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A game-theory based optimization strategy to configure inter-plant heat integration schemes

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HIGHLIGHTS

- A game-theory based strategy is used to configure inter-plant heat integration schemes.
- The HEN designs are generated according to a sequential optimization procedure.
- This strategy facilitates global minimization of the overall energy cost.
- This strategy also allows every participant to gain maximum achievable benefit.
- The effectiveness of this approach is demonstrated in two examples.

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ABSTRACT

The conventional heat exchanger network synthesis method is useful only for achieving maximum energy recovery (or minimum total annual cost) within a single chemical plant. If the same approach is applied to the hot and cold process streams in more than one plant on an industrial park, the resulting cost savings may be distributed unfairly among all involved parties. A systematic design procedure is developed to circumvent this drawback in the present study on the basis of game theory. Specifically, the inter-plant heat integration scheme is generated in four consecutive steps to determine (1) the lowest acceptable overall utility cost, (2) the proper heat flows between every pair of plants and also their fair trade prices (under the constraints of a lowest acceptable total utility cost and Nash equilibrium), (3) the minimum number of matches and the corresponding heat duties, and (4) the optimal network configuration. This sequential strategy allows every plant to maximize its own financial benefit at every step while simultaneously striving for the largest cost saving for the entire site. The case studies concerning a vinyl chloride process are also presented to demonstrate the feasibility of the proposed approach.

1. Introduction

Heat exchange network (HEN) synthesis in a single chemical plant is a matured research issue which has received considerable attention in the recent decades. Several critical reviews of the rich literature in this area have already been published, e.g., see Biegler et al. (1997) and Furman and Sahinidis (2002). From these reviews, it can be noted that a number of rigorous mathematical programming models are available for creating the optimal structure of heat-recovery system in any given process. Since it is generally believed that a greater level of energy/cost saving can be achieved by expanding the scope of integration to the entire industrial park, the emphases of some of the recent studies, e.g., Bagajewicz and Rodera (2002) and Kralj (2008), were shifted to the development of heat-exchange schemes across plant boundaries. In a study on industrial cluster, Matsuda et al. (2009) showed that even a highly integrated plant can further improve its energy efficiency via total-site heat integration (TSHI). Klemeš et al. (2013a, 2013b) later also demonstrated that TSHI can be very effective for enhancing the energy utilization efficiency of a large industrial site. On the other hand, Chew et al. (2013) pointed out that various other practical issues should also be considered for a successful implementation of TSHI methods, e.g., controllability, operability, reliability/availability and economic viability. Both graphic-based and mathematical

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programming approaches have been adopted for synthesizing the TSHI schemes, and the corresponding literature reviews are briefly summarized below:

- Graphical tools, such as the total-site profiles (TSP), the site composite curves (SCC), the site-utility grand composite curve (SUGCC), were first developed by Klemeš et al. (1997) for TSHI system analysis. In order to further reduce carbon emission, Perry et al. (2008) expanded the scope of integration by incorporating services within the residential areas as additional heat sinks and also the renewable energy sources. Varbanov et al. (2012) addressed issues concerning the minimum temperature difference (ΔT_{min}) for total-site targeting, and suggested that ΔT_{min} should be specified individually for each process on site. It was shown that this selection strategy could facilitate identification of more realistic heat recovery targets.
- Several mathematical programming models are available for generating optimal TSHI structures. Mavromatis and Kokossis (1998) developed the model of a steam system for TSHI. The resulting optimal design specifications include (1) the pressure levels and (2) the unit configuration at each level. Bandyopadhyay et al. (2010) proposed to use a simple method for targeting the cogeneration potential based on rigorous energy balances at the steam headers. Kapil et al. (2012) introduced a new model based on the assumption of isentropic steam expansion within the turbines. The obtained targets were found to be favorable compared with the results from the detailed isentropic design methods. Notice that this method also included an optimization study, which systematically determined the levels of the steam mains, subject to economic parameters and constraints. Becker and Maréchal (2012) proposed an inter-process heat integration method which incorporated additional options for not only the forbidden matches but also the intermediate heat transfer units. Liew et al. (2012) developed a computation procedure, known as the total-site problem-table algorithm (TS-PTA), for targeting the site-wide utility requirement. Recently, Liew et al. (2014) further extended TS-PTA to target the steam production rate for any site utility facility with a centralized condensate system.

In all aforementioned studies, the common objective was to minimize total energy usage of the entire site. Such an all-encompassing approach tried essentially to maximize the overall benefit, while neglected the economic incentives of individual plants. Consequently, the resulting heat-exchange arrangements may not always be acceptable to all participating parties.

To facilitate inter-plant heat integration in a realistic environment, each and every business entity must be allowed to pursue its own benefit as much as possible. In such a multi-agent system, if all behavioral patterns of interactions between agents are the results of players acting according to the available strategies in a game, then the mathematical tools of game theory are certainly applicable for analyzing the decisions of agents (Johansson, 1999). In fact, the cooperative game theory has already been utilized in a recent study (Hiete et al., 2012) for drawing up the benefit sharing plan in inter-plant heat integration. Specifically, a three-step design procedure was followed in this work, i.e.,

- Step 1: A single linear *transportation* problem (Cerda et al., 1983) was first solved to determine the overall energy cost target and also the corresponding heat flow pattern.
- Step 2: The network design was then produced *manually* by applying the traditional Pinch method (Linnhoff and Hindmarsh, 1983).
- Step 3: Having obtained the network design, the corresponding overall cost savings was distributed among all participants on the basis of the cooperative game theory.

Since manual heuristic manipulations are still required in this procedure, there is a strong incentive for developing a more rigorous approach for generating the optimal solutions systematically.

An optimal single-plant HEN design can be produced with either a simultaneous (Yee et al., 1990) or a sequential (Papoulias and Grossmann, 1983; Floudas et al., 1986) design strategy. The former usually yields a better trade-off between energy and capital costs since the sum of these two costs, i.e., the total annual cost (TAC), is minimized in a single step, but the computational effort needed for solving the corresponding mixed-integer nonlinear programming (MINLP) model may be overwhelming. On the other hand, although only suboptimal solutions can be obtained in the latter case, implementing a step-by-step procedure is expected to be much easier. Specifically, the following three simpler optimization problems are considered one-at-a-time consecutively:

- a linear programming (LP) model for determining the minimum total utility cost (Papoulias and Grossmann, 1983),
- a mixed integer linear programming (MILP) model for identifying the minimum number of matches and the corresponding heat duties (Papoulias and Grossmann, 1983), and
- a nonlinear programming (NLP) model for synthesizing the cost-optimal network (Floudas et al., 1986).

In principle, the above two approaches can both be applied to produce near “optimal” HEN designs if all process streams on site are handled indiscriminately. Since the resulting inter-plant heat integration schemes may not be acceptable to all participants, it is our intention here to address the practical issues encountered in distributing cost savings by properly modifying the latter method. This is due to not only a lighter computation load but also the fact that the game theoretic models can be more naturally incorporated into a sequential design practice when, in each step, the same type of decision variables can be evaluated on a consistent basis.

To achieve the aforementioned research goal, a four-step design procedure is devised in the present study for configuring the total-site heat-integration structure. After determining the *global* lower bound of total utility cost with the LP model used in the first step of the conventional approach, a novel NLP model can then be built to identify the feasible inter-plant heat flows in the given system. Since *the commodities to be traded* in the present applications are essentially various grades of energies, this model is formulated as a nonzero-sum matrix game, in which the proportion of heat flow to/from every temperature interval is regarded as a game strategy. With this view, the Nash equilibrium constraints (Nash, 1950; Rabin, 1993) can be imposed for solving the game while still keeping the overall utility cost at a target level (which can be determined in the first step). By incorporating the resulting inter-plant heat flows into the energy balance constraints for characterizing the integrated multi-plant heat flow cascade, the MILP model adopted in the second step of the conventional procedure can be slightly modified for identifying the minimum total number of both inner- and inter-plant matches and also the corresponding heat duties. Finally, after fixing the matches and heat duties, the approach to produce an optimal HEN structure should be no different from that suggested in the third step of conventional procedure. The corresponding superstructure and the model constraints can obviously be built in the same way as before, while an alternative objective function is adopted in this work for maximizing the individual TAC savings simultaneously.

Note that the first two steps mentioned above not only facilitates identification of the upper limit of overall energy cost but also allows every participant to gain maximum achievable benefit under the most acceptable price structure for inter-plant energy trades. On the basis of the resulting heat flow pattern, the next two design steps are then adopted to ensure optimal allocations of the matches and their duties and also maximize the TAC saving of each and every plant. It should be noted that there are really no fundamental differences between these latter two steps and their conventional counterparts. For clarity, a brief summary of the proposed design method is provided in the sequel:

2. The sequential optimization strategy

As mentioned previously, the traditional sequential approach for HEN synthesis has been modified in the present work. A series of four distinct optimization problems are solved consecutively as follows:

1. The minimum acceptable total utility cost of the entire site is first determined with a linear program, which can be formulated by modifying the conventional transshipment model (Papoulias and Grossmann, 1983).
2. By incorporating the constraints of minimum acceptable overall utility cost (obtained in Step 1) and also Nash equilibrium in a nonlinear program, the heat flows between every pair of plants on site and also their fair trade prices can be calculated accordingly.
3. By fixing the inter-plant heat-flow patterns determined in Step 2, the minimum total number of both inner- and inter-plant matches and the corresponding heat duties can be determined with an extended version of the conventional MILP model (Papoulias and Grossmann, 1983).
4. By following the approach suggested by Floudas et al. (1986) the superstructure of total-site HEN designs (in which all possible flow configurations are embedded) can be built to facilitate the matches identified in Step 3. A nonlinear programming model (Floudas et al., 1986; Floudas and Grossmann, 1987) can then be constructed for generating the optimal HEN configuration that maximizes the individual TAC saving of every plant.

