system considered in example 5.1.

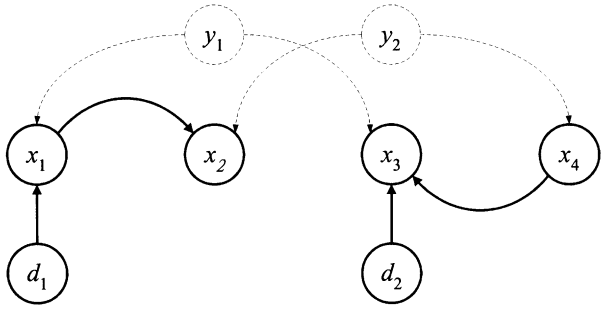

Fig. 3. Modified state diagram of Fig. 2.

and measurement models are not really needed for building a state diagram.

6. Selection of appended states

There are in general several ways that one can make use of the appended states in an observer. As mentioned before, a popular method to construct an output-injection observer for estimating both states and disturbances is to treat the latter as appended states. Also, one may wish to consider the measurements of the *true* inputs of a physical system as *outputs* in the observer due to the fact that they are noisy. This goal can be achieved by treating the variables in **u** as appended states and augmenting the resulting equation, i.e.

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathbf{0} \tag{6.1}$$

with the Eq. (2.1) to form the governing equations of the observers. Thus, the corresponding measurement matrix $\bar{\mathbf{H}}$ becomes

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{6.2}$$

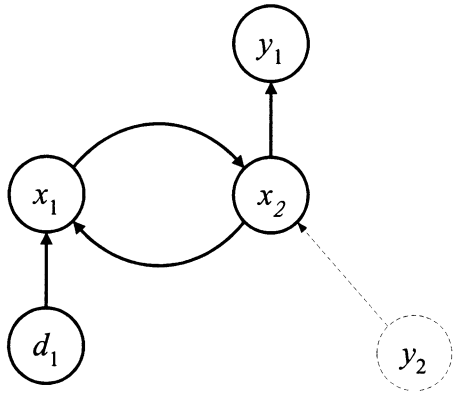

Fig. 4. State diagram of a sub-observer identified in Fig. 3.

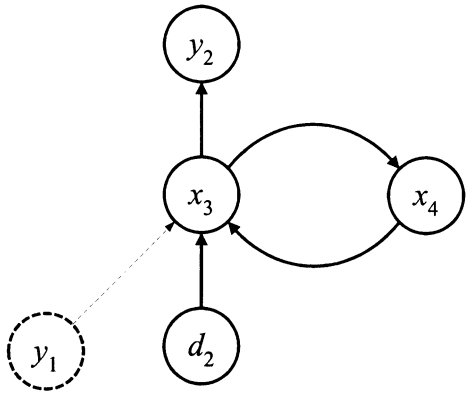

Fig. 5. State diagram of another sub-observer identified in Fig. 3.

A similar procedure can be followed to derive the observer system matrix. It can be shown that the estimates of **u** can be made to converge independently and the error dynamics of the states and disturbances can be forced to follow that of Eq. (2.9). The detailed derivation is omitted in this paper for the sake of brevity.

Finally, from the discussions in the previous sections, one can observe that the identification of decoupled open-loop observers is a critical step in decomposing any output-injection observer. Since the success of this manipulation depends mainly on the inherent model structure, the resulting sub-observers may still be too large to be implemented on-line. This problem can sometimes be circumvented by deliberately treating some of the real states as appended states. More specifically, the following procedure can be performed on Eq. (2.1):

(1) remove the corresponding rows and then columns of matrix **A**.
(2) remove the corresponding rows in matrices **B** and **D**.
(3) insert the columns removed from **A** into **D** and add additional elements to the vector **d**.

As a result, additional opportunities for system decomposition can be identified. Let us use another simple example to illustrate this point:

Example 6.1. Let us consider the same system described in Example 3.1 except there is only one measurement signal y and one disturbance d. In particular,

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & -0.8301 & 0 \end{bmatrix}^T$$

The corresponding state diagram is presented in Fig. 6.

Since there is only one measurement signal and it is not possible to construct an output-injection observer without at least one output, the given system really can

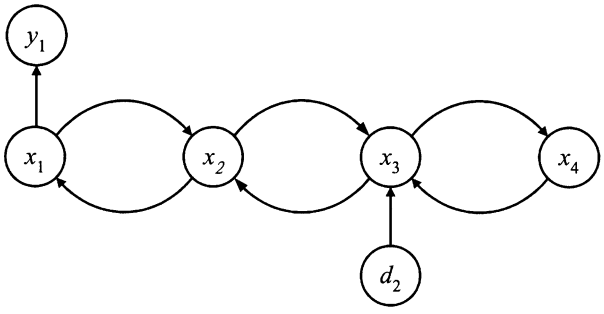

Fig. 6. State diagram of the system considered in Example 6.1.

not be decomposed by treating y as an input. However, if x_2 is used as an appended state, one can obtain

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 0.004 & -0.3652\end{bmatrix}\begin{pmatrix}x_1\\ x_3\\ x_4\end{pmatrix}+\begin{bmatrix}-1 & 0\\ 1 & -0.8301\\ 0 & 0\end{bmatrix}\begin{pmatrix}x_2\\ d\end{pmatrix}$$

The corresponding state diagram can be obtained by removing the incoming arcs of x_2 (Fig. 7). Notice that x_1 and x_2 are now unaffected by d, x_3 and x_4. Thus, a smaller observer can be constructed to estimate x_1 and x_2, i.e.

$$\dot{\boldsymbol{x}}_1=-x_2 \quad \dot{x}_2=0 \quad y=x_1$$

7. Extension to stochastic systems

The extension of the proposed methods to stochastic systems is straightforward. Let us consider the linear case first, i.e.

$$\dot{\tilde{\boldsymbol{x}}}=\tilde{\mathbf{A}}\tilde{\mathbf{x}}+\mathbf{B}\mathbf{u}+\mathbf{G}\omega \tag{7.1}$$

$$\mathbf{y}=\mathbf{H}\tilde{\mathbf{x}}+\nu \tag{7.2}$$

where $\omega\sim(\mathbf{0},\mathbf{Q})$ and $\nu\sim(\mathbf{0},\mathbf{R})$ are white noises uncorrelated with each other. It has been well established that the optimal observer for this process is the Kalman filter. This observer can also be written in the general form of Eq. (2.8) in which $\mathbf{L}$ is replaced by a time-variant Kalman gain $\mathbf{K}(t)$. The corresponding error dynamics can be described with

$$\dot{\tilde{\mathbf{e}}}=\left[\tilde{\mathbf{A}}-\mathbf{K}\mathbf{H}\right]\tilde{\mathbf{e}}+\mathbf{G}\omega-\mathbf{K}\nu \tag{7.3}$$

From this equation, it is important to realize that the estimation errors are not affected by the inputs $\mathbf{u}$. It has also been shown that, as long as the observer system matrix $\mathbf{A}_o$ tends to an asymptotically stable matrix at $t\to\infty$ and the noises are bounded, then in the limit the estimates converges to the *expected* true states [7]. Consequently, the decomposition techniques used in the deterministic case can be adopted here without modifications.

