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An Improved Mathematical Programming Model for Generating Optimal Heat-Exchanger Network (HEN) Designs

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ABSTRACT: The traditional model-based approach to synthesize the heat-exchanger networks (HENs) was developed on the basis of several unrealistic assumptions. An improved mixed-integer nonlinear programming (MINLP) model is thus developed in this work to circumvent these drawbacks. Specifically, a well-established empirical relation is introduced in the improved model formulation to account for the variation in heat-transfer coefficient with respect to flow rate, and, furthermore, the concept of heat-exchanger efficiency is incorporated to produce an accurate estimate of the heat-transfer area in each exchanger. As a result, it is possible to produce more cost-effective HEN designs with this model. The optimization results obtained in various case studies also show that the process conditions of individual exchangers in the optimal network can generally be tuned simultaneously to achieve a proper compromise between high efficiency and low irreversibility.

■ INTRODUCTION

Because of the increasing need for energy conservation in the chemical industry, heat exchanger network (HEN) design has become an active research issue in recent decades. Generally speaking, the conventional model-based HEN synthesis strategies can be classified as one of two types: the sequential and simultaneous approaches.[1] In the former case, three mathematical programming models—i.e., a linear program (LP), a mixed-integer linear program (MILP), and a nonlinear program (NLP)—are solved sequentially in consecutive steps to determine the minimum utility cost,[2] the minimum number of matches and their heat duties,[3] and the optimal HEN configuration and the heat-transfer areas of all exchangers embedded in this network.[4] On the other hand, an optimal HEN design can also be produced directly with a mixed-integer nonlinear programming (MINLP) model if the simultaneous approach is adopted.[5] A better solution can obviously be obtained with the latter strategy, because the inevitable tradeoffs in sequential steps can be avoided. However, it should also be noted that this assertion is only valid if a rigorous solution algorithm is available to overcome the inherent computational challenges.[6]

Although satisfactory results have been reported in the literature, it should be noted that the aforementioned MINLP model was formulated on the basis of several unrealistic assumptions. Specifically,

(1) The outlet streams from the exchangers in each stage of the proposed superstructure were assumed to be mixed isothermally;
(2) The overall heat-transfer coefficient of every exchanger in HEN was assumed to be identical; and
(3) Ideal heat-transfer behavior was assumed in evaluating the heat duty of every exchanger.

While these model simplifications were quite effective for the purpose of promoting computation efficiency, the corresponding optimal solutions may not be suitable for direct applications in practice. Many subsequent studies have thus been carried out to improve the validity of the original model by removing the isothermal mixing assumption.[7] However, very few published works addressed issues concerning the other two assumptions.

In this study, a well-established empirical relation is introduced in the model formulation to account for the variation in heat-transfer coefficient, with respect to flow rate; furthermore, the concept of heat-exchanger efficiency[8] is incorporated to produce an accurate estimate of the heat-transfer area in each exchanger. As a result, it is possible to generate better HEN designs by solving the improved mathematical program. To further demonstrate the merit of this approach, the entropy production rate associated with every heat exchanger in HEN design is also calculated and used as an alternative performance index for irreversibility assessment. It can be observed from the optimization results obtained in various case studies that, not only can a cost-effective HEN design be generated for any given problem, but also the process conditions of individual exchangers in the optimal network can generally be tuned simultaneously to achieve a proper compromise between high efficiency and low irreversibility.

■ EXISTING MODEL FORMULATION

On the basis of the Nomenclature section provided at the end of this paper, the existing model formulation[1,4,5] is first outlined here to facilitate clear illustration of the proposed modifications in the next section. For the sake of simplicity, in this model, it is also assumed that only one hot utility and one cold utility are available and their temperatures are selected to meet all possible heating and cooling requirements, respectively.

- Overall heat balances:

$$(Tin_i - Tout_i)fh_i = \sum_{k \in ST} \sum_{j \in CP} q_{i,j,k} + qcu_i \qquad i \in HP \qquad (1)$$

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$$(Tout_j - Tin_j)fc_j = \sum_{k \in ST} \sum_{i \in HP} q_{i,j,k} + qhu_j \qquad j \in CP \tag{2}$$

- Stage-wise heat balances:

$$(th_{i,k} - th_{i,k+1})fh_i = \sum_{j \in CP} q_{i,j,k} \qquad i \in HP,\ k \in ST \tag{3}$$

$$(tc_{j,k} - tc_{j,k+1})fc_j = \sum_{i \in HP} q_{i,j,k} \qquad j \in CP,\ k \in ST \tag{4}$$