A simple example is utilized throughout this paper to illustrate the above steps (see Tables 1 and 2) and, for convenience, will be referred to as Example 1. Fig. 1 shows the heat-flow cascades and pinch points of all plants in this example, which can be independently determined without considering inter-plant heat integration. The hot utility consumption rates of plants P1, P2 and P3 can be found to be 800 kW, 100 kW and 255 kW respectively, while the cold utility consumption rates are 210 kW, 160 kW and 670 kW respectively. The corresponding energy costs of these plants are 66,100 USD/yr, 6600 USD/yr and 30,300 USD/yr, respectively.

Table 1
The process data used in Example 1.

Plant	Stream	T_{in} (°C)	T_{out} (°C)	F_{cp} (kW/°C)
P1	H1	150	40	7
P2	H1	200	70	5.5
P3	H1	370	150	3.0
P3	H2	200	40	5.5
P1	C1	60	140	9
P1	C2	110	190	8
P2	C1	30	110	3.5
P2	C2	140	190	7.5
P3	C1	110	360	4.5

Table 2
The utility data used in Example 1.

Plant	Utility	T (°C)	Cost (USD/kW yr)	Upper bound (kW)
P1	Cooling water	25	10	1000
P1	HPS (240 psig)	200	90	1000
P1	Fuel	500	80	1000
P2	Cooling water	25	22.5	1000
P2	HPS (240 psig)	200	30	1000
P2	Fuel	500	120	1000
P3	Cooling water	25	30	1000
P3	HPS (240 psig)	200	60	1000
P3	Fuel	500	40	1000

3. The minimum acceptable site-wide utility cost

The minimum acceptable total utility cost of the industrial park is first determined on the basis of a modified version of the traditional transshipment model. To construct such a model, the entire temperature range is partitioned according to the inlet and outlet temperatures of *all* process streams on site (Papoulias and Grossmann, 1983). The heat flows into and out of every temperature interval in *each* plant are depicted in Fig. 2, and the corresponding linear program (LP) can be formulated as

$$\overline{ZT} = \min \sum_{p=1}^{P} \hat{Z}_p^U \tag{1}$$

subject to

$$R_k^p - R_{k-1}^p - \sum_{m_p \in S_k^p} Q_{m_p}^S - \sum_{\substack{q=1 \\ q \neq p}}^{P} Q_k^{qp} + \sum_{n_p \in W_k^p} Q_{n_p}^W + \sum_{\substack{q'=1 \\ q' \neq p}}^{P} Q_k^{pq'} = \Delta H_k^p \tag{2}$$

$$\Delta H_k^p = \sum_{i_p \in H_k^p} Q_{i_p k}^H - \sum_{j_p \in C_k^p} Q_{j_p k}^C = \left(\sum_{i_p \in H_k^p} F_{i_p}^H - \sum_{j_p \in C_k^p} F_{j_p}^C \right) \Delta T_k \tag{3}$$

$$\hat{Z}_p^U = \sum_{m_p \in S^p} c_{m_p} Q_{m_p}^S + \sum_{n_p \in W^p} c_{n_p} Q_{n_p}^W \tag{4}$$

$$\overline{Z}_p^U - \hat{Z}_p^U \geq 0 \tag{5}$$

$$R_0^p = 0; \quad R_K^p = 0; \quad R_1^p, R_2^p \cdots, R_{K-1}^p \geq 0 \tag{6}$$

$$Q_k^{qp}, Q_k^{pq'} \geq 0 \tag{7}$$

where $k\ (=1,2,3,\ldots,K)$ is the numerical label used for identifying a temperature interval, and p, q and $q'(=1,2,3,\ldots,P)$ are those for plants; $\overline{Z}_p^U$ is the lower bound of utility cost of plant p obtained via inner-plant heat integration only; $\hat{Z}_p^U$ is the minimum utility cost of plant p obtained via both inner- and inter-plant heat exchanges without treating the latter as energy trades. Note that the energy balance around the temperature interval in Fig. 2 is described mathematically with Eqs. (2) and (3). For the sake of brevity, the formal definitions of all other symbols in this model are placed in the nomenclature section. Finally, note that Eqs. (1)–(4), (6) and (7) are essentially reduced to the conventional transshipment formulation if $P=1$ and the extra constraint, i.e., Eq. (5), is imposed primarily to ensure the individual cost saving achieved by inter-plant heat integration is acceptable (i.e., nonnegative).

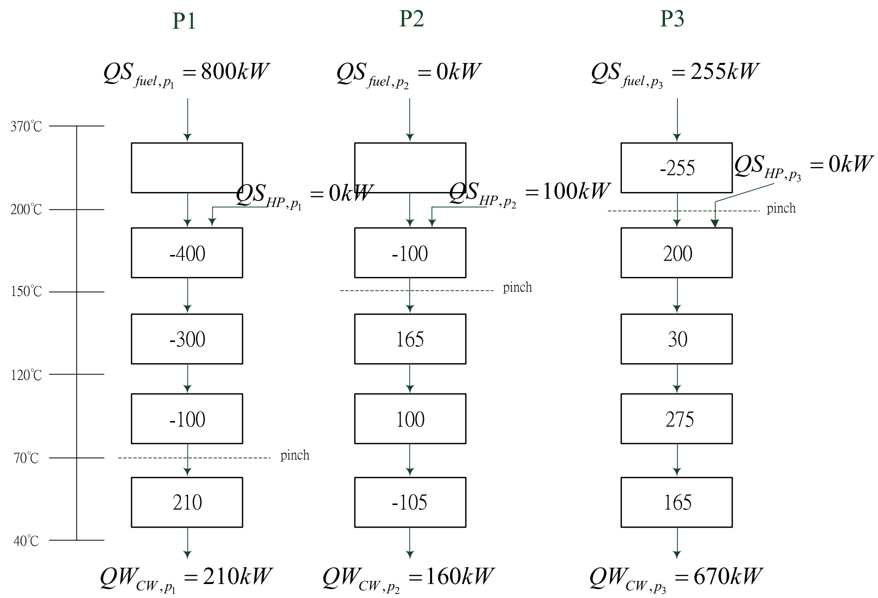

Fig. 1. The heat-flow cascades obtained without inter-plant heat integration in Example 1.

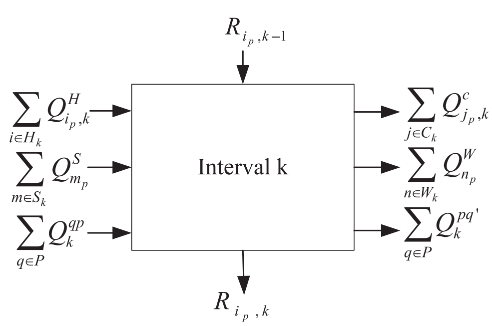

Fig. 2. The heat flows around interval k in plant p.

For Example 1, the integrated heat-flow cascade was obtained by solving the model described above (see Fig. 3). The minimum consumption rates of heating utility in P1, P2 and P3 were found to be 0, 220 and 440 kW respectively, while those of the cooling utilities were 485, 0 and 60 kW. The corresponding utility cost savings achieved by the three plants in this case can be calculated accordingly, i.e., 61,250, 0 and 10,900 USD/yr. A more detailed cost analysis is presented in Table S1 in Supplementary material. Notice from Fig. 3 that, although the hot utility of plant P2 is cheaper than those in the other plants, its consumption rate cannot exceed 220 kW since there is a need to satisfy Eq. (5).

4. The feasible inter-plant heat flows and their fair trade prices

The feasible inter-plant heat flows and their fair trade prices can *both* be determined in the second step according to a nonlinear program formulated by imposing Nash equilibrium constraints in another transshipment model. This approach takes into account of not only the fairness in energy trade but also heat-transfer efficiency, while only financial arrangements can be considered with the available savings allocation methods, e.g., see Hiete et al. (2012). Before presenting the corresponding results in Example 1, let us first consider the key components in this NLP model, i.e., the energy balances, the payoff matrices, the game strategies, the equilibrium constraints, the objective function, and the upper limits of total utility cost and consumption rates.

- Energy balances:
  In the present model, the energy balances around every temperature interval in each plant should also be treated as constraints, i.e., Eqs. (2), (3), (6) and (7).
- Payoffs:
  In the proposed multi-player game, every player (say plant p) can select one or more strategy from four alternatives, i.e., exporting heat at a temperature above or below the pinch (denoted as UD and LD) or importing heat at a temperature above or below the pinch (denoted as UA and LA). The structure of the payoff matrix $\mathbf{A}_p$ $(=[\mathbf{A}_{pq_1}|\cdots|\mathbf{A}_{pq_N}])$ for plant p can be expressed as

$$
\begin{array}{cc}
 & \begin{array}{cccccccccc} & \text{plant } q_1 & & & & \cdots & \text{plant } q_N & & & \\ UA & LA & UD & LD & & UA & LA & UD & LD \end{array} \\
\begin{array}{c} UD \\ LD \\ UA \\ LA \end{array} &
\left[\begin{array}{cccc|c|cccc}
\mathfrak{R}^{pUq_1U} & \mathfrak{R}^{pUq_1L} & NA & NA & & \mathfrak{R}^{pUq_NU} & \mathfrak{R}^{pUq_NL} & NA & NA \\
\mathfrak{R}^{pLq_1U} & \mathfrak{R}^{pLq_1L} & NA & NA & & \mathfrak{R}^{pLq_NU} & \mathfrak{R}^{pLq_NL} & NA & NA \\
NA & NA & \mathfrak{R}^{q_1UpU} & \mathfrak{R}^{q_1LpU} & \cdots & NA & NA & \mathfrak{R}^{q_NUpU} & \mathfrak{R}^{q_NLpU} \\
NA & NA & \mathfrak{R}^{q_1UpL} & \mathfrak{R}^{q_1LpL} & & NA & NA & \mathfrak{R}^{q_NUpL} & \mathfrak{R}^{q_NLpL}
\end{array}\right]
\end{array}
$$

