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A Model-Based Search Strategy for Exhaustive Identification of Alternative Water Network Designs

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ABSTRACT: A systematic procedure is proposed in this paper to identify all optimal single-contaminant water-using networks by solving a series of three mathematical programming models. In particular, a linear programming (LP) model is adopted in this procedure to minimize the operating costs incurred from freshwater consumption and wastewater treatment, and a mixed-integer linear program (MILP) is then utilized for minimizing the total number of interconnections in the network while keeping the total operating cost at its lowest level. The third model is also a MILP model, which is used to minimize the total throughput and generate all alternative designs under the conditions of *fixed* minimum operating costs and interconnection number. Notice that, since the nonlinear mass-balance constraints are all converted to linear form in the proposed models according to the necessary conditions of optimality (Savelski and Bagajewicz, 2000), the convergence of the corresponding optimization processes can always be guaranteed. The *solution pool* technique provided by the CPLEX II solver within GAMS environment has been adopted to search for all optima. Four examples are presented in this paper to demonstrate the effectiveness of the proposed approach.

1. INTRODUCTION

Water conservation in a chemical plant can be realized with an optimally configured water network.[1] Usually it is possible to identify more than one configuration which consumes the least amount of freshwater.[2−4] Additional criteria (such as the capital costs, operability, and safety) may also be incorporated to facilitate final selection of the network design. Two distinct types of design strategies have been reported in the literature, that is, the pinch-based methods[1,5] and the model-based methods.[4,6,7] These different approaches can be used to produce an optimal base-case design for any given system. Prakash and Shenoy[8] first developed a so-called source-shift algorithm (SSA) to manually generate alternative configurations from this base-case solution. Ng and Foo[9] later improved this algorithm (ISSA) to eliminate the iterative nature of SSA. Li and Chang[10] recently proposed a generalized source-shift procedure, which is in essence still a manual evolution technique.

Although manual evolution allows the designer(s) to take full charge of the design process and also gain practical insights, it is sometimes laborious, time-consuming, and incomprehensive. Thus, a systematic model-based procedure is proposed in this paper to synthesize all alternative single-contaminant water-using networks by solving a series of three mathematical programs sequentially within the GAMS environment.[11] In particular, a linear programming (LP) model is adopted in this procedure to minimize the operating costs incurred from freshwater consumption and wastewater treatment, and a mixed-integer linear program (MILP) is then utilized for minimizing the total interconnection number while keeping the total operating cost at its lowest possible level. The third model is also a MILP model, which is used to minimize the total throughput and identify all alternative network designs under the conditions of *fixed* minimum operating costs and interconnection number. Notice that, since the nonlinear mass-balance constraints are all converted to linear form in the proposed models according to the necessary conditions of optimality,[12] the convergence of the corresponding optimization processes can always be guaranteed. The *solution pool* technique provided by the CPLEX II solver within GAMS environment[13] has been used to search for all optima.

It should be noted that similar versions of the aforementioned mathematical programming models have already been developed by Bagajewicz and Savelski[4] and Das et al.[14] These models are not suitable for the present applications since they can be used to produce only one single optimal solution, and also there are additional drawbacks which could seriously hamper their feasibility in realistic problems, that is, (i) only one external source and one external sink are allowed, (ii) the fixed-flow rate operations are ignored in the former study, and (iii) the fixed-load operations are ignored in the latter.

The proposed procedure has been tailored specifically for solving the exhaustive enumeration problem posed in this paper. To facilitate clear presentation, this paper is organized as follows: A design problem is first formally defined in the next section. The general model constraints are then formulated in section 3, while the search strategy of the alternative network structures is outlined

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in section 4. The effectiveness of the proposed method is demonstrated with four examples adopted from literature. On the basis of these results, favorable conclusions are drawn in the end of this paper.

2. PROBLEM DEFINITION

For the design problem under consideration, the process parameters of all essential units in a given single-contaminant water-using system are assumed to be given. Specifically, these available data are concerned with (a) a set of fixed-load operations which are specified by their contaminant loads to be removed and the corresponding maximum inlet and outlet concentrations; (b) a set of *internal* water sources and/or sinks (i.e., the fixed-flow rate operations) which are specified by their flow rates and concentrations in the former case and their flow rates and concentration upper bounds in the latter case; (c) a set of *external* water sources which are specified by their contaminant concentrations and unit costs; (d) a set of *external* sinks which are specified by their unit processing costs and upper limits for water flow rates and contaminant concentrations.