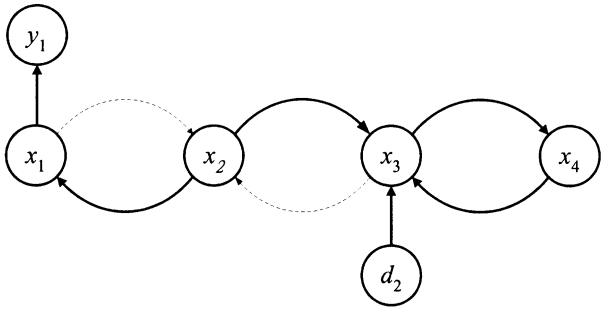

Fig. 7. State diagram obtained by treating x_2 as appended state in Fig. 6.

Furthermore, since the state diagram can also be used to describe the information flows in nonlinear systems and the proposed graphic manipulations for observer decomposition involve only *role-switching operations* of inputs and outputs, the techniques developed in the previous sections can be extended to nonlinear stochastic systems. Although convergence is not always guaranteed, performance of of the resulting extended Kalman filters (EKFs) is in general satisfactory in our extensive simulation studies.

Example 7.1. Let us consider the system of two identical storage tanks connected in series (see Fig. 8). An accurate system model is assumed to be available, i.e.

$$A_1\frac{\mathrm{d}h_1}{\mathrm{d}t}=q_i-q_1-\Delta cl_1\sqrt{h_1}$$

$$\frac{\mathrm{d}q_1}{\mathrm{d}t}=\frac{\pi d_1^2}{4\rho l_1}\left[\rho g(h_1-h_2)-\frac{8f_1l_1\rho q_1^2}{\pi^2d_1^5}\right]$$

$$A_2\frac{\mathrm{d}h_2}{\mathrm{d}t}=q_1-q_2-\Delta cl_2\sqrt{h_2}$$

$$\frac{\mathrm{d}q_2}{\mathrm{d}t}=\frac{\pi d_2^2}{4\rho l_2}\left[\rho gh_2-\frac{8f_2l_2\rho q_2^2}{\pi^2d_2^5}\right]$$

where, ρ ($=1000.0$ kg/m^3) is the density of liquid, h_k and A_k ($=1.0$ m^2) denote respectively the height of liquid level in and the cross-sectional area of tank k ($k=1$, 2), q_k, d_k ($=0.0508$ m), l_k ($=5.0$ m) and f_k ($=2.509\times10^{-3}$) represent respectively the volumetric flow rate in and the diameter, length and friction factor of the outlet pipeline from tank k ($k=1$, 2). Also notice that the *unknown* parameters Δcl_ks ($k=1$, 2) are associated with the assumed faults, i.e. leakage in tanks 1 and 2, respectively. Obviously, only positive parameter values are allowed for Δcl_ks and each can be considered as a measure of the corresponding leak size.

Numerical simulation studies were carried out to produce the on-line measurement data. It was assumed that the system was operated at its normal steady state initially. The initial heights of liquid levels h_1 and h_2 were chosen to be 1.378 and 0.689 m respectively and the corresponding flow rates are $q_i=q_1=q_2=0.015$ m^3/s. Let us

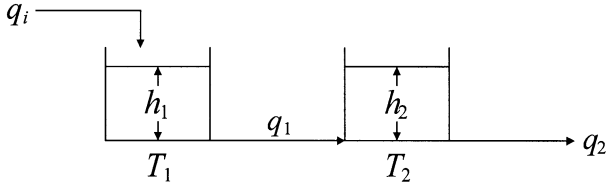

Fig. 8. The simplified process flow diagram of a two-tank system.

assume that the above mentioned faults occurred at $t = 50$ s. For the sake of convenience, the change in Δcl_k was introduced in simulation according to:

$$\Delta cl_k = C_k\{1 - \alpha(t-50)\}u(t-50) \qquad k = 1, 2$$

and

$$u(t-50) = \begin{cases} 1 & t \geqslant 50 \\ 0 & t < 50 \end{cases}$$

where C_1 (=0.00175), C_2 (=0.0015) and α (=0.05 s^{-1}) are constants.

The above six equations were integrated together to simulate the transient behavior of the state variables. In this example, it was further assumed that the state variables q_1 and h_2 can be measured on-line. The measurement noises were produced with a random number generator and then added to the simulated values of these two variables to obtain the simulated on-line measurements.

By implementing the proposed graphic approach, two smaller EKFs can be constructed to estimate Δcl_1 and Δcl_2 on-line on the basis of the measurement data of q_1 and h_2. Their respective observer models should be:

- EKF1

$$A_1\frac{dh_1}{dt} = q_i - q_1 - \Delta cl_1\sqrt{h_1}$$
$$\frac{dq_1}{dt} = \frac{\pi d_1^2}{4\rho l_1}\left[\rho g(h_1 - h_2) - \frac{8f_1 l_1 \rho q_1^2}{\pi^2 d_1^5}\right]$$
$$\frac{d\Delta cl_1}{dt} = 0$$

where the measurement of h_2 should be treated as input and the measurement of q_1 is the output.

- EKF2

$$A_2\frac{dh_2}{dt} = q_1 - q_2 - \Delta cl_2\sqrt{h_2}$$
$$\frac{dq_2}{dt} = \frac{\pi d_2^2}{4\rho l_2}\left[\rho g h_2 - \frac{8f_2 l_2 \rho q_2^2}{\pi^2 d_2^5}\right]$$
$$\frac{d\Delta cl_2}{dt} = 0$$

where the measurement of q_1 should be treated as input and the measurement of h_2 is the output.