where, $N = P-1$; $q_i \in \{1, 2, ..., p-1, p+1, ..., P\}$ and $i = 1, 2, \cdots, N$. Notice that, in this case, plant p and plant q_i are treated respectively as the row and column players of the ith submatrix $\mathbf{A}_{pq_i}$. Notice also that the symbol NA denotes the corresponding heat exchange is forbidden and the remaining payoff values (of plant p) are calculated according to the following formulas:

$$\mathfrak{R}^{pUq_iU} = -C_p^{HU} - C_{trd}^{pUq_iU} \tag{8}$$

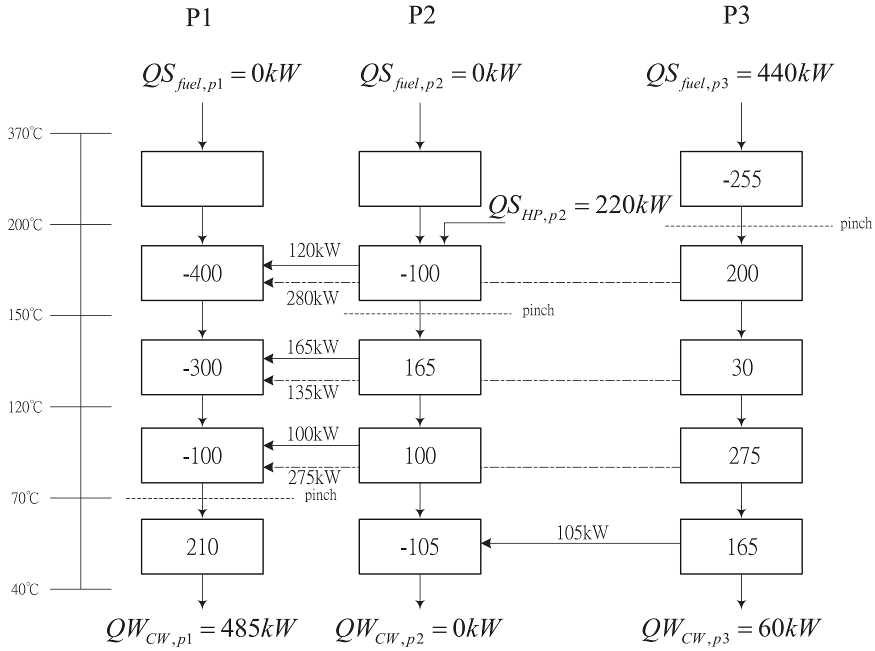

Fig. 3. The integrated heat-flow cascade obtained without energy trades in Example 1.

$$\Re^{pUq_iL} = -C_p^{HU} - C_{trd}^{pUq_iL} \tag{9}$$

$$\Re^{pLq_iU} = C_p^{CU} - C_{trd}^{pLq_iU} \tag{10}$$

$$\Re^{pLq_iL} = C_p^{CU} - C_{trd}^{pLq_iL} \tag{11}$$

$$\Re^{q_iUpU} = C_p^{HU} + C_{trd}^{q_iUpU} \tag{12}$$

$$\Re^{q_iLpU} = C_p^{HU} + C_{trd}^{q_iLpU} \tag{13}$$

$$\Re^{q_iUpL} = -C_p^{CU} + C_{trd}^{q_iUpL} \tag{14}$$

$$\Re^{q_iLpL} = -C_p^{CU} + C_{trd}^{q_iLpL} \tag{15}$$

In these equations, C_p^{HU} and C_p^{CU} denote the unit costs of hot and cold utilities of plant p, respectively, and their values should always be positive and *a priori* given. On the other hand, the second term in every formula represents the unknown unit trade price of the corresponding heat flow between plant p and plant q_i, and this flow is identified with superscript, e.g., $C_{trd}^{pLq_iU}$ denotes the unit trade price for the heat flow from below the pinch in plant p to above the pinch in plant q_i and $C_{trd}^{q_iUpL}$ denotes that from above the pinch in plant q_i to below the pinch in plant p, etc. To facilitate consistent model formulation, a positive cash flow is chosen in this study to coincide with the heat flow, i.e., a fee should be paid by source and received by sink.

If all trade prices are zero, only half of the above heat exchanges are beneficial to plant p. Specifically, the payoffs in Eqs. (8), (9), (14) and (15) should all be negative without the second terms on the right sides and, thus, extra costs are incurred in these scenarios. On the other hand, the other four heat transfers should all result in positive payoffs and extra savings under the condition that no fee can be assessed to plant p.

Introducing a nonzero trade price obviously changes the payoffs of the two plants involved in the corresponding heat exchange. This design option is adopted in the present study only for the purpose of (a) shifting the extra saving of one party partially to make up for a portion of the extra cost incurred to the other, or (b) redistributing the extra savings (or costs) if both benefited (or suffered) from such a heat transfer. These design purposes can be achieved by imposing the following inequality constraints:

$$-\max(C_p^{HU}, C_{q_i}^{HU}) \le C_{trd}^{pUq_iU} \le -\min(C_p^{HU}, C_{q_i}^{HU}) \tag{16}$$

$$-C_p^{HU} \le C_{trd}^{pUq_iL} \le C_{q_i}^{CU} \tag{17}$$

$$-C_{q_i}^{HU} \le C_{trd}^{pLq_iU} \le C_p^{CU} \tag{18}$$

$$\min(C_p^{CU}, C_{q_i}^{CU}) \le C_{trd}^{pLq_iL} \le \max(C_p^{CU}, C_{q_i}^{CU}) \tag{19}$$

$$-\max(C_{q_i}^{HU}, C_p^{HU}) \le C_{trd}^{q_iUpU} \le -\min(C_{q_i}^{HU}, C_p^{HU}) \tag{20}$$

$$-C_{q_i}^{HU} \le C_{trd}^{q_iUpL} \le C_p^{CU} \tag{21}$$

$$-C_p^{HU} \le C_{trd}^{q_iLpU} \le C_{q_i}^{CU} \tag{22}$$

$$\min(C_{q_i}^{CU}, C_p^{CU}) \le C_{trd}^{q_iLpL} \le \max(C_{q_i}^{CU}, C_p^{CU}) \tag{23}$$

For the sake of illustration brevity, let us consider only two examples, i.e., the first and third constraints. Notice that transferring a unit of heat from above the pinch in plant p to above the pinch in plant q_i inevitably results in a cost of C_p^{HU} for the former and a saving of $C_{q_i}^{HU}$ for the latter. On the basis of Eq. (16), plant q_i should pay plant p a fee of $-C_{trd}^{pUq_iU}$ per unit of transferred heat so as to make the payoff of plant p in Eq. (8) less negative if $C_p^{HU} \ge C_{q_i}^{HU}$ or positive if otherwise. On the other hand, by transferring a unit of heat from below the pinch in plant p to above the pinch in plant q_i, both parties can achieve savings. In this scenario, the overall saving per unit of transferred heat is $C_p^{CU} + C_{q_i}^{HU}$ and it is redistributed on the basis of Eq. (18). It can also be observed from Eq. (10) that the allowed payoff of plant p is bounded between zero and this upper limit.

- Strategy allocation:
For plant p, the weights placed upon the aforementioned four row strategies can be expressed as

$$PR_p^{UD} = \frac{1}{Q_p^E}\left(\sum_{k \in K_p^U} \sum_{\substack{q=1 \\ q \neq p}}^{P} Q_k^{pq}\right) \tag{24}$$

$$PR_p^{LD} = \frac{1}{Q_p^E}\left(\sum_{k \in K_p^L} \sum_{\substack{q=1 \\ q \neq p}}^{P} Q_k^{pq}\right) \tag{25}$$

$$PR_p^{UA} = \frac{1}{Q_p^E}\left(\sum_{k \in K_p^U} \sum_{\substack{q=1 \\ q \neq p}}^{P} Q_k^{qp}\right) \tag{26}$$

$$PR_p^{LA} = \frac{1}{Q_p^E}\left(\sum_{k \in K_p^L} \sum_{\substack{q=1 \\ q \neq p}}^{P} Q_k^{qp}\right) \tag{27}$$

$$PR_p^{UD} + PR_p^{LD} + PR_p^{UA} + PR_p^{LA} = 1 \tag{28}$$

where Q_k^{pq} and Q_k^{qp} respectively denote the heat flow transferred from interval k in plant p to interval k in plant q and vice versa; $Q_p^E = \sum_{k \in K} \sum_{q=1, q \neq p}^{P} (Q_k^{pq} + Q_k^{qp})$ is the total amount of heat exchanged externally by plant p; K_p^U and K_p^L denote the sets of temperature intervals above and below the pinch point of plant p. It is also clear that $K_p^U \cap K_p^L = \varnothing$ and $K_p^U \cup K_p^L = K$.
- Nash constraints:
The Nash equilibrium constraints (Quintas, 1989) can be expressed as

$$\mathbf{x}_p^T \sum_{\substack{q=1 \\ q \neq p}}^{P} \mathbf{A}_{pq}\mathbf{x}_q = \alpha_p \tag{29}$$

$$\sum_{\substack{q=1 \\ q \neq p}}^{P} \mathbf{A}_{pq}\mathbf{x}_q \leq \alpha_p \mathbf{J}_p \tag{30}$$

$$\mathbf{x}_p^T \mathbf{J}_p = 1 \tag{31}$$

$$\mathbf{J}_p = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \tag{32}$$

where $\mathbf{x}_p^T = [PR_p^{UD}, PR_p^{LD}, PR_p^{UA}, PR_p^{LA}]$ is the strategy vector of plant p; $\mathbf{x}_p^T = [PR_q^{UA}, PR_q^{LA}, PR_q^{UD}, PR_q^{LD}]$ denotes the strategy vector of plant q; $\mathbf{A}_{pq}$ is a sub-matrix of the payoff matrix $\mathbf{A}_p$ in which the payoff values between plant p and plant q are specified; and α_p denotes the average payoff value of plant p.
The above constraints are incorporated into the proposed NLP model to ensure that the heat exchanges in the inter-plant integration scheme are acceptable to all involved parties. Since no plant can gain any expected payoff if one or more party in the game chooses to deviate from the Nash equilibrium, the corresponding set of strategies adopted by each plant should be the best against those of the other parties.
- Objective function:
The objective function of the maximization problem in Step 2 is formulated as

$$\max \prod_{p=1}^{P} S_p^U \tag{33}$$

In this equation, $S_p^U \geq 0$ denotes the utility cost saving achieved by plant p after inter-plant integration with energy trades, and its value is calculated with the following formula:

$$S_p^U = \overline{Z}_p^U - Z_p'^U + pf_p \tag{34}$$

where $\overline{Z}_p^U$ denotes the minimum utility costs of plant p obtained with only inner-plant heat integration; $Z_p'^U$ denotes the total utility cost of plant p obtained via both inner- and inter-plant heat integration with *nonzero* trade prices; pf_p represents the revenue gained by plant p via inter-plant energy trades, i.e.