It is the objective of this design task to search for *all* possible optimal network configurations, and each must feature (1) the lowest total operating costs, (2) the smallest total number of pipeline connections, and (3) the minimum total throughput.

3. GENERAL MODEL CONSTRAINTS

3.1. Superstructure. To develop a general model formulation, it is necessary to first build a superstructure in which all possible flow connections are embedded. The superstructure presented here is essentially a modified version of that given in Chang and Li.[6] Specifically, a distinct label is assigned to each source, demand, and fixed-load unit; that is,

$$\mathbb{W} = \{w \,|\, w \text{ is the label of a water source}\} \tag{1}$$

$$\mathbb{D} = \{d \,|\, d \text{ is the label of a water demand}\} \tag{2}$$

$$\mathbb{U} = \{u \,|\, u \text{ is the label of a fixed-load unit}\} \tag{3}$$

Notice that the water sources in $\mathbb{W}$ can be further classified into (1) external waters acquired from environment and (2) internal waters obtained from fixed-flow rate operations within plant, that is, $\mathbb{W} = \mathbb{W}_1 \cup \mathbb{W}_2$ and

$$\mathbb{W}_1 = \{w_1 \,|\, w_1 \text{ is the label of an external source}\} \tag{4}$$

$$\mathbb{W}_2 = \{w_2 \,|\, w_2 \text{ is the label of an internal source}\} \tag{5}$$

Similarly, the water demands in $\mathbb{D}$ can be further classified as external and internal sinks. The former sinks exist outside the plant boundary, which are usually the wastewater treatment facilities. The latter are often associated with the fixed-flow rate operations in the water network. More specifically, they can be defined as

$$\mathbb{D}_1 = \{d_1 \,|\, d_1 \text{ is the label of an external demand}\} \tag{6}$$

$$\mathbb{D}_2 = \{d_2 \,|\, d_2 \text{ is the label of an internal demand}\} \tag{7}$$

and $\mathbb{D} = \mathbb{D}_1 \cup \mathbb{D}_2$.

On the basis of the above definitions, a superstructure can be constructed according to the following steps:

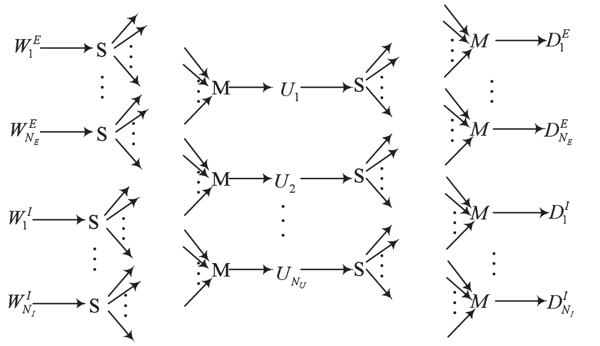

Figure 1. General superstructure of water-using network.

(1) Place a mixing node at the inlet of every unit in $\mathbb{U}$ and every demand in $\mathbb{D}$.
(2) Place a splitting node after every source in $\mathbb{W}_1$. The split branches from this node are connected to all mixing nodes before the units in $\mathbb{U}$ and the demands in $\mathbb{D}_2$.
(3) Place a splitting node after every source in $\mathbb{W}_2$. The split branches from this node are connected to all mixing nodes before the units in $\mathbb{U}$ and the demands in $\mathbb{D}$.
(4) Place a splitting node after every unit in $\mathbb{U}$. The split branches from this node are connected to all mixing nodes before the units in $\mathbb{U}$ (except the one before the same unit) and the demands in $\mathbb{D}$.

This scheme can be represented by Figure 1, in which the symbols S and M denote the splitting and mixing node, respectively. Notice that an external water source in $\mathbb{W}_1$ is not allowed to be sent to any of the external demands in $\mathbb{D}_1$ since direct dilution before discharge is considered to be an inappropriate design practice. Also, the self-recycle stream around a fixed-load unit is excluded in this study because adding such a stream only raises throughput and inlet concentration, but contributes nothing to the mass load.