The feasibility of these two EKFs has been confirmed with numerically simulated data. The results can be found in Figs. 9–14.

8. An application example

Let us consider the process shown in Fig. 15 in which reactants A and B are converted to product C by the reaction The reactants are fed to the process in two different immiscible phases (A in phase I and B in phase II), and the reaction takes place in the mixer of the mixer–settler pairs. The mathematical model of this system can be found in Himmelblau and Bischoff [13]:

$$(1-\alpha)V\frac{dy_1}{dt} = F_1(Y - y_1) - k_1\alpha V x_1 y_1 \tag{8.1}$$

$$\alpha V\frac{dx_1}{dt} = F_0 x_0 + F_3 x_5 - F_2 x_1 - 2k_1\alpha V x_1 y_1 \tag{8.2}$$

$$V_1'\frac{dy_2}{dt} = F_1(y_1 - y_2) \tag{8.3}$$

$$V_2'\frac{dx_2}{dt} = F_2(x_1 - x_2) \tag{8.4}$$

$$V_3\frac{d\xi}{dt} = F_2(x_2 - \xi) \tag{8.5}$$

$$(1-\alpha)V\frac{dy_3}{dt} = F_1(y_0 - y_3) - k_2\alpha V x_3 y_3 \tag{8.6}$$

$$\alpha V\frac{dx_3}{dt} = F_2(\xi - x_3) - 2k_2\alpha V x_3 y_3 \tag{8.7}$$

$$V_1'\frac{dY}{dt} = F_1(y_3 - Y) \tag{8.8}$$

$$V_2'\frac{dx_4}{dt} = F_2(x_3 - x_4) \tag{8.9}$$

$$(v_0 + S\lambda)\frac{dx_5}{dt} = F_2(x_4 - x_5) - W[(\alpha_1 - 1)x_5 + \beta] \tag{8.10}$$

$$S\frac{d\lambda}{dt} = F_2 - W - F_3 \tag{8.11}$$

$$\frac{dW}{dt} = \frac{K_1}{S}(F_2 - W - F_3) + K_2\lambda \tag{8.12}$$

$$\frac{dF_0}{dt} = \frac{F_2 - F_0}{x_0 - X}\frac{\bar{X} - X}{\tau} \tag{8.13}$$

$$\frac{dF_3}{dt} = \frac{F_2 - F_0}{x_0 - X}\frac{\bar{X} - X}{\tau} \tag{8.14}$$

Most of the notation used in this model is presented in the block diagram shown in Fig. 16. Additional explanations can be found in the Nomenclature section. It is assumed that there are two possible disturbances in the process, i.e. a change in the inlet concentration of reactant A and/or reactant B. The parameters used to

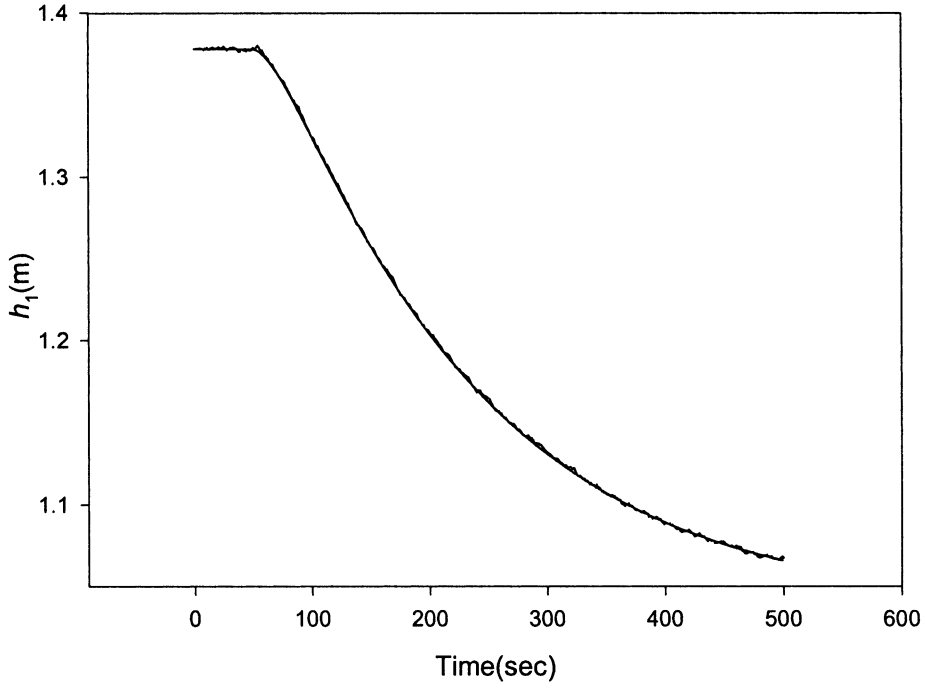

Fig. 9. EKF1 estimates of h_1 in Example 7.1.