$$\begin{aligned} pf_p = & -\sum_{\substack{q'=1 \\ q' \neq p}}^{P} \left(C_{trd}^{pUq'U} \sum_{k \in K_p^U \cap K_{q'}^U} Q_k^{pq'} + C_{trd}^{pUq'L} \sum_{k \in K_p^U \cap K_{q'}^L} Q_k^{pq'} \right) \\ & -\sum_{\substack{q'=1 \\ q' \neq p}}^{P} \left(C_{trd}^{pLq'U} \sum_{k \in K_p^L \cap K_{q'}^U} Q_k^{pq'} + C_{trd}^{pLq'L} \sum_{k \in K_p^L \cap K_{q'}^L} Q_k^{pq'} \right) \\ & +\sum_{\substack{q=1 \\ q \neq p}}^{P} \left(C_{trd}^{qUpU} \sum_{k \in K_p^U \cap K_q^U} Q_k^{qp} + C_{trd}^{qUpL} \sum_{k \in K_p^L \cap K_q^U} Q_k^{qp} \right) \\ & +\sum_{\substack{q=1 \\ q \neq p}}^{P} \left(C_{trd}^{qLpU} \sum_{k \in K_p^U \cap K_q^L} Q_k^{qp} + C_{trd}^{qLpL} \sum_{k \in K_p^L \cap K_q^L} Q_k^{qp} \right) \end{aligned} \tag{35}$$

Notice that $Z'_p\hat{U}$ can be computed in the same way as $\hat{Z}_p^U$ in the previous model, i.e., by using Eq. (4), and S_p^U should always be nonnegative because, if otherwise, there are really no incentives for plant p to take part in the inter-plant heat integration scheme.
- Upper limits of total utility cost and consumption rates:
To ensure a reasonable pricing structure, the minimum total utility cost obtained in the previous step is used as an upper bound in the present step, i.e.,

$$\sum_{p=1}^{P} Z_p'^U \leq \overline{ZT} \tag{36}$$

It is also assumed that the supply rate of each utility generated on site cannot be unlimited, i.e.

$$\sum_{p=1}^{P} Q_{m_p}^S \leq \overline{QS}_m \quad m \in S \tag{37a}$$

$$\sum_{p=1}^{P} Q_{n_p}^W \leq \overline{QW}_n \quad n \in W \tag{37b}$$

where $\overline{QS}_m$ and $\overline{QW}_n$ are the chosen upper bounds of the hot and cold utility consumption rates.
By solving the above model for Example 1, one can obtain the following strategy vectors and the corresponding payoff matrices:

$$\mathbf{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}; \quad \mathbf{x}_2 = \begin{bmatrix} 0.415 \\ 0.361 \\ 0 \\ 0.224 \end{bmatrix}; \quad \mathbf{x}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{A}_{12} & \mathbf{A}_{13} \end{bmatrix} = \left[\begin{array}{cccc|cccc} -60 & 0 & NA & NA & 0 & 0 & NA & NA \\ 40 & 0 & NA & NA & 70 & 0 & NA & NA \\ NA & NA & 52.2 & 105 & NA & NA & 0 & 51.7 \\ NA & NA & 0 & 12.5 & NA & NA & -70 & 0 \end{array}\right]$$

$$\mathbf{A}_2 = \begin{bmatrix} \mathbf{A}_{21} & \mathbf{A}_{23} \end{bmatrix}$$

Table 3
The utility cost savings achieved in Example 1.

Plant	Trade revenue (USD/yr)	Net saving (USD/yr)
P1	−26,781	34,469
P2	12,413	5513
P3	14,368	34,468

Table 4
The utility consumption rates needed for inter-plant heat integration with energy trades in Example 1.

Plant	Fuel (kW)	Steam (kW)	CW (kW)
P1	0	0	485
P2	0	405	60
P3	255	0	0

Table 5
The inter-plant heat flows obtained with energy trades in Example 1.

Interval	P2→P1 (kW)	P3→P1 (kW)	P3→P2 (kW)
1	0	0	0
2	305	95	0
3	165	135	0
4	100	275	0
5	0	0	165

$$= \left[\begin{array}{cccc|cccc} 7.5 & -40 & NA & NA & 0 & -10.7 & NA & NA \\ 7.5 & 0 & NA & NA & 82.5 & 0 & NA & NA \\ NA & NA & 0 & 0 & NA & NA & -30 & 0 \\ NA & NA & -112.5 & -12.5 & NA & NA & -82.5 & 7.5 \end{array}\right]$$

$$\mathbf{A}_3 = \begin{bmatrix} \mathbf{A}_{31} & \mathbf{A}_{32} \end{bmatrix}$$

$$= \left[\begin{array}{cccc|cccc} 30 & 0 & NA & NA & 0 & 0 & NA & NA \\ 68.3 & 20 & NA & NA & 60 & 0 & NA & NA \\ NA & NA & -30 & 0 & NA & NA & 30 & 0 \\ NA & NA & -120 & -20 & NA & NA & -49.3 & -7.5 \end{array}\right]$$

The resulting unit trade prices can be found in Table S2 in Supplementary material. The average payoffs received by the plants can be respectively determined to be 111.4 USD/yr, 7.5 USD/yr and 68.3 USD/yr, which indicate that plant P1 benefits the most from inter-plant heat integration. The required utility costs of every plant before and after integration are presented in Table S3 in Supplementary material, and the corresponding cost savings are also listed in the same table. The total revenue received by each plant via energy trades and the resulting net saving in utility cost can be found in Table 3. To provide further insights into the optimal integration scheme, the utility consumption rates of each plant and the inter-plant heat flows are also presented in Tables 4 and 5 respectively. Notice that these data will also be used in the next step.

5. The minimum number of matches and their heat duties

In the third step of the proposed procedure, the minimum number of exchangers and the corresponding heat duties are determined by solving a modified version of the conventional MILP model (Floudas and Ciric, 1989). This model can be formulated according to the generalized heat-flow pattern in Fig. 4, which is in essence a zoom-in picture of Fig. 2. The detailed energy balances associated with the corresponding temperature interval can be established in a straightforward fashion by considering the input and output heat flows

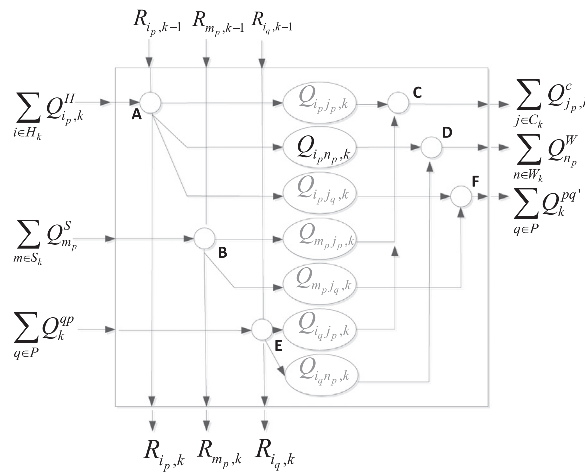

Fig. 4. The generalized heat flow pattern around and within interval k in plant p.

around every node in Fig. 4 one by one. Specifically, these model constraints are listed exhaustively in the sequel:

- *Node A*:

$$R_{i_p,k}-R_{i_p,k-1}+\sum_{j_p\in C_k^p}Q_{i_pj_pk}+\sum_{n_p\in W_k^p}Q_{i_pn_pk}+\sum_{\substack{q'=1\\ q'\neq p}}^{P}\sum_{j_{q'}\in C_k^{q'}\cup W_k^{q'}}Q_{i_pj_{q'}k}=Q^H_{i_p,k}$$
$$i_p\in H_k^p \tag{38}$$

Note that the heat delivered to interval k from hot stream i_p in plant p, i.e., $Q^H_{i_p,k}$, is a model parameter which can be calculated according to the given stream data.

- *Node B:*

$$R_{m_p,k}-R_{m_p,k-1}+\sum_{j_p\in C_k^p}Q_{m_pj_pk}+\sum_{\substack{q'=1\\ q'\neq p}}^{P}\sum_{j_{q'}\in C_k^{q'}}Q_{m_pj_{q'}k}=Q^S_{m_p}$$
$$m_p\in S_k^p \tag{39}$$

It should be noted that the consumption rate of hot utility m_p (i.e., $Q^S_{m_p}$) in interval k is a given parameter in the present model, and its value can be determined in Step 2 with a nonlinear program.

- *Node C:*

$$\sum_{i_p\in H_k^p}Q_{i_pj_pk}+\sum_{m_p\in S_k^p}Q_{m_pj_pk}+\sum_{\substack{q=1\\ q\neq p}}^{P}\sum_{i_q\in H_k^q\cup S_k^q}Q_{i_qj_pk}=Q^C_{j_p,k}$$
$$j_p\in C_k^p \tag{40}$$

Note that the heat transported from interval k to cold stream j_p in plant p, i.e., $Q^C_{j_p,k}$, is a model parameter which can be calculated according to the given stream data.