- Heat balance for each exchanger:

$$fhd_{i,j,k}(thl_{i,j,k} - thr_{i,j,k}) = q_{i,j,k} \qquad i \in HP,\ j \in CP,\ k \in ST \tag{5}$$

$$fcd_{i,j,k}(tcl_{i,j,k} - tcr_{i,j,k}) = q_{i,j,k} \qquad i \in HP,\ j \in CP,\ k \in ST \tag{6}$$

$$th_{i,k} = thl_{i,j,k} \qquad i \in HP,\ j \in CP,\ k \in ST \tag{7}$$

$$fc_j tc_{j,k} = \sum_{i \in HP} fcd_{i,j,k} tcl_{i,j,k} \qquad j \in CP,\ k \in ST \tag{8}$$

$$fh_i th_{i,k+1} = \sum_{j \in CP} fhd_{i,j,k} thr_{i,j,k} \qquad i \in HP,\ k \in ST \tag{9}$$

$$tc_{j,k+1} = tcr_{i,j,k} \qquad i \in HP,\ j \in CP,\ k \in ST \tag{10}$$

- Sum of stream splitting:

$$fh_i = \sum_{j \in CP} fhd_{i,j,k} \qquad i \in HP,\ k \in ST \tag{11}$$

$$fc_j = \sum_{i \in HP} fcd_{i,j,k} \qquad j \in CP,\ k \in ST \tag{12}$$

- Temperature feasibility constraints:

$$th_{i,1} = Tin_i \qquad i \in HP \tag{13}$$

$$tc_{j,NOK+1} = Tin_j \qquad j \in CP \tag{14}$$

$$th_{i,k} \geq th_{i,k+1} \qquad i \in HP,\ k \in ST \tag{15}$$

$$tc_{j,k} \geq tc_{j,k+1} \qquad j \in CP,\ k \in ST \tag{16}$$

$$Tout_i \leq th_{i,NOK+1} \qquad i \in HP \tag{17}$$

$$Tout_j \geq tc_{j,1} \qquad j \in CP \tag{18}$$

- Utility loads:

$$(th_{i,NOK+1} - Tout_i)fh_i = qcu_i \qquad i \in HP \tag{19}$$

$$(Tout_j - tc_{i,1})fc_j = qhu_j \qquad j \in CP \tag{20}$$

- Heat-load constraints:

$$q_{i,j,k} - \Omega z_{i,j,k} \leq 0 \qquad i \in HP,\ j \in CP,\ k \in ST \tag{21}$$

$$qcu_i - \Omega zcu_i \leq 0 \qquad i \in HP \tag{22}$$

$$qhu_j - \Omega zhu_j \leq 0 \qquad j \in CP \tag{23}$$

- Approach temperatures:

$$dtl_{i,j,k} \leq thl_{i,j,k} - tcl_{i,j,k} + \Gamma(1 - z_{i,j,k}) \qquad i \in HP,\ j \in CP,\ k \in ST \tag{24}$$

$$dtr_{i,j,k} \leq thr_{i,j,k} - tcr_{i,j,k} + \Gamma(1 - z_{i,j,k}) \qquad i \in HP,\ j \in CP,\ k \in ST \tag{25}$$

$$dthu_j \leq Tout_{HU} - tc_{j,1} + \Gamma(1 - zhu_j) \qquad j \in CP \tag{26a}$$

$$dthu_j \leq Tin_{HU} - Tout_j + \Gamma(1 - zhu_j) \qquad j \in CP \tag{26b}$$

$$dtcu_i \leq th_{i,NOK+1} - Tout_{CU} + \Gamma(1 - zcu_i) \qquad i \in HP \tag{27a}$$

$$dtcu_i \leq Tout_i - Tin_{CU} + \Gamma(1 - zcu_i) \qquad i \in HP \tag{27b}$$

- Approach temperature bounds

$$dtl_{i,j,k},\ dtr_{i,j,k},\ dthu_j,\ dtcu_i \geq EMAT \qquad i \in HP,\ j \in CP,\ k \in ST \tag{28}$$

- Heat-transfer areas for streams and utility matches:

For stream matches:

$$A_{i,j,k} = \frac{q_{i,j,k}}{U_{i,j}\Delta t^{LMTD}_{i,j,k}} \qquad i \in HP,\ j \in CP,\ k \in ST \tag{29}$$