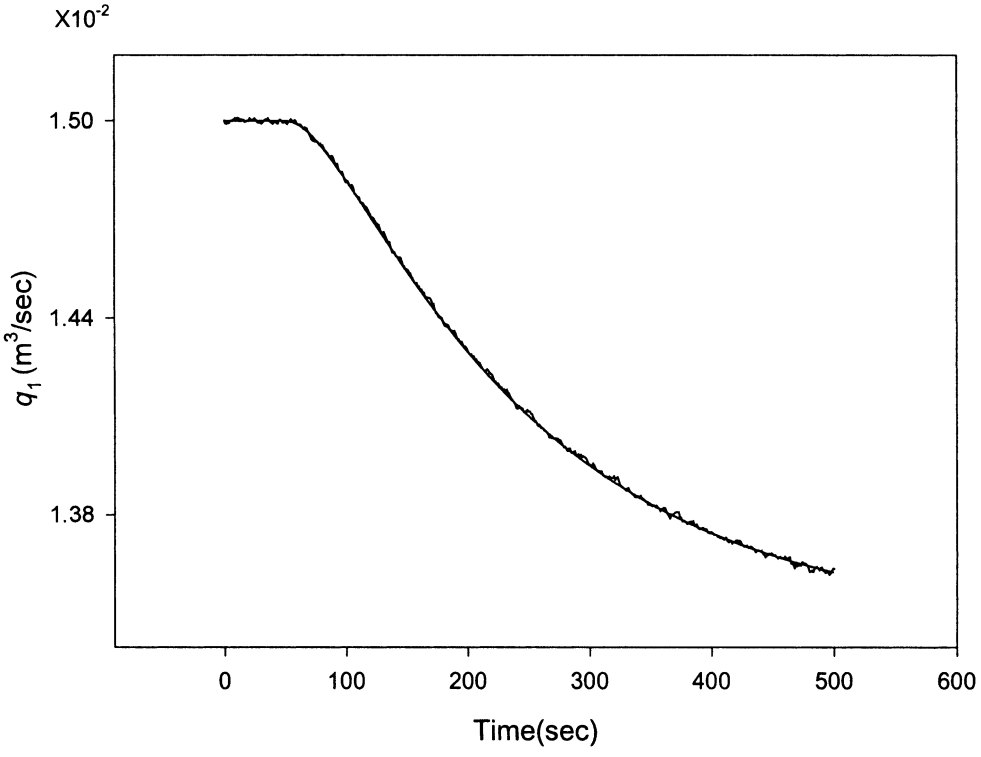

Fig. 10. EKF1 estimates of q_1 in Example 7.1.

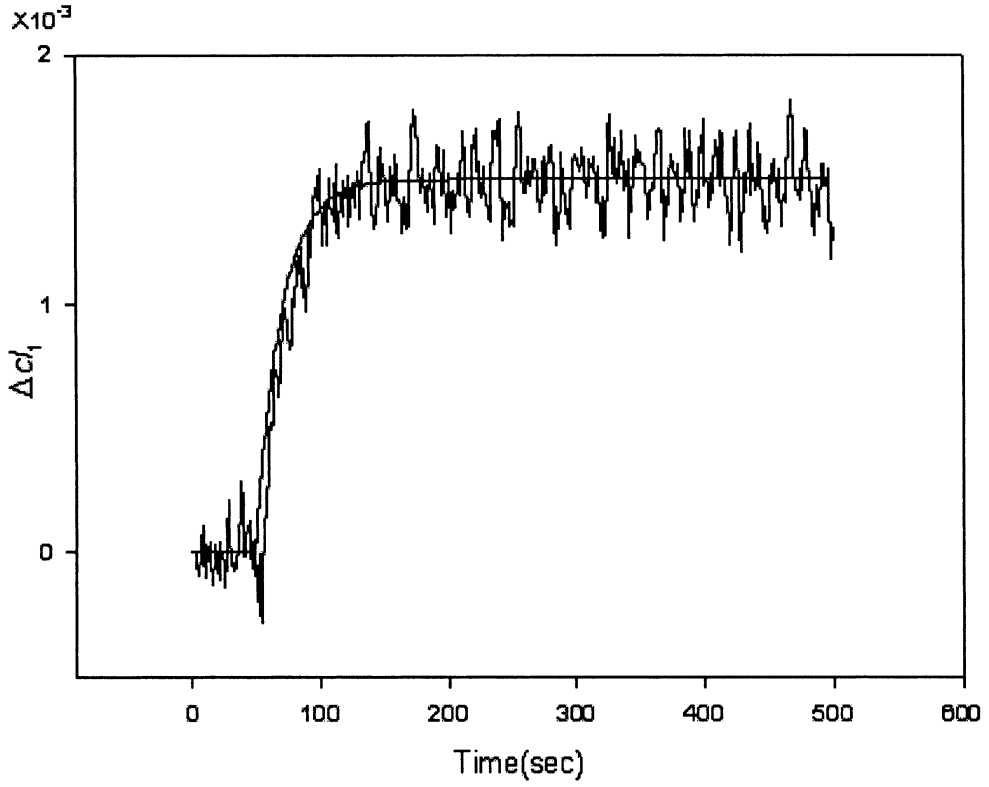

Fig. 11. EKF1 estimates of Δcl_1 in Example 7.1.

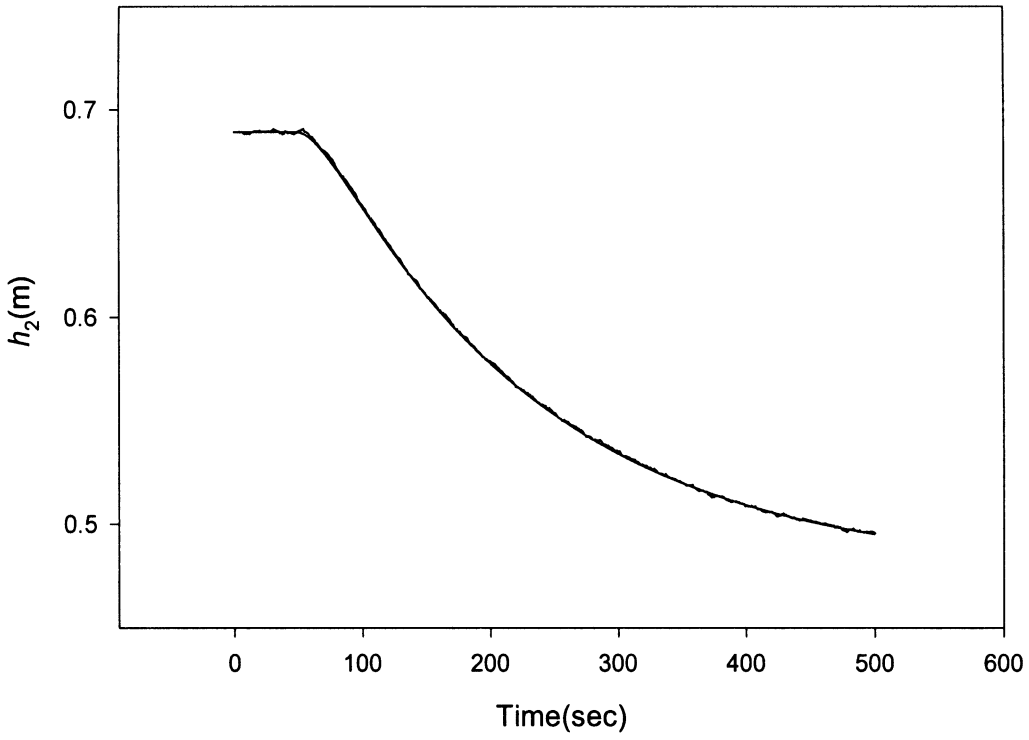

Fig. 12. EKF2 estimates of h_2 in Example 7.1.