- *Node D:*

$$\sum_{i_p\in H_k^p}Q_{i_pn_pk}+\sum_{\substack{q=1\\ q\neq p}}^{P}\sum_{i_q\in H_k^q\cup S_k^q}Q_{i_qn_pk}=Q^W_{n_p}$$
$$n_p\in W_k^p \tag{41}$$

Note that the consumption rate of cold utility n_p (i.e., $Q^W_{n_p}$) in interval k is a given parameter which can be determined in Step 2.

Table 6
The optimal inner- and inter-plant matches obtained in Example 1.

Match #	Hot stream	Cold stream	Heat duty (kW)
1	P1_H1	P1_C1	285
2	P1_H1	P1_CW	485
3	P2_H1	P1_C1	160
4	P2_H1	P1_C2	380
5	P2_H1	P2_C1	175
6	P3_H1	P3_C1	660
7	P3_H2	P1_C1	275
8	P3_H2	P1_C2	230
9	P3_H2	P2_C1	105
10	P3_H2	P3_C1	210
11	P3_H2	P2_CW	60
12	P2_HP	P1_C2	30
13	P2_HP	P2_C2	375
14	P2_Fuel	P3_C1	255

- *Node E:*

$$R_{i_q,k}-R_{i_q,k-1}+\sum_{j_p \in C_k^p} Q_{i_qj_pk}+\sum_{n_p \in W_k^p} Q_{i_qn_pk}=Q_{i_q,k}^{HE_1} \quad i_q \in H_k^q \tag{42}$$

$$R_{i_q,k}-R_{i_q,k-1}+\sum_{j_p \in C_k^p} Q_{i_qj_pk}=Q_{i_q,k}^{HE_2} \quad i_q \in S_k^q \tag{43}$$

$$\sum_{i_q \in H_k^q} Q_{i_q,k}^{HE_1}+\sum_{i_q \in S_k^q} Q_{i_q,k}^{HE_2}=Q_k^{qp} \tag{44}$$

where $q=1,2,\cdots,p-1,p+1,\cdots,P$, and the inter-plant heat flow Q_k^{qp} is a given parameter which should be determined in Step 2.

- *Node F:*

$$\sum_{i_p \in H_k^p} Q_{i_pj_{q'}k}+\sum_{m_p \in S_k^p} Q_{m_pj_{q'}k}=Q_{j_{q'},k}^{CE_1} \quad j_{q'} \in C_k^{q'} \tag{45}$$

$$\sum_{i_p \in H_k^p} Q_{i_pj_{q'}k}=Q_{j_{q'},k}^{CE_2} \quad j_{q'} \in W_k^{q'} \tag{46}$$

$$\sum_{j_{q'} \in C_k^{q'}} Q_{j_{q'},k}^{CE_1}+\sum_{j_{q'} \in W_k^{q'}} Q_{j_{q'},k}^{CE_2}=Q_k^{pq'} \tag{47}$$

where $q'=1,2,\ldots,p-1,p+1,\cdots,P$, and the inter-plant heat flow $Q_k^{pq'}$ is a given parameter which should be determined in Step 2.

Let us next define a set of binary variables:

$$z_{i_pj_q}=\begin{cases}1 & \text{if there is heat exchange between } i_p \text{ and } j_q\\ 0 & \text{otherwise}\end{cases} \tag{48}$$

where $i_p \in H_k^p \cup S_k^p, j_q \in C_k^q \cup W_k^q$ and $p,q=1,2,\cdots,P$. The following inequality constraints can then be imposed accordingly:

$$\sum_{k \in K} Q_{i_pj_qk}-z_{i_pj_q}U_{i_pj_q} \le 0 \tag{49}$$

where $U_{i_pj_q}$ is the maximum heat exchange rate between hot stream i_p in plant p and cold stream j_q in plant q, and $Q_{i_pj_qk}$ is the rate of this heat exchange within interval k.

The objective function of the proposed MILP model can be expressed as

$$\overline{NU}=\min\left(\sum_{p=1}^{P}\sum_{i_p \in H^p}\sum_{j_p \in C^p} z_{i_pj_p}+\sum_{p=1}^{P}\sum_{i_p \in S^p}\sum_{j_p \in C^p} z_{i_pj_p}+\sum_{p=1}^{P}\sum_{i_p \in H^p}\sum_{j_p \in W^p} z_{i_pj_p}+\sum_{p=1}^{P}\sum_{\substack{q=1\\q\neq p}}^{P}\sum_{i_p \in H^p}\sum_{j_p \in C^p} z_{i_pj_q}+\sum_{p=1}^{P}\sum_{\substack{q=1\\q\neq p}}^{P}\sum_{i_p \in S^p}\sum_{j_p \in C^p} z_{i_pj_q}+\sum_{p=1}^{P}\sum_{\substack{q=1\\q\neq p}}^{P}\sum_{i_p \in H^p}\sum_{j_p \in W^p} z_{i_pj_q}\right) \tag{50}$$

After constructing the MILP model for Example 1 and carrying out the corresponding optimization run, the minimum unit number can be found to be 14 and the optimal matches are shown in Table 6.

6. The optimal network configuration

Since only the matches and their heat duties are fixed in Step 3, further information about the network structure and the design specifications of each exchanger must be obtained for calculating the total investment cost of a HEN design. This task has been traditionally accomplished with a superstructure-based NLP model (Floudas et al., 1986). Essentially the same approach is taken in the present study to build the model constraints, while a different objective function is adopted to facilitate reasonable distribution of TAC savings. Specifically, the design objective of this step is to maximize the product of total cost savings of all plants, i.e.

$$\max \prod_{p=1}^{P} S_p^T \tag{51}$$

and each individual TAC saving can be determined according to the following formula:

$$S_p^T=S_p^U+Af\left(\overline{\overline{Z}}_p^C-\hat{Z}_p^C-\sum_{\substack{q'=1\\q'\neq p}}^{P} SC_{pq'}^p-\sum_{\substack{q=1\\q\neq p}}^{P} SC_{qp}^p\right) \tag{52}$$

where S_p^U denotes the utility cost saving achieved by plant p (which can be calculated in Step 2); Af is the annualization factor; $\overline{\overline{Z}}_p^C$ is the minimum total capital cost of HEN in plant p *without* inter-plant heat integration (which can be determined independently with a traditional sequential design procedure); $\hat{Z}_p^C$ is the total capital cost of all inner-plant heat exchangers in plant p after inter-plant heat integration; $SC_{pq'}^p$ is the total capital cost shared by plant p to facilitate inter-plant heat exchanges between the hot streams in plant p and the cold streams in plant q'; SC_{qp}^p is the total capital cost shared by plant p to facilitate inter-plant heat exchanges between the hot streams in plant q and the cold streams in plant p. Notice that only the last three costs are adjustable in the present model and they can be evaluated with the following formulas:

$$\hat{Z}_p^C=\sum_{i_p \in H^p \cup S^p}\sum_{j_p \in C^p \cup W^p} z_{i_pj_p}c_{i_pj_p}\left\{\frac{Q_{i_pj_p}}{U_{i_pj_p}\left[\theta_{i_pj_p}^1\theta_{i_pj_p}^2(\theta_{i_pj_p}^1+\theta_{i_pj_p}^2)/2\right]^{\frac{1}{3}}}\right\}^{\beta} \tag{53}$$

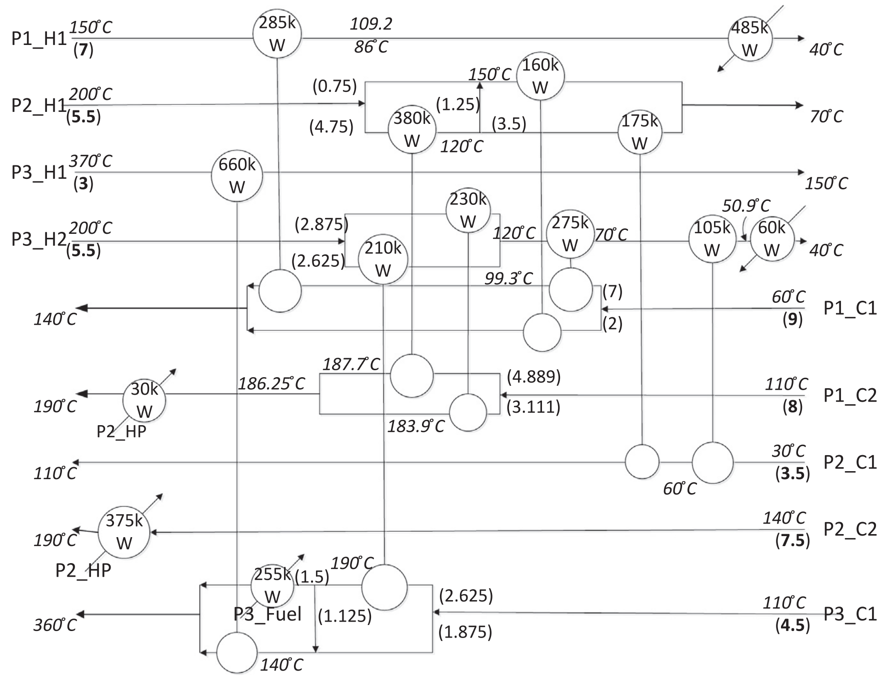

Fig. 5. The optimal inter-plant HEN design in Example 1.

$$SC_{pq'} = \sum_{i_p \in H^p \cup S^p} \sum_{j_{q'} \in C^{q'} \cup W^p} z_{i_p j_{q'}} \gamma^p_{i_p j_{q'}} c_{i_p j_{q'}} \left\{ \frac{Q_{i_p j_{q'}}}{U_{i_p j_{q'}} \left[\theta^1_{i_p j_{q'}} \theta^2_{i_p j_{q'}} (\theta^1_{i_p j_{q'}} + \theta^2_{i_p j_{q'}})/2 \right]^{\frac{1}{3}}} \right\}^{\beta} \tag{54}$$

$$SC_{qp} = \sum_{i_q \in H^q \cup S^q} \sum_{j_p \in C^p \cup W^p} z_{i_q j_p} \gamma^p_{i_q j_p} c_{i_q j_p} \left\{ \frac{Q_{i_q j_p}}{U_{i_q j_p} \left[\theta^1_{i_q j_p} \theta^2_{i_q j_p} (\theta^1_{i_q j_p} + \theta^2_{i_q j_p})/2 \right]^{\frac{1}{3}}} \right\}^{\beta} \tag{55}$$