3.2. Model Constraints. The following model constraints can then be formulated according to this superstructure.

3.2.1. At the Splitting Nodes after the Water Sources. The water balances can be expressed as

$$\mathrm{sr}_w = \sum_{u\in} f^{\mathrm{W}}_{w,u} + \sum_{d\in_w} f^{\mathrm{W}}_{w,d} \qquad w\in\mathbb{W} \tag{8}$$

where, sr_w is the water supply rate from source w; $f^{\mathrm{W}}_{w,u}$ and $f^{\mathrm{W}}_{w,d}$ denote the flow rates of waters (from source w) which are consumed by unit u and demand d, respectively; $_w$ is a set defined as

$$\mathbb{D}_w = \begin{cases} \mathbb{D}_2 & \text{if } w\in\mathbb{W}_1 \\ \mathbb{D} & \text{if } w\in\mathbb{W}_2 \end{cases} \tag{9}$$

Since the internal waters must be completely consumed and their supply rates are assumed to be constants in this study, the following constraints should also be imposed

$$\mathrm{sr}_w = S_w \qquad w\in\mathbb{W}_2 \tag{10}$$

where S_w is a given design parameter.

3.2.2. At the Splitting Node at the Outlet of a Fixed-Load Unit. The water balance at the splitting node at the outlet of a fixed-load unit u can be written as

$$f^{\mathrm{out}}_u = \sum_{u'\in\mathbb{U},\, u'\neq u} f_{u,u'} + \sum_{d\in\mathbb{D}} f^{\mathrm{D}}_{u,d} \qquad u\in\mathbb{U} \tag{11}$$

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where f_u^{out} is the water flow rate at the outlet of unit u; $f_{u,u'}$ represents the water flow rate from unit u to unit u'; $f_{u,d}^{\mathrm{D}}$ is the flow rate of wastewater generated by unit u and sent to demand d.

3.2.3. At the Inlet of Each Fixed-Load Unit. The water balance around the mixing node can be written as

$$f_u^{\mathrm{in}} = \sum_{u' \in \mathbb{U}, u' \neq u} f_{u',u} + \sum_{w \in \mathbb{W}} f_{w,u}^{\mathrm{W}} \qquad u \in \mathbb{U} \qquad (12)$$

where, f_u^{in} is the total flow rate at the mixing node of unit u; $f_{u',u}$ is the water flow rate from unit u' to unit u. The corresponding mass balance of contaminant should be

$$f_u^{\mathrm{in}} c_u^{\mathrm{in}} = \sum_{u' \in \mathbb{U}, u' \neq u} f_{u',u} c_{u'}^{\mathrm{out}} + \sum_{w \in \mathbb{W}} f_{w,u}^{\mathrm{W}} C_w \qquad u \in \mathbb{U} \qquad (13)$$

where, c_u^{in} and $c_{u'}^{\mathrm{out}}$ denote the contaminant concentrations at the inlet of unit u and outlet of unit u', respectively; C_w denotes the contaminant concentration from water source w, which is a given parameter.

Since a upper bound ($\overline{C}_u^{\mathrm{in}}$) must be imposed upon the inlet concentration of each fixed-load unit, and also, as required by the necessary optimal conditions,[12] the corresponding outlet concentration must reach the maximum allowable limit ($\overline{C}_u^{\mathrm{out}}$), eq 13 can be reformulated as

$$f_u^{\mathrm{in}} \overline{C}_u^{\mathrm{in}} \geq \sum_{u' \neq u, u' \in \mathbb{U}} f_{u',u} \overline{C}_{u'}^{\mathrm{out}} + \sum_{w \in \mathbb{W}} f_{w,u}^{\mathrm{W}} C_w \qquad u \in \mathbb{U} \qquad (14)$$

Obviously, the advantage of this formulation is that the nonlinear constraint in eq 13 can be converted to a linear form.[4]

3.2.4. At the Mixing Node of Each Sink. The water and contaminant balances are

$$f_d^{\mathrm{in}} = \sum_{u \in \mathbb{U}} f_{u,d}^{\mathrm{D}} + \sum_{w \in \mathbb{W}_d} f_{w,d}^{\mathrm{W}} \qquad d \in \mathbb{D} \qquad (15)$$

$$f_d^{\mathrm{in}} \overline{C}_d^{in} \geq \sum_{u \in \mathbb{U}} f_{u,d}^{\mathrm{D}} \overline{C}_u^{\mathrm{out}} + \sum_{w \in \mathbb{W}_d} f_{w,d}^{\mathrm{W}} C_w \qquad d \in \mathbb{D} \qquad (16)$$

where, f_d^{in} is the total water flow rate delivered to demand d; $\overline{C}_d^{\mathrm{in}}$ is the maximum concentration allowed at sink d; the set $\mathbb{W}_d$ is defined as

$$\mathbb{W}_d = \begin{cases} \mathbb{W}_2 & \text{if } d \in \mathbb{D}_1 \\ \mathbb{W} & \text{if } d \in \mathbb{D}_2 \end{cases} \qquad (17)$$