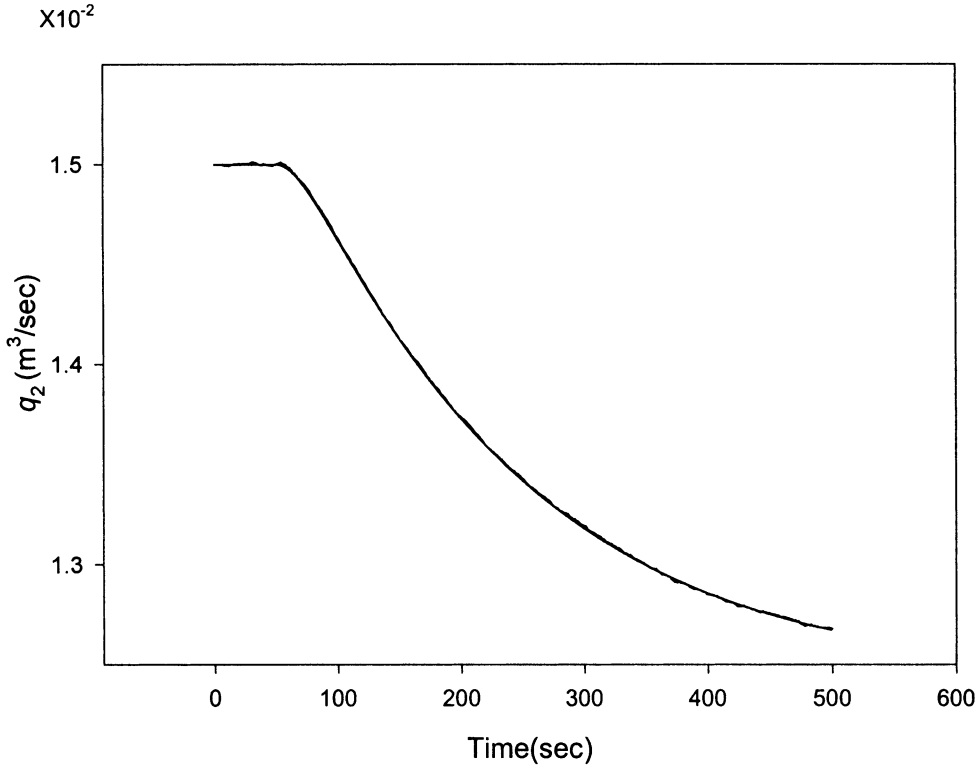

Fig. 13. EKF2 estimates of q_2 in Example 7.1.

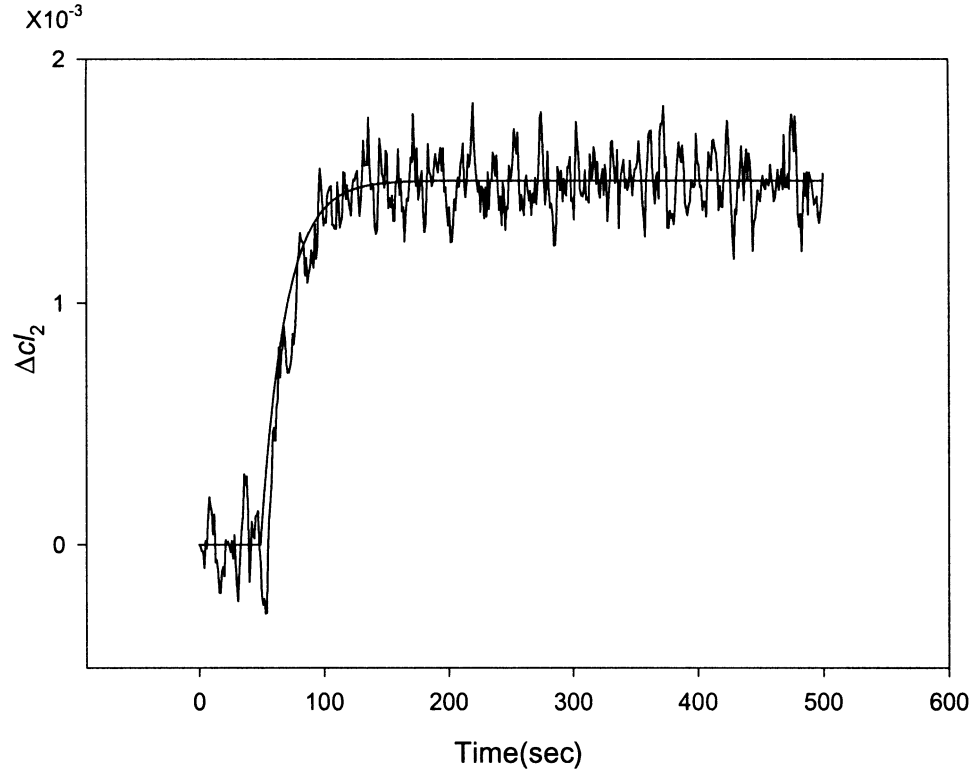

Fig. 14. EKF2 estimates of Δcl_2 in Example 7.1.

described their effects are Δy_0 and Δx_0 respectively. In EKF, they are treated as appended states and their respective model equations are

$$\frac{d\Delta y_0}{dt} = 0 \tag{8.15}$$

$$\frac{d\Delta x_0}{dt} = 0 \tag{8.16}$$

The above mathematical model, i.e. Eqs. (8.1)–(8.16), can be converted to a state diagram (Fig. 17) to facilitate implementation of the proposed decomposition strategy.

Case A. The measurement variables selected for this case are x_1, x_4 and Y. First, let us construct an open-loop observer by treating all measurements as inputs. To obtain the corresponding state diagram, one can simply remove all outward edges of the nodes representing the measured variables in Fig. 17. This is due to the fact that, in the open-loop observer, these information flows should be replaced by the on-line measurement data. The results are presented in Figs. 18 and 19. Notice that the dotted lines represent the removed edges and the squares denote the measured variables. One can clearly see that there are two separate components in this open-loop observer.