In these cost models, each term is associated with a match between hot stream i and cold stream j, and Q_{ij}, c_{ij}, U_{ij}, θ^1_{ij} and θ^2_{ij} respectively denote the heat duty, the cost coefficient, the overall heat transfer coefficient, and the temperature differences at the hot and cold ends of the corresponding heat exchanger. Note also that γ^p_{ij} represents the proportion of capital cost that is paid by plant p for match (i,j), and the following constraints should also be incorporated in the NLP model:

$$\gamma^p_{i_p j_{q'}} + \gamma^{q'}_{i_p j_{q'}} = 1 \tag{56}$$

$$\gamma^q_{i_q j_p} + \gamma^p_{i_q j_p} = 1 \tag{57}$$

A distinct superstructure has been constructed according to Table 6 for each process stream in Example 1 and the material and energy balances in this structure were then formulated accordingly. In Example 1, the following model parameters were chosen for the purpose of demonstrating the usefulness of the proposed approach:

$c_{ij} = 670\ USD/m^{1.66}$
$U_{ij} = 1\ W/m^2\ K$
$\beta = 0.83$
$Af = 0.1349$
$\Delta T_{min} = 5^\circ C$

Table 7
The capital costs of inter-plant heat exchangers in the optimal HEN design for Example 1.

Hot stream	Cold stream	Area (m^2)	Capital cost (USD)
P2_H1	P1_C1	16	13,291
P2_H1	P1_C2	34.25	19,185
P2_HP	P1_C2	1.375	7473
P3_H2	P1_C1	18.698	14,215
P3_H2	P1_C2	17.970	13,968
P3_H2	P2_C1	7.102	10,010
P3_H2	P2_CW	3.372	8438

From Table 3, note that the net utility cost savings achieved by P1, P2 and P3 are 34,469 USD/yr (S^U_1), 5513 USD/yr (S^U_2) and 34,468 USD/yr (S^U_3). By applying the traditional sequential design strategy to generate the HENs for the three plants individually, the optimal annualized capital costs were found to be 5891 USD/yr ($Af \cdot \overline{Z}^C_1$), 5861 USD/yr ($Af \cdot \overline{Z}^C_2$) and 7865 USD/yr ($Af \cdot \overline{Z}^C_3$), respectively. The resulting optimal inter-plant HEN design is given in Fig. 5. The capital costs of all inter-plant exchangers in this design can be found in Table 7. Note that, in this work, the capital cost of every inter-plant unit is shared by the two parties involved in the corresponding heat exchange. The optimized proportions of their payments are shown in Table 8. Based on the above data, the individual TAC savings of the three plants can be determined to be 30,099, 8039 and 30,099 USD/yr, respectively. A more detailed economic analysis is also given in Table S4 in Supplementary material. It can be observed that, although the inter-plant heat integration scheme results in an increase in the capital cost, the reduction in the utility cost is more than enough to justify the extra investment. The proposed optimization procedure also ensures fair distribution of financial benefits among all participating members. Finally, it should be noted that the additional energy saving achieved with inter-plant integration also implies that the corresponding CO_2 emission rate is much less.

7. An additional example: the vinyl chloride process

The vinyl chloride monomer (VCM) is traditionally produced with ethylene, chlorine and oxygen. In this example, let us assume that three different firms are interested in a joint venture to build manufacturing facilities and produce VCM on an industrial park. Due to the unique technical expertise and financial constraint of each company, this process is divided into three separate plants (see Fig. 6) for the participating parties to invest and run independently. The process data used in this example (see Table 9) were taken from Lakshmanan et al. (1999). Since the utilities are assumed to be provided by the same systems on site, their prices are identical for all plants (see Table 10).

To facilitate realistic cost estimation, the following models have been utilized to replace Eqs. (53)–(55) (Douglas, 1988):

$$\hat{Z}_p^C = \sum_{i_p \in H^p \cup S_H^p} \sum_{j_p \in C^p \cup W^p} z_{i_p j_p} \left(\frac{M\&S}{280}\right) 101.3 A_{i_p j_p}^{0.65} (2.29 + F_c) + \sum_{i_p \in S_{fuel}^p} \sum_{j_p \in C^p} z_{i_p j_p} \left(\frac{M\&S}{280}\right) (5.07 \times 10^3) Q_{i_p j_p}^{0.85} (1.23 + F_c^{fuel}) \qquad (58)$$

$$SC_{pq'} = \sum_{i_p \in H^p \cup S_H^p} \sum_{j_{q'} \in C^{q'} \cup W^{q'}} z_{i_p j_{q'}} \gamma_{i_p j_{q'}}^p \left(\frac{M\&S}{280}\right) 101.3 A_{i_p j_{q'}}^{0.65} (2.29 + F_c) + \sum_{i_p \in S_{fuel}^p} \sum_{j_{q'} \in C^{q'}} z_{i_p j_{q'}} \gamma_{i_p j_{q'}}^p \left(\frac{M\&S}{280}\right) (5.07 \times 10^3) Q_{i_p j_{q'}}^{0.85} (1.23 + F_c^{fuel}) \qquad (59)$$

$$SC_{qp} = \sum_{i_q \in H^q \cup S_H^q} \sum_{j_p \in C^p \cup W^p} z_{i_q j_p} \gamma_{i_q j_p}^p \left(\frac{M\&S}{280}\right) 101.3 A_{i_q j_p}^{0.65} (2.29 + F_c) + \sum_{i_q \in S_{fuel}^q} \sum_{j_p \in C^p} z_{i_q j_p} \gamma_{i_q j_p}^p \left(\frac{M\&S}{280}\right) (5.07 \times 10^3) Q_{i_q j_p}^{0.85} (1.23 + F_c^{fuel}) \qquad (60)$$

In the above models, S_H^p denotes the set of hot utilities in plant p and S_{fuel}^p denotes the set of fuels in plant p. Note that $S_H^p \cap S_{fuel}^p = \varnothing$ and $S_H^p \cup S_{fuel}^p = S^p$. Note also that the Marshall & Swift (M&S) index is chosen to be 914 (at 1989), because this year's utility costs

Table 8
The optimal pay proportions for the capital costs of inter-plant heat exchangers in Example 1.

Hot stream	Cold stream	Plant 1	Plant 2	Plant 3
P2_H1	P1_C1	1	0	–
P2_H1	P1_C2	1	0	–
P2_HP	P1_C2	1	0	–
P3_H2	P1_C1	0.411	–	0.589
P3_H2	P1_C2	0.054	–	0.946
P3_H2	P2_C1	–	0	1
P3_H2	P2_CW	–	0	1

Table 9
The process data used in the VCM example.

Stream	T_{in} (°C)	T_{out} (°C)	F_{CP} (kW/°C)
P1_H1	192.6	57.3	1.37
P1_C1	25.0	193.1	1.19
P2_H1	120.9	119.9	608.14
P2_H2	147.0	146.0	1600.24
P2_H3	499.6	57.4	6.46
P2_C1	207.9	208.9	12.32
P2_C2	158.5	159.5	2560.38
P2_C3	199.8	498.8	0.06
P3_H1	36.1	35.1	667.81
P3_C1	86.6	87.6	2699.9
P3_C2	157.3	158.3	1392.21

Table 10
The utility data used in the VCM example.

Utility	Temperature (°C)	Cost (USD/ kW yr)	Maximum level (kW)
P1_Steam (240 psig)	200	150	5000
P1_Cooling water	20	60	1000
P2_Steam (240 psig)	200	150	5000
P2_Fuel oil	600	130	5000
P2_Cooling water	20	60	1000
P3_Steam (240 psig)	200	150	5000
P3_Cooling water	20	60	1000

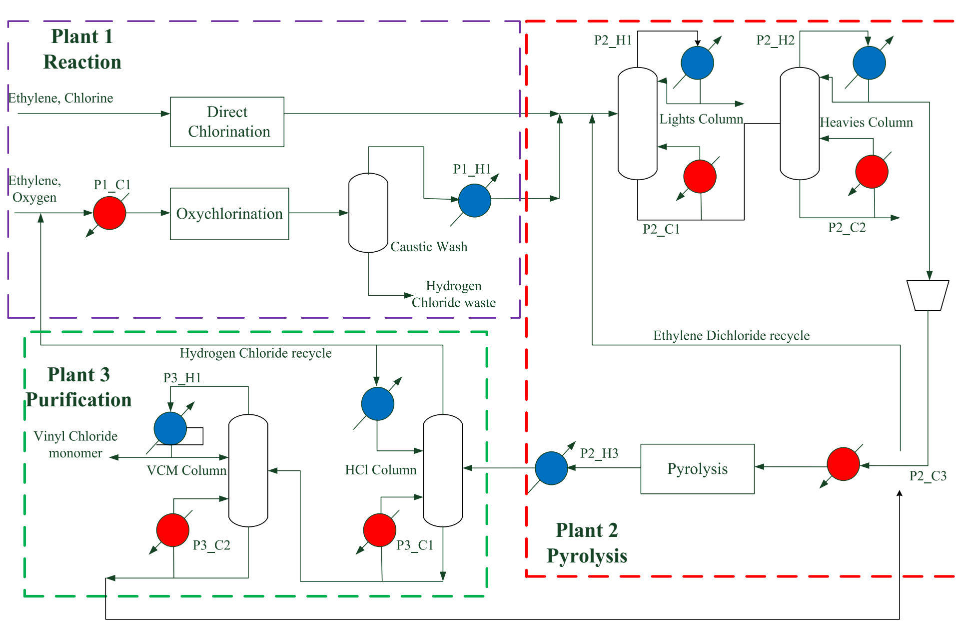

Fig. 6. The VCM process.

are also adopted in the present example. Finally, F_c and $F_c^{fuel}(=1.0)$ respectively denote the correction factors for heat exchangers and furnaces. The following formulas for computing the heat-transfer areas of two types of heat exchangers, i.e., the floating-head and kettle reboilers, have been adopted:

- Floating head ($F_c = 1.0$):

$$A_{ij} = \left[\frac{Q_{ij}}{U_{ij}\left[\theta^1_{ij}\theta^2_{ij}(\theta^1_{ij}+\theta^2_{ij})/2\right]^{1/3}} \right] \tag{61}$$

- Kettle ($F_c = 1.36$):

$$A_{ij} = \left[\frac{Q_{ij}}{U_{ij}\left[\frac{2(\theta^1_{ij}\theta^2_{ij})^{1/2}}{3} + \frac{(\theta^1_{ij}+\theta^2_{ij})}{6}\right]} \right] \tag{62}$$

All overall heat-transfer coefficients are again chosen to be 1, while the inlet and outlet temperature difference of the cold utility stream of every cooler is set to be 5 °C.