3.2.5. The Performance of Each Fixed-Load Unit. The process performance of a fixed-load unit can be characterized as

$$\sum_{u' \in \mathbb{U}, u' \neq u,} f_{u',u} \overline{C}_{u'}^{\mathrm{out}} + \sum_{w \in \mathbb{W}} f_{w,u}^{\mathrm{W}} C_w + M_u = f_u^{\mathrm{out}} \overline{C}_u^{\mathrm{out}} \qquad u \in \mathbb{U} \qquad (18)$$

where, $\overline{C}_{u'}^{\mathrm{out}}$ and $\overline{C}_u^{\mathrm{out}}$, respectively, represent the maximum outlet concentrations of fixed-load units u' and u; M_u denotes the mass load of unit u, and it is a given parameter. In this study, the possibility of water loss is not considered in any of the fixed-load unit. Therefore,

$$f_u^{\mathrm{in}} = f_u^{\mathrm{out}} \qquad u \in \mathbb{U} \qquad (19)$$

4. EXHAUSTIVE SEARCH STRATEGY

To search for all alternative configurations, three distinct tasks are performed sequentially in this study with LP and MILP models. These models can be solved with CPLEX 11[13] in GAMS environment.[11] A summary of the search procedure is given below:

4.1. Determine the Minimum Total Operating Cost with a LP Model. The objective function of the proposed LP model is expressed as

$$\min \mathrm{tc} = \sum_{w \in \mathbb{W}_1} \alpha_w \mathrm{sr}_w + \sum_{d \in \mathbb{D}_1} \beta_d f_d^{\mathrm{in}}$$

where α_w and β_d, respectively, represent the purchase cost of a unit of freshwater from source w and the processing cost of a unit of wastewater at demand d; tc denotes the total operating cost. Equations 8−12 and 14−19 are used as the constraints of this LP model. The resulting minimum value of tc is referred to as $\underline{\mathrm{TC}}$ later in this paper.

4.2. Determine the Minimum Total Connection Number with a MILP Model. If no fixed-load operations are involved in the design problem, this MILP model should be solved with the *solution pool* function in CPLEX 11 to identify all alternative solutions. Otherwise, all three steps are necessary.

The objective function in this case is

$$\min \mathrm{tn} = \sum_{w \in \mathbb{W}} \sum_{u \in \mathbb{U}} n_{w,u}^{\mathrm{W}} + \sum_{w \in \mathbb{W}} \sum_{d \in \mathbb{D}} n_{w,d}^{\mathrm{W}} + \sum_{u' \in \mathbb{U}} \sum_{u \in \mathbb{U}} n_{u',u} + \sum_{u \in \mathbb{U}} \sum_{d \in \mathbb{D}} n_{u,d}^{\mathrm{D}}$$

where $n_{w,u}^{\mathrm{W}}$, $n_{w,d}^{\mathrm{W}}$, $n_{u',u}$, and $n_{u,d}^{\mathrm{D}}$ are binary variables and each is used to reflect whether or not the corresponding branch stream exists. In particular, they are constrained in the proposed model with the following inequalities:

$$f_{w,u}^{\mathrm{W}} \leq n_{w,u}^{\mathrm{W}} M \quad f_{w,d}^{\mathrm{W}} \leq n_{w,d}^{\mathrm{W}} M \qquad w \in \mathbb{W},\ u \in \mathbb{U},\ d \in \mathbb{D} \quad (20)$$

$$f_{u',u} \leq n_{u',u} M \quad f_{u,d}^{\mathrm{D}} \leq n_{u,d}^{\mathrm{D}} M \qquad u, u' \in \mathbb{U},\ d \in \mathbb{D} \qquad (21)$$

where M is a large enough positive number, which should be greater than the largest possible value of $f_{w,u}^{\mathrm{W}}$, $f_{w,d}^{\mathrm{W}}$, $f_{u',u}$, or $f_{u,d}^{\mathrm{D}}$. The following constraint is also needed to maintain the total operating cost at the minimum level:

$$\underline{\mathrm{TC}} \geq \sum_{w \in \mathbb{W}_1} \alpha_w \mathrm{sr}_w + \sum_{d \in \mathbb{D}_1} \beta_d f_d^{\mathrm{in}} \qquad (22)$$

Thus, the constraints of the present MILP model include eqs 20−22 and those in the previous LP model. Finally, the resulting minimum value of tn is referred to as $\underline{\mathrm{TN}}$.