To establish an output-based correction mechanism, it is only necessary to consider the measured variables remained in each component. Their measurement data should be used as outputs of the corresponding sub-observer. Specifically, the EKF model associated with

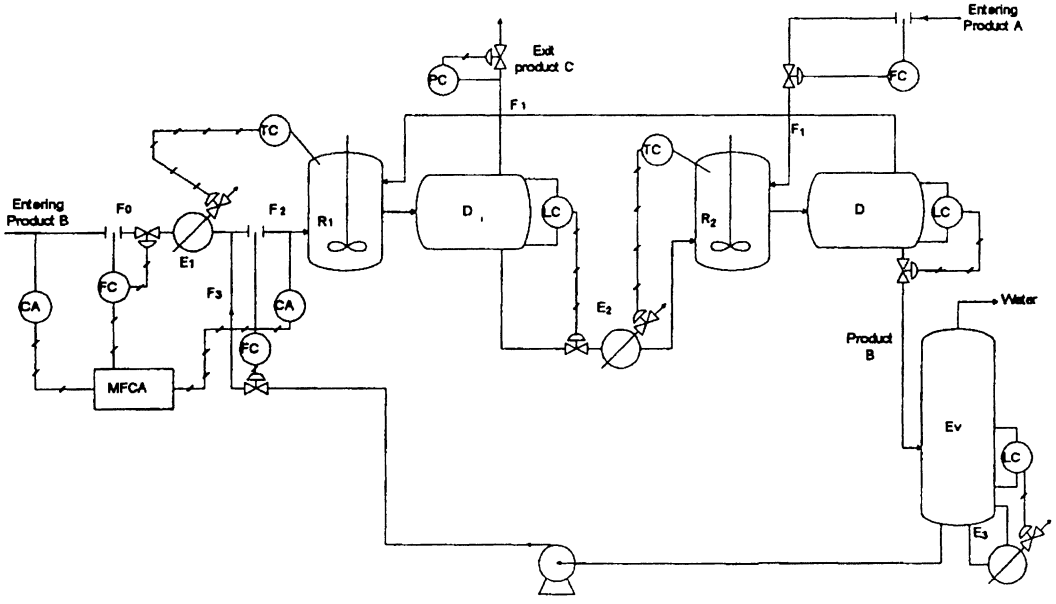

Fig. 15. Flow diagram of a two-phase reaction system.

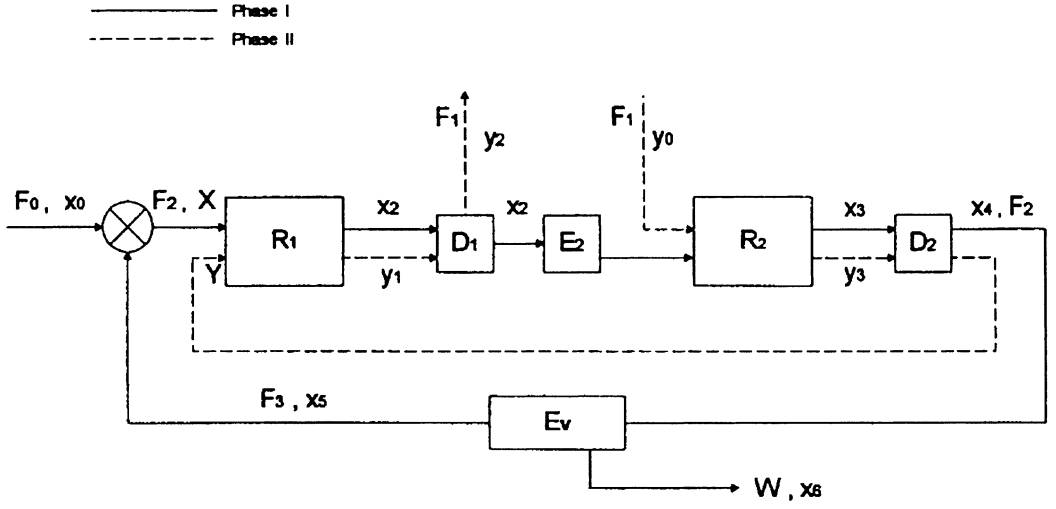

Fig. 16. Block diagram of the two-phase reaction process.

Fig. 18 should consist of Eqs. (8.4)–(8.9) and (8.15). The outputs of this sub-observer are the measurement signals of x_4 and Y, and the input is the measurement data of x_1. On the other hand, the EKF model corresponding to the second component in Fig. 19 can be built with Eqs. (8.1)–(8.3), (8.10)–(8.14) and (8.16). The output used in this EKF is the measurement of x_1 and the inputs should be the measurements of x_4 and Y. The feasibility and correctness of these two EKFs have been confirmed with extensive simulation studies [14]. A sample of the results can be found in Figs. 20 and 21.

The benefit of observer decomposition for nonlinear stochastic systems is demonstrated in this study by counting the number of differential equations that are required to be integrated numerically. This number N_{eq} can be computed according to

$$N_{eq} = \frac{(n+q)(n+q+1)}{2} + n$$

where n is the number of system states and q is the number of appended states. Notice that the first term on the right is the number of error covariance propagation equations or Riccati equations. This number can be significantly reduced by replacing the full-size observer with several smaller ones. Notice that, if the entire set of model equations are used in a single EKF, it is necessary to integrate 14 state estimate propagation equations and 136 error covariance propagation equations. On the other hand, one only needs to integrate 28 and 45 error covariance equations respectively in the two smaller EKFs suggested above. Thus, approximately 42% of the original computation load can be reduced.

Case B. Let us next try to decompose the state diagram into more components by selecting a different set of measurement variables. In this case, x_1, x_3, x_5, y_3 and F are chosen. By following the proposed decomposition procedure, four components can be identified (Figs. 22–25). Notice that the computation load now can be decreased to roughly 40% of the original level. Specifically, the total number of error covariance propagation equations used in the four corresponding EKFs is only 47. Again, based upon the results of our simulation studies, it can be observed that the performance of these EKFs is quite satisfactory [14].