Case 1. In this case, we would like to find out whether it is necessary to involve all three companies in the inter-plant heat integration scheme. After implementing the aforementioned sequential optimization strategy, the network structure in Fig. 7 can be obtained. The required utility consumption rates before and after inter-plant heat integration are summarized in Table 11, while a comparison of utility cost savings, capital cost savings and TAC savings can be found in Table 12. Since the utility cost saving of plant P1 is 0 USD/yr and the capital cost saving is only 107 USD/yr, the decision-maker of plant P1 may feel that the incentives are not enough. Therefore, it is necessary to evaluate the feasibility of inter-plant heat integration between P2 and P3 only.

Case 2. Before inter-plant heat integration, the minimum consumption rate of fuel in plant P2 is 0.553 kW and those of hot utilities in plant P2 and P3 are 450.988 kW and 4092.110 kW, while the cold utility consumption rates of P2 and P3 are 2925.698 kW and 667.81 kW. On the basis of these results, the inter-plant HEN structure in Fig. 8 can be synthesized according to the proposed procedure. The corresponding economic analyses for these two plants are presented in Tables 13 and 14, and their TACs can be found to be 280,621 USD/yr and 280,622 USD/yr. By comparing these values with those achieved in the aforementioned three-plant integration scheme, it can be concluded that the TACs of plants P2 and P3 are virtually unchanged and, therefore, the participation of plant P1 is in fact unnecessary.

Table 11
The utility consumption rates before and after inter-plant heat integration in Case 1 of the VCM example.

Plant	Before integration (kW)			After integration (kW)		
	HP	Fuel	CW	HP	Fuel	CW
P1	14.7	–	0	12.6	–	0
P2	451.0	0.553	2925.7	1853.2	0.553	233.6
P3	4092.1	–	667.8	0	–	667.8

Table 12
A comparison of the utility cost savings, capital cost savings and TAC savings achieved in Case 1 of the VCM example.

Plant	Total utility saving (USD/yr)	Capital cost saving (USD/yr)	TAC saving (USD/yr)
P1	0.002	107	107
P2	282,932	− 1480	281,452
P3	282,400	1094	283,494

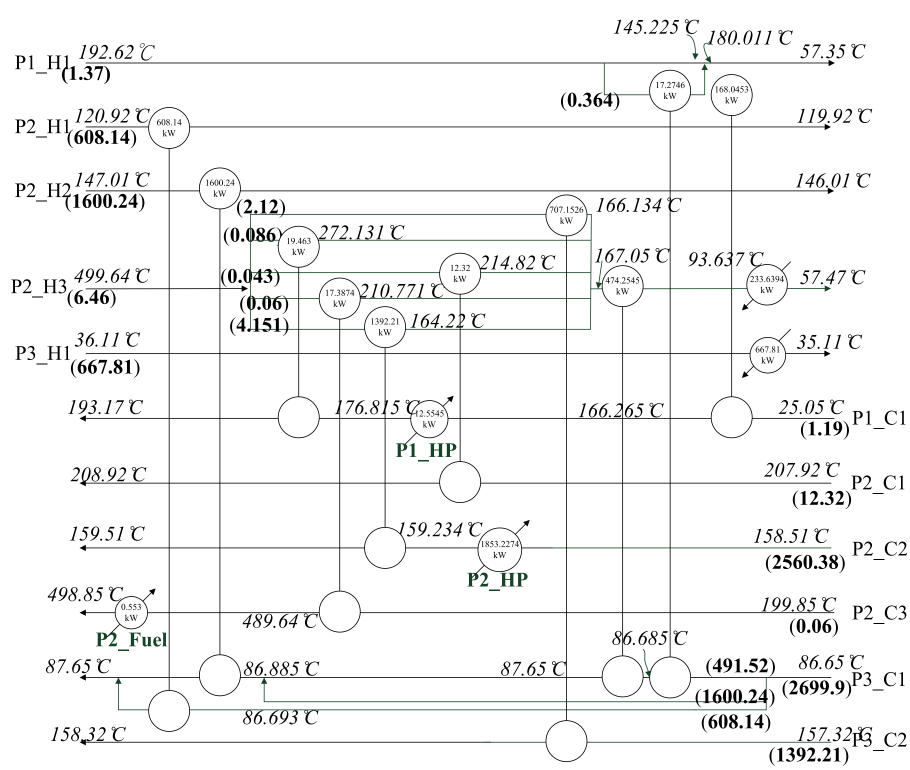

Fig. 7. The optimal inter-plant HEN design in Case 1 of the VCM example.

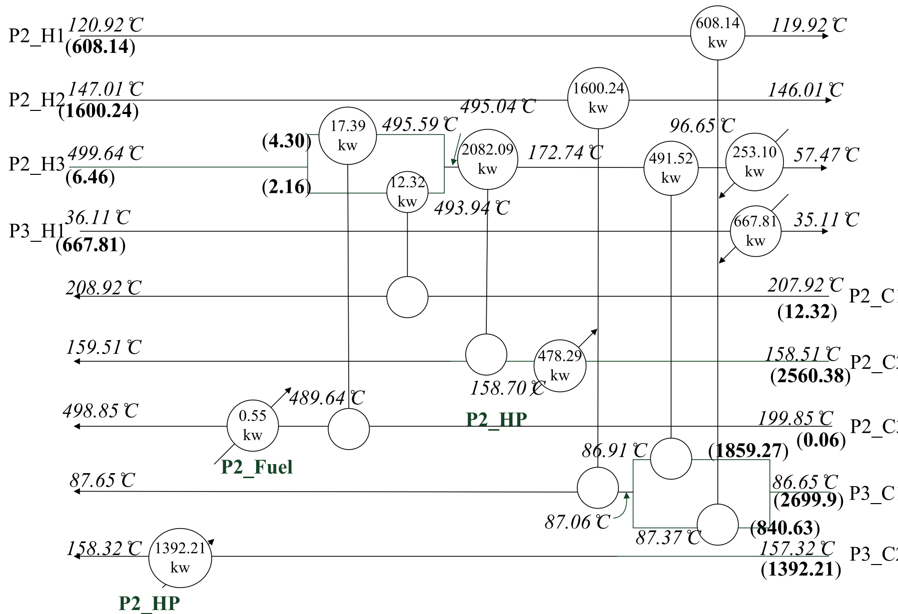

Fig. 8. The optimal inter-plant HEN design in Case 2 of the VCM example.

Table 13
The utility consumption rates before and after inter-plant heat integration in Case 2 of the VCM example.

Plant	Before integration (kW)			After integration (kW)		
	HP	Fuel	CW	HP	Fuel	CW
P2	451.0	0.553	2925.7	1870.5	0.553	253.1
P3	4092.1	–	667.8	0	–	667.8

Table 14
A comparison of the utility cost savings, capital cost savings and TAC savings achieved in Case 2 of the VCM example.

Plant	Total utility saving (USD/yr)	Capital cost saving (USD/yr)	TAC saving (USD/yr)
P2	280,623	−2	280,621
P3	280,623	−1	280,622

8. Conclusions

A game-theory based optimization strategy is presented in this paper for the purpose of configuring the optimal inter-plant heat integration schemes. This HEN design can be generated by following four consecutive steps to determine (1) the minimum acceptable overall utility cost, (2) the heat flows between every pair of plants and also their fair trade prices, (3) the minimum number of heat-exchanger units and their duties, and (4) the optimal network configuration. A simple example is adopted to illustrate the proposed procedure. It can also be observed from the optimization results obtained in the more complex case studies that the resulting inter-plant heat integration scheme is practically feasible. Finally, based on the discussions presented in this paper, we can see that the typical decisions made by a process designer may often be unacceptable to some of the participants. This is of course due to the global objective usually taken in the traditional engineering design. The proposed optimization strategy is shown to be quite effective for developing an alternative approach to address the concerns (or needs) of multiple decision makers.