4.3. Determine the Minimum Total Throughput of Fixed-Load Units with Another MILP Model. Determine the minimum total throughput of fixed-load units with another MILP model and identify all alternative solutions with the *solution pool* function in CPLEX 11.

Since the input and/or output flow rate of every fixed-flow rate operation must be kept unchanged, the total throughput of a water network can be minimized by minimizing that of all fixed-load units. Thus, the objective function of this model can be written as

$$\min \mathrm{tt} = \sum_{u \in \mathbb{U}} f_u^{\mathrm{in}}$$

Table 1. Limiting Data for Example 1[15]

sources			demands		
S_i	flow rate F_i (t/h)	concentration C_i (ppm)	D_j	flow rate F_j (t/h)	concentration C_j (ppm)
S1	50	50	D1	50	20
S2	100	100	D2	100	50
S3	70	150	D3	80	100
S4	60	250	D4	70	200

Table 2. Design I Represented as Matching Matrix for Example 1

	streams	D1	D2	D3	D4{200}	WW{∞}
F{C}		50{20}	100{50}	80{100}	70{**164.3**}	50{**250**}
70{0}	S0	30	40			
50{50}	S1	20	30			
100{100}	S2		20	80		
70{150}	S3		10		60	
60{250}	S4				10	50

where tt is the total throughput of all fixed-load operations. The following constraint is imposed in the present model to maintain the minimum total connection number:

$$\mathrm{TN} \geq \sum_{w \in \mathrm{W}} \sum_{u \in \mathrm{U}} n_{w,u}^{\mathrm{W}} + \sum_{w \in \mathrm{W}} \sum_{d \in \mathrm{D}} n_{w,d}^{\mathrm{W}} + \sum_{u' \in \mathrm{U}} \sum_{u \in \mathrm{U}} n_{u',u} + \sum_{u \in \mathrm{U}} \sum_{d \in \mathrm{D}} n_{u,d}^{\mathrm{D}} \tag{23}$$

Thus, the model constraints are eqs 20−23 and those in the aforementioned LP model.

As mentioned above, the *solution pool* function of CPLEX 11 is adopted in this work to produce all optimal solutions of a given MILP model. Notice that there may be an infinite number of possible values for a continuous variable, and it is really not practical to enumerate all of them on a finite-precision computer. Therefore, only one solution is produced for each set of binary and/or integer variables, even though there may be several solutions having the same values for all binary and integer variables but different values for some of the continuous variables.[13] Since all integer variables in the aforementioned MILP models are binary, every possible network configuration can be identified with the *solution pool* function. More specifically, the following options should be selected before calling the CPLEX solver: (1) Set the relative gap parameter *SolnPoolGap* = 0 or the absolute gap parameter *SolnPoolAGap* = 0. These parameters set the tolerance on the objective bound for the solutions in the solution pool. A value of 0 requires that all solutions should have the same objective value. (2) Set the intensity parameter *SolnPoolIntensity* = 4. This setting forces CPLEX to find all practical solutions. (3) Set the pool population parameter *SolnPoolPop* = 2. This setting executes the "populate" procedure to generate multiple solutions.

5. APPLICATIONS

To demonstrate the effectiveness of the proposed strategy, the search results of four case studies are presented in the sequel.

Table 3. Design II Represented as Matching Matrix for Example 1

	streams	D1	D2	D3	D4{200}	WW{∞}
F{C}		50{20}	100{50}	80{100}	70{**164. 3**}	50{**250**}
70{0}	S0	40	30			
50{50}	S1		50			
100{100}	S2	10	10	80		
70{150}	S3		10		60	
60{250}	S4				10	50

Table 4. Design III Represented as Matching Matrix for Example 1

	streams	D1	D2	D3	D4{200}	WW{∞}
F{C}		50{20}	100{50}	80{100}	70{**164.3**}	50{**250**}
70{0}	S0	43.33	26.67			
50{50}	S1		50			
100{100}	S2		20	80		
70{150}	S3	6.67	3.33		60	
60{250}	S4				10	50

Example 1 is a design problem which involves only fixed-flow rate operations, while Example 2 and 3 are concerned with only fixed-load operations. The last case (Example 4) is a general hybrid problem. The process data of all four examples are taken from literature.