For hot utility matches:

$$Ahu_j = \frac{qhu_j}{Uhu_j \Delta t^{LMTD}_{HU,j}} \qquad j \in CP \tag{30}$$

For cold utility matches:

$$Acu_i = \frac{qcu_i}{Ucu_i \Delta t^{LMTD}_{i,CU}} \qquad i \in HP \tag{31}$$

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- Objective function

$$\mathrm{TAC} = \sum_{i \in HP} C_{CU} qcu_i + \sum_{j \in CP} C_{HU} qhu_j + C_F\left(\sum_{i \in HP}\sum_{j \in CP}\sum_{k \in ST} z_{i,j,k} + \sum_{j \in CP} zhu_j + \sum_{i \in HP} zcu_i\right) + C_A\left(\sum_{i \in HP}\sum_{j \in CP}\sum_{k \in ST} A_{i,j,k}^{\beta} + \sum_{j \in CP} Ahu_j^{\beta} + \sum_{i \in HP} Acu_i^{\beta}\right) \quad (32)$$

■ IMPROVED FORMULATION

As mentioned previously, it is the intention of this study to develop an improved model formulation by removing the original simplification assumptions. Since the isothermal mixing constraints have already been relaxed in the published model (i.e., eqs 1−32), let us consider the other two assumptions in the sequel.

Overall Heat-Transfer Coefficients. It has been widely recognized (e.g., see Nie[9] and Balkan[10]) that the individual heat-transfer coefficient is proportional to the 0.8th power of the flow rate or heat-capacity flow rate (if the heat capacity is a constant), i.e.,

$$h_{new} = h_{old}\left(\frac{\dot{m}_{new}}{\dot{m}_{old}}\right)^{0.8} \quad (33)$$

On the basis of this relation, the overall heat-transfer coefficient in an exchanger can be considered to be a function of hot and cold heat-capacity flow rates. More specifically, the following constraints should be inserted into the existing model:

$$hhk_{i,j,k}\overline{fh}_i^{0.8} = \overline{hh}_i fhd_{i,j,k}^{0.8} \qquad i \in HP,\ j \in CP,\ k \in ST \quad (34)$$

$$hck_{i,j,k}\overline{fc}_j^{0.8} = \overline{hc}_j fcd_{i,j,k}^{0.8} \qquad i \in HP,\ j \in CP,\ k \in ST \quad (35)$$

$$\frac{1}{U_{i,j,k}} = \frac{1}{hhk_{i,j,k}} + \frac{1}{hck_{i,j,k}} \qquad i \in HP,\ j \in CP,\ k \in ST \quad (36)$$

The same approach can also be adopted to compute the overall heat-transfer coefficients for utility users and these variable coefficients should then be used in eqs 29−31 in the improved model.

Heat-Exchanger Efficiency. Conceptually, the heat-exchanger efficiency can be regarded as the ratio between the actual heat-transfer rate and an ideal one,[8] i.e.,

$$\eta = \frac{q}{q_{ideal}} = \frac{q}{UA\Delta t^{AMTD}} \quad (37)$$

For each possible match in HEN, this efficiency can be expressed accordingly as

$$\eta_{i,j,k} = \frac{2q_{i,j,k}}{U_{i,j,k}A_{i,j,k}(thl_{i,j,k} + thr_{i,j,k} - tcl_{i,j,k} - tcr_{i,j,k})} \qquad i \in HP,\ j \in CP,\ k \in ST \quad (38)$$

Fakheri[8] further suggested the use of the *fin analogy number* (Fa) as a means to determine the efficiencies of various different types of exchangers. For the convenience of illustration, the formula for counter-flow exchangers is presented below:

$$\eta_{i,j,k} = \frac{\tanh(Fa_{i,j,k})}{Fa_{i,j,k}} \qquad i \in HP,\ j \in CP,\ k \in ST \quad (39)$$

where $Fa_{i,j,k}$ ($I \in HP$, $j \in CP$, $k \in ST$) represents the fin analogy number for match (i,j,k). Combining eq 38, eq 39, and the definition of the fin analogy number (given in the Nomenclature section) yields

$$A_{i,j,k} = \frac{2C_{i,j,k}^{\min}}{U_{i,j,k}(1 - Cr_{i,j,k})}\tanh^{-1}\frac{q_{i,j,k}(1 - Cr_{i,j,k})}{C_{i,j,k}^{\min}(dtl_{i,j,k} + dtr_{i,j,k})} \qquad i \in HP,\ j \in CP,\ k \in ST \quad (40)$$

This expression can be used in the improved model to replace eq 29. The efficiency of each exchanger can be recalculated according to eq 37 when the optimal solution becomes available.

The formulas for computing heat-transfer areas of the utility users can be derived in a similar fashion. In the heaters, since the hot utility is usually steam and the corresponding heat-capacity flow rate can be regarded as infinitely large, it is reasonable to set $Cr = 0$