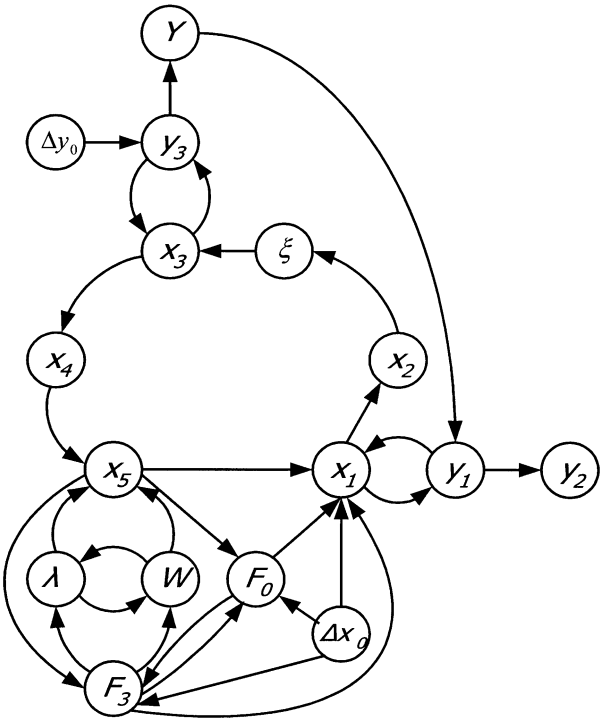

Fig. 17. The state diagram of system model.

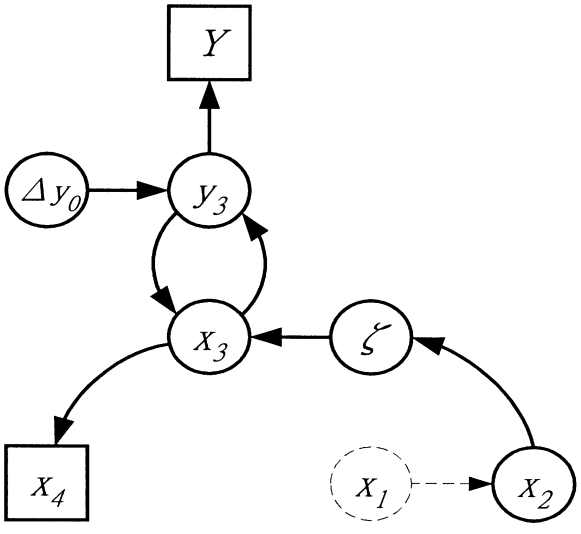

Fig. 18. A component in the open-loop observer of case A.

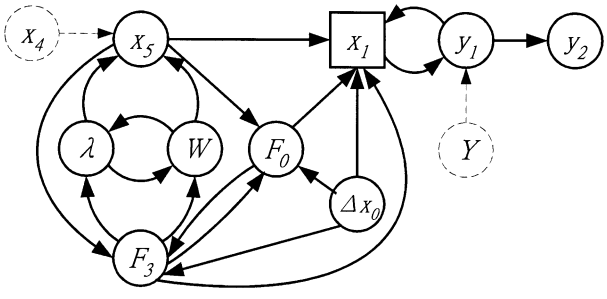

Fig. 19. Another component in the open-loop observer of Case A.

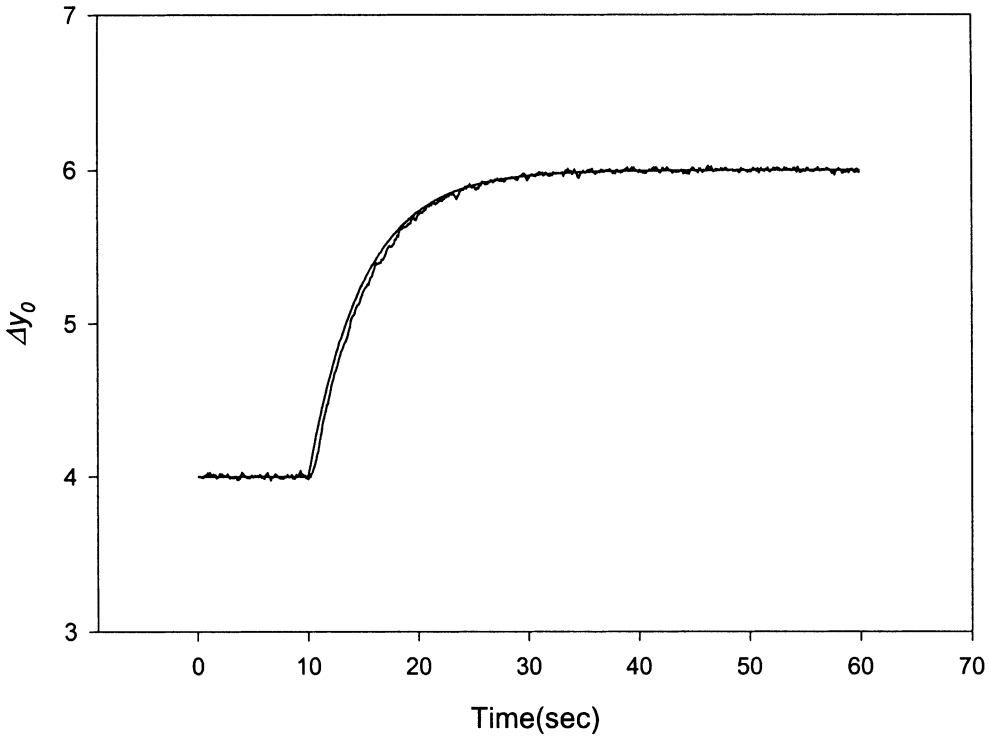

Fig. 20. Estimates of the inlet concentration disturbance of reactant A.