All models were solved within GAMS environment (version 23.1)[11] on a PC with an Intel Core CPU at 2.66 GHz. Solver CPLEX 11[13] was adopted to solve the LP models, while both CPLEX 11 and OSL[13] were used to perform the optimization runs according to the proposed MILP models for comparison and verification purposes. In each case study, all optimal solutions are found within 0.8 CPU second.

5.1. Example 1. Consider a design problem which was originally studied by Polley and Polley.[15] The process data needed in this problem are given in Table 1. It is assumed that there are only one freshwater source (in which the contaminant concentration is 0 ppm) and one wastewater sink (without the upper contaminant concentration limit).

There are 13 constraints and 25 continuous variables in the corresponding LP model. In this case, the objective function can be reduced to the freshwater consumption rate because this rate always differs from the wastewater discharge rate by a constant value. On the other hand, the third step in the proposed search procedure can be ignored since there are no fixed-load units in this system. In the MILP model used in the second step, there are 38 constraints, 25 continuous variables, and 24 binary variables. The total number of alternative solutions can be found to be 3.

The identified solutions are presented in the form of *matching matrix*[8] in Tables 2, 3, and 4. The rows and columns of the *matching matrix* are associated with sources (S0−S4) and sinks (D1−D4 and WW), respectively, while their process data are given in the cells in the far-left column and in the top row. In each cell, the concentration in ppm and the water flow rate in ton/h are provided within and outside of the brace, respectively. The first row of this matching matrix represents a freshwater (S0) source, whereas the last column is the wastewater (WW). In addition, the actual concentration of wastewater is given in boldface font because it is not the same as the allowed upper limit (infinity). Such convention is adopted throughout this paper.

Table 5. Design Specifications of Water-Using Units for Example 2[1]

unit	$\overline{C}^{in}$ (ppm)	$\overline{C}^{out}$ (ppm)	mass load (kg/h)	F^{lim} (t/h)
P1	0	100	2	20
P2	50	100	5	100
P3	50	800	30	40
P4	400	800	4	10

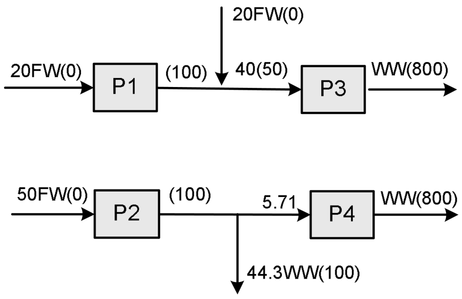

Figure 2. Final optimal solution for Example 2.

Notice that, in each of the optimal design mentioned above, the minimum freshwater usage is 70 ton/h and the minimum total interconnection number is 10, which are the same as those reported in literature.[5,15] Although these available designs[8] are as good as the ones given in this example, it should be noted that they were found in a much more tedious iterative computation process.

5.2. Example 2. The process data for the four fixed-load operations in this example are taken from Wang and Smith[1] (see Table 5). Assume that there are only one water freshwater source and one wastewater sink, the contaminant concentration of the freshwater is 0 ppm, and there is no upper contaminant concentration limit for the wastewater sink.

Similar to Example 1, the objective function of the LP model (with 17 constraints and 29 continuous variables) is simplified to the freshwater consumption rate only. The resulting minimum freshwater usage is 90 ton/h, which is the same as the reported value.[1] The corresponding two MILP models both contain 42 continuous variables, 12 binary variables, and approximately 43 constraints. From their solutions, the minimum number of interconnections can be found to be 8 and the minimum total throughput is 115.714 ton/h. It happens in this example that there is only one optimal solution (see Figure 2), which is as good as the reported one by Prakash and Shenoy.[5]

5.3. Example 3. Consider the process data in Table 6, which are adopted from Olesen and Polley.[16] It is assumed that there are only one freshwater source (in which the contaminant concentration is 0 ppm) and one wastewater sink (without the upper contaminant concentration limit).