Nomenclature

Af the annualization factor, (dimensionless)

$\mathbf{A}_{pq}$ a sub-matrix of the payoff values between plant p and plant q

$\mathbf{A}_p$ the payoff matrix

$C_{trd}^{pUq_iU}$ the unit trade price of heat transferred from a temperature above the pinch point of plant p to a temperature above the pinch point of plant q_i, (USD/kW yr)

$C_{trd}^{pUq_iL}$ the unit trade price of heat transferred from a temperature above the pinch point of plant p to a temperature below the pinch point of plant q_i, (USD/kW yr)

$C_{trd}^{pLq_iU}$ the unit trade price of heat transferred from a temperature below the pinch point of plant p to a temperature above the pinch point of plant q_i, (USD/kW yr)

$C_{trd}^{pLq_iL}$ the unit trade price of heat transferred from a temperature below the pinch point of plant p to a temperature below the pinch point of plant q_i, (USD/kW yr)

$C_{trd}^{q_iUpU}$ the unit trade price of heat transferred from a temperature above the pinch point of plant q_i to a temperature above the pinch point of plant p, (USD/kW yr)

$C_{trd}^{q_iLpU}$ the unit trade price of heat transferred from a temperature below the pinch point of plant q_i to a temperature above the pinch point of plant p, (USD/kW yr)

$C_{trd}^{q_iUpL}$ the unit trade price of heat transferred from a temperature above the pinch point of plant q_i to a temperature below the pinch point of plant p, (USD/kW yr)

$C_{trd}^{q_iLpL}$ the unit trade price of heat transferred from a temperature below the pinch point of plant q_i to a temperature below the pinch point of plant p, (USD/kW yr)

$C_k^{q'}$ denote the sets of cold streams in interval k of plant q'

C_k^{p} denote the sets of cold streams in interval k of plant p

C_p^{HU} the known unit cost of the hot utility of plant p, (USD/kW yr)

C_p^{CU} the known unit cost of the cold utility of plant p, (USD/kW yr)

$C_{q_i}^{HU}$ the known unit cost of the hot utility of plant q_i, (USD/kW yr)

$C_{q_i}^{CU}$ the known unit cost of the cold utility of plant q_i, (USD/kW yr)

$c_{i_pj_p}$ the coefficients in the cost model of heat exchanger, (USD/kW yr).
$c_{i_pj_{q'}}$ the coefficients in the cost model of heat exchanger, (USD/kW yr)
$c_{i_qj_p}$ the coefficients in the cost model of heat exchanger, (USD/kW yr)
c_m the unit price of the hot utility m, (USD/kW y)
c_n the unit price of the cold utility n, (USD/kW yr)
c_{ij} the cost coefficient of the corresponding heat exchanger, (USD/kW yr)
$F^C_{j_p}$ the flow capacity of cold utility j_p, (kW/K)
$F^H_{i_p}$ the flow capacity of hot utility i_p, (kW/K)
H^p_k the set of hot process streams in interval k of plant p, (kW).
K^U_p the sets of temperature intervals above the pinch point of plant p
K^L_p the sets of temperature intervals below the pinch point of plant p
PR^{UD}_p the proportion of exporting heat above pinch point of plant p in total inter-plant exchanged heat, (dimensionless)
PR^{LD}_p the proportion of exporting heat below pinch point of plant p in total inter-plant exchanged heat, (dimensionless)
PR^{UA}_p the proportion of importing heat above pinch point of plant p in total inter-plant exchanged heat, (dimensionless)
PR^{LA}_p the proportion of importing heat below pinch point of plant p in total inter-plant exchanged heat, (dimensionless)
Q_{ij} the heat duty of the corresponding heat exchanger, (kW).
Q^S_m the heat supplied by hot utility m, (kW)
Q^W_n the heat rejected to cold utility n, (kW)
Q^{qp}_k the heat flow transferred from interval k in plant q to interval k in plant p, (kW)
$Q^{pq'}_k$ the heat flow transferred from interval k in plant p to interval k in plant q', (kW)
$Q^C_{j_pk}$ the heat transported from interval k to cold stream j_p in plant p, (kW)
$Q_{i_pj_pk}$ the amount of heat exchanged between hot stream i_p and cold stream j_p in interval k of plant p, (kW)
$Q_{i_pn_pk}$ the amount of heat exchanged between hot stream i_p and cold utility n_p in interval k of plant p, (kW)
$Q_{m_pj_pk}$ the amount of heat exchanged between hot stream m_p and cold utility j_p in interval k of plant p, (kW)
$Q_{m_pj_{q'}k}$ the amount of heat exchanged between hot stream m_p and cold utility j_q in interval k of plant p, (kW)
$Q_{i_pj_{q'}k}$ the amount of heat exchanged between hot stream i_p in interval k of plant p and cold process or utility stream $j_{q'}$ in interval k of plant q', (kW)
$Q_{i_pj_qk}$ the heat exchange rate between hot stream i of plant p and cold stream j of plant q in interval k, (kW)
$Q_{i_qj_pk}$ the heat exchange rate between hot stream i of plant q and cold stream j of plant p in interval k, (kW)
$Q_{i_qn_pk}$ the heat exchange rate between hot stream i of plant q and cold stream n of plant p in interval k, (kW)
$Q^H_{i_p,k}$ the amount of heat supplied by hot stream i_p in interval k of plant p, (kW)
$Q^S_{m_p}$ the consumption rate of cold utility m in plant p, (kW)
$Q^{pq'}_k$ the flow rate of heat transferred from interval k in plant p to interval k in plant q', (kW)
$Q^S_{m_p}$ the hot utility consumption rate in interval m of plant p, (kW)
$Q^W_{n_p}$ the cold utility consumption rate in interval n of plant p, (kW)
$\overline{QS}_m$ the upper bounds of the hot utility consumption rates in interval m, (kW)
$\overline{QW}_n$ the upper bounds of the cold utility consumption rates in interval n, (kW)
pf_p the revenue received by plant p via energy trades, (USD/yr)
R^p_k the heat residue of the interval k in plant p, (kW)
$R_{i_p,k}$ the heat residue from hot utility i in interval k of plant p, (kW)
$R_{i_q,k}$ the heat residue from hot utility i in interval k of plant q, (kW)
$R_{m_p,k}$ the heat residue from hot utility m in interval k of plant p, (kW)
$\mathfrak{R}^{pUq_iU}$ The unit payoff value of exporting heat at a temperature above the pinch in plant p to a temperature above the pinch in plant q_i, (USD/kW yr)
$\mathfrak{R}^{pUq_iL}$ The unit payoff value of exporting heat at a temperature above the pinch in plant p to a temperature below the pinch in plant q_i, (USD/kW yr)
$\mathfrak{R}^{pLq_iU}$ The unit payoff value of exporting heat at a temperature below the pinch in plant p to a temperature above the pinch in plant q_i, (USD/kW yr)
$\mathfrak{R}^{pLq_iL}$ The unit payoff value of exporting heat at a temperature below the pinch in plant p to a temperature below the pinch in plant q_i, (USD/kW yr)
$\mathfrak{R}^{q_iUpU}$ The unit payoff value of exporting heat at a temperature above the pinch in plant q_i to a temperature above the pinch in plant p, (USD/kW yr)
$\mathfrak{R}^{q_iLpU}$ The unit payoff value of exporting heat at a temperature below the pinch in plant q_i to a temperature above the pinch in plant p, (USD/kW yr)
$\mathfrak{R}^{q_iUpL}$ The unit payoff value of exporting heat at a temperature above the pinch in plant q_i to a temperature below the pinch in plant p, (USD/kW yr)
$\mathfrak{R}^{q_iLpL}$ The unit payoff value of exporting heat at a temperature below the pinch in plant q_i to a temperature below the pinch in plant p, (USD/kW yr)
S^p_k the set of hot utilities in interval k of plant p
S^U_p the utility cost saving realized by plant p after inter-plant heat integration, (USD/yr)
S^T_p the total cost saving realized by plant p after inter-plant heat integration, (USD/yr)
$SC^p_{pq'}$ the total capital cost shared by plant p to facilitate inter-plant heat exchanges between the hot streams in plant p and the cold streams in plant q', (USD/yr)
SC^p_{qp} the total capital cost shared by plant p to facilitate inter-plant heat exchanges between the hot streams in plant q and the cold streams in plant p, (USD/yr)
$U_{i_pj_q}$ the upper bound of the heat exchange value between hot stream i_p and cold stream j_q, (kW)
U_{ij} the overall heat transfer coefficient, (kW/m^2 K)
W^p_k the sets of cold utilities in interval k of plant p
$W^{q'}_k$ the sets of cold utilities in interval k of plant q'
$\mathbf{x}^T_p$ the strategy vector of plant p
$\mathbf{x}_q$ the strategy vector of plant q
$z_{i_pj_p}$ binary parameters to denote if the corresponding matches are present in HEN
$z_{i_pj_{q'}}$ binary parameters to denote if the corresponding matches are present in HEN
$z_{i_qj_p}$ binary parameters to denote if the corresponding matches are present in HEN
$z_{i_pj_q}$ binary parameters to denote if the corresponding matches are present in HEN.

$\overline{Z}_p^U$ the minimum utility cost of plant p after inner-plant heat integration, (USD/yr)
$\overline{Z}_p^C$ the minimum total capital cost of plant p without inter-plant heat integration, (USD/yr)
$\hat{Z}_p^U$ the minimum utility cost of plant p after inter-plant heat integration, (USD/yr)
$\hat{Z}_p^C$ the minimum total capital cost of plant p after inter-plant heat integration, (USD/yr)
α_p the average payoff value of plant p.
β the coefficients in the cost model of heat exchanger, (dimensionless)
γ_{ij}^p the proportion of capital cost for match (i, j) that is paid by plant p, (dimensionless)
$\gamma_{i_p j_{q'}}^p$ the proportions of capital cost shared by plant p for the heat exchanger facilitating heat export from hot stream i_p to cold stream $j_{q'}$, (dimensionless)
$\gamma_{i_p j_{q'}}^{q'}$ the proportions of capital cost shared by plant q' for the heat exchanger facilitating heat export from hot stream i_p to cold stream $j_{q'}$, (dimensionless)
$\gamma_{i_q j_p}^p$ the proportions of capital cost shared by plant p for the exchanger facilitating heat import from hot stream i_q to cold stream j_p, (dimensionless)
$\gamma_{i_q j_p}^q$ the proportions of capital cost shared by plant q for the exchanger facilitating heat import from hot stream i_q to cold stream j_p, (dimensionless)
θ_{ij}^1 the hot end temperature differences of the corresponding heat exchanger, (°C).
θ_{ij}^2 the cold end temperature differences of the corresponding heat exchanger, (°C).
ΔH_k^p the enthalpy difference of the hot and cold process streams in interval k of plant p, (kW).
ΔT_k the temperature difference in interval k, (°C).

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.ces.2014.07.001.

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