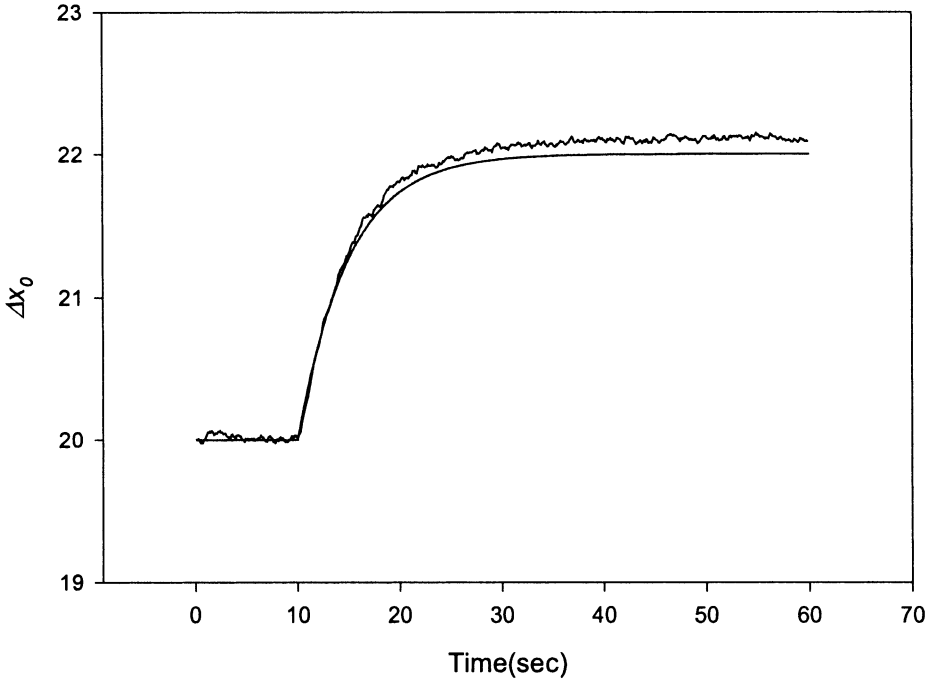

Fig. 21. Estimates of the inlet concentration disturbance of reactant B.

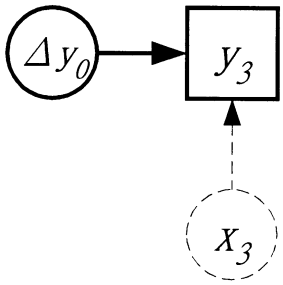

Fig. 22. Component 1 in Case B.

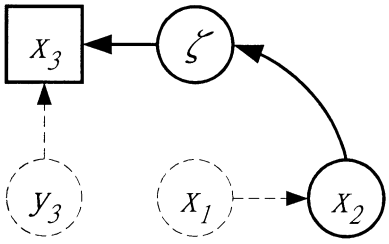

Fig. 23. Component 2 in Case B.

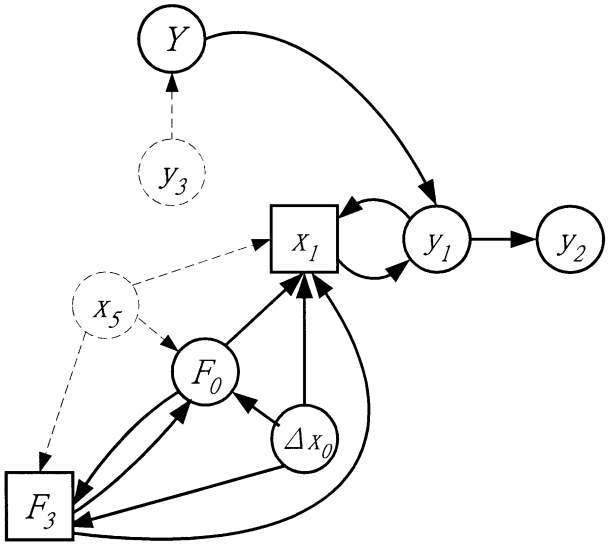

Fig. 24. Component 3 in Case B.

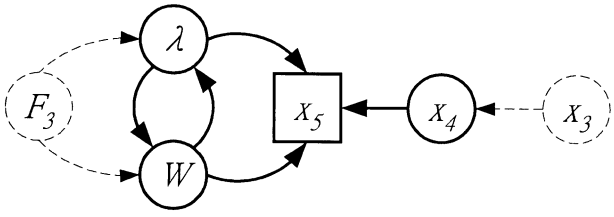

Fig. 25. Component 4 in Case B.

9. Conclusions

From the above discussions, it is clear that the classification of measurement signals in an output-injection observer does not really have to be consistent with the roles of inputs and outputs in the given physical system. On the basis of this proposition, a systematic procedure is proposed in this study to properly decompose any observer into several smaller ones according to the model structure. The potential benefits of using appended states to reduce observer size are also analyzed in this paper. The proposed decomposition and size-reduction techniques can be adopted to identify subsystems of interest and to relieve the on-line computation load of output-injection observers. In particular, they should be especially useful to those associated with nonlinear stochastic systems.

References

[1] K. Watanabe, D.M. Himmelblau, Fault diagnosis in nonlinear chemical processes—part I. Theory, AIChE Journal 29 (1983) 243.
[2] K. Watanabe, D.M. Himmelblau, Fault diagnosis in nonlinear chemical processes—part II. Application to a chemical reactor, AIChE Journal 29 (1983) 250.
[3] K. Watanabe, D.M. Himmelblau, Incipient fault diagnosis of nonlinear processes with multiple causes of faults, Chem. Eng. Sci. 39 (1984) 491.
[4] D.T. Dalle Molle, D.M. Himmelblau, Fault-detection in a single-stage evaporator via parameter estimation using the Kalman filter, Ind. Eng. Chem. Res. 84 (1987) 2482.
[5] C.T. Chang, K.N. Mah, C.S. Tsai, A simple design strategy for fault monitoring systems, AIChE Journal 39 (1993) 1146.
[6] M. Mangold, G. Lauschke, J. Schaffner, M. Zeitz, E.-D. Gilles, State and parameter estimation for adsorption columns by nonlinear distributed parameter state observers, J. Proc. Cont. 4 (1994) 163.
[7] F.L. Lewis, Applied Optimal Control and Estimation: Digital Design and Implementation, Prentice Hall, Englewood Cliffs, NJ, 1992.
[8] D.G. Luenberger, An introduction to observers, IEEE Trans. Auto. Cont. AC-16 (1971) 596.
[9] B. Gopinath, On the control of linear multiple input-output systems, The Bell Sys. Tech. Journal 50 (1971) 1063.
[10] C.T. Chen, Linear System Theory and Design, Rinehart and Winston, New York, 1984.
[11] J.Y. Keller, M. Darouach, Reduced-order Kalman filter with unknown inputs, Automatica 34 (1998) 1463.
[12] B.C. Kuo, Automatic Control Systems, Prentice Hall, Englewood Cliffs, NJ, 1975.
[13] D.M. Himmelblau, K.B. Bischoff, Process Analysis and Simulation, John Wiley & Sons, New York, 1968.
[14] Y. S. Lin, A Study Concerning the use of Inputs, Outputs and Appended States in Observers, MS thesis, National Cheng Kung University, Tainan, Taiwan, 1999.