A minimum freshwater usage of 157.143 ton/h can be found by solving the LP model (with 25 constraints and 55 continuous variables) in the first step, which is the same as the reported result.[3,5,16] There are 79 continuous variables, 25 binary variables, and approximately 75 constraints in each MILP model used for the next two steps. The resulting minimum number of interconnections is 13 and the minimum total throughput is 193.571 ton/h, which are both identical to those found by Prakash and Shenoy.[5] After completing step 3, four alternative optimal configurations can be found, and they are presented in Figures 3, 4, 5, and 6, respectively. It should be noted that all of them simultaneously feature minimum freshwater usage, minimum total interconnection number, and minimum total throughput. Such superior solutions have never been reported before.[3,5,16]

Table 6. Design Specifications of Water-Using Units for Example 3[16]

unit	$\overline{C}^{in}$ (ppm)	$\overline{C}^{out}$ (ppm)	mass load (kg/h)	F^{lim} (t/h)
P1	25	80	2	36.36
P2	25	100	5	66.67
P3	25	200	4	22.86
P4	50	100	5	100
P5	50	800	30	40
P6	400	800	4	10

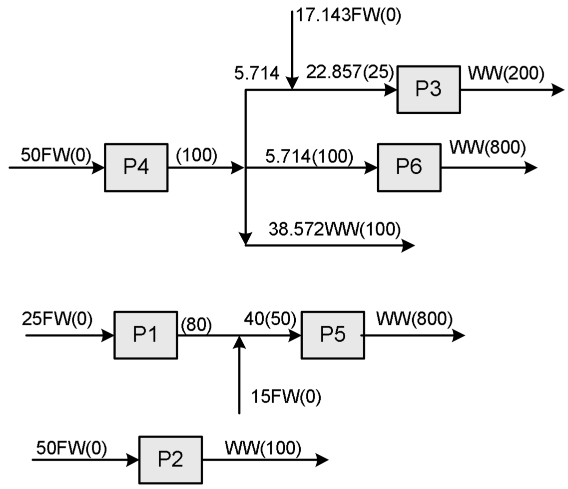

Figure 3. Optimal design I for Example 3.

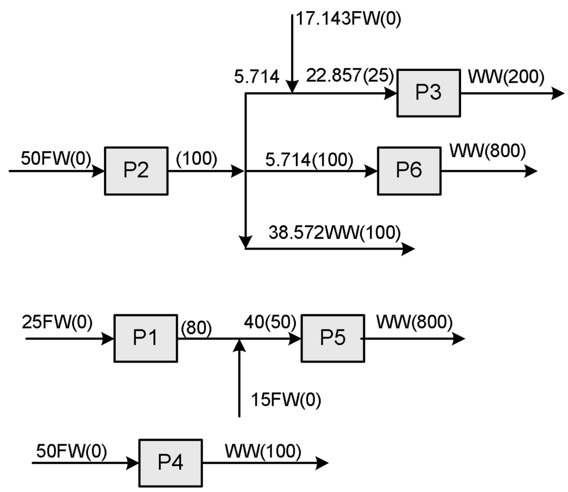

Figure 4. Optimal design II for Example 3.

Finally, it should be noted that OSL[13] rather than CPLEX should be adopted to solve the MILP model for minimizing the total interconnection number in this example. Otherwise, a local minimum of 14 will be found.

5.4. Example 4. A hybrid design problem can be formed by combining the process data of Examples 1 and 2. Assume that there are only one freshwater source (in which the contaminant

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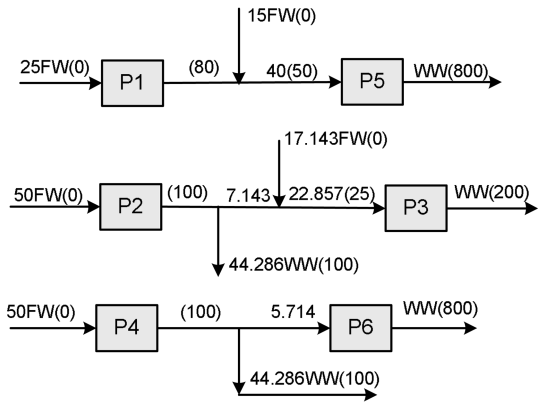

Figure 5. Optimal design III for Example 3.

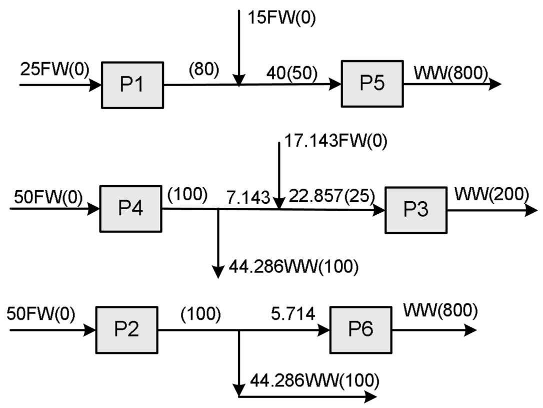

Figure 6. Optimal design IV for Example 3.

Table 7. An Optimal Solution for Example 4

	streams	$P1_{in}$	$P2_{in}$	D1	$P3_{in}$	D2	D3	$P4_{in}$	D4	WW
F (t/h)		20	50	50	40	100	80	5.714	70	135
{C ppm}		{0}	{0}	{20}	{50}	{50}	{100}	{100}	{200}	{∞}
155{0}	FW	20	50	35		50				
50{50}	S1			10	40					
20{100}	$P1_{out}$			5				5.714		9.286
100{100}	S2						80			20
50{100}	$P2_{out}$					50				
70{150}	S3								70	
60{250}	S4									50
40{800}	$P3_{out}$									40
5.714{800}	$P4_{out}$									5.714

concentration is 0 ppm) and one wastewater sink (without the upper contaminant concentration limit).

Without cost data, the design objective in the present case study is also reduced to minimization of freshwater usage in the first step. The minimum freshwater consumption rate was found to be 155 ton/h by solving a LP model with 29 constraints and 85 continuous variables. This result is the same as that obtained by Prakash and Shenoy.[5] The corresponding MILP models contain 104 continuous variables, 62 binary variables, and approximately 111 constraints. From the optimal solutions of these models, it can be determined that the minimum total interconnection number is 16 and the minimum total throughput is 115.714 ton/h as obtained in Example 2. There are 8 optimal alternative solutions for this problem. Only one of them is provided in the form of *matching matrix*[8] in Table 7 for brevity. If compared with the solution given in Prakash and Shenoy,[5] one can see that the total number of interconnections is reduced by 3 while both the freshwater usage and total throughput are kept at their minimum levels.

6. CONCLUSIONS

A systematic design procedure is proposed in this paper to automatically generate all alternative single-contaminant water-using networks by solving a series of three mathematical programming models sequentially. This procedure is considered to be more general than any available model in the sense that both fixed-load and fixed-flow rate operations are considered in the implementation steps. It should also be noted that, in the proposed models, the nonlinear mass-balance constraints are all converted to linear form according to the necessary conditions of optimality. Consequently, the convergence of the corresponding solution processes can be guaranteed. In addition, the *solution pool* function provided by CPLEX II solver within the GAMS environment can be utilized to find all optimal solutions. Four examples are adopted in this paper to illustrate the effectiveness of the proposed approach.

7. DISCUSSION

During the review period of this paper, Faria and Bagajewicz[17] published a similar work on searching multiple degenerate solutions for water/wastewater network. The main differences between this work (which will be referred to as paper 1) and Faria and Bagajewicz's paper (which will be referred to as paper 2) are summarized below:

(1) The focuses of these two works are different although both tried to obtain multiple optimal solutions for each water network design problem.

Specifically, it should be noted that paper 1 concentrates on the single-contaminant water-using networks with both fixed-load and fixed-flow rate operations. On the other hand, paper 2 deals with water network consisting of fixed-load water users and/or regeneration operations, and it is not limited to single-contaminant system.

(2) The optimization goals and also qualities of alternative solutions are different.

The design goal of paper l is to find all optimal water-using networks of *the same quality*, that is, each network features minimum operating costs, minimum interconnection number, and minimum total throughput of fixed-load water users simultaneously. Both *global optimum* and *all alternative solutions* are guaranteed to be found because the proposed models are limited to be LP or MILP models. This is also the reason why the multicontaminant cases are not considered in present work. As stated in the conclusion section of paper 2, its goal is to manifest the advantages of a program-based technique over the pinch-based method for the purpose of finding a given number of degenerate or suboptimal solutions. Obviously, the number of alternative solutions is specified before the search process begins and the corresponding qualities should be different. As admitted by the authors, a global optimum cannot always be achieved since their model is a mixed integer nonlinear program (MINLP).

(3) The model formulations and solution strategies used in these two works are different.

The model formulations proposed in paper 1 are limited to one LP and two MILPs. These models are solved in sequence with the *solution pool* technique provided by CPLEX 11[13] to automatically identify the *global optimum* and also *all alternative solutions*. On the other hand, a series of MINLPs are solved in paper 2. Additional constraints are introduced after obtaining a new degenerate solution. The added constraints are used to exclude the feasible solutions which have already been found in the previous runs. This solution strategy is repeated until a given number of degenerate solutions can be found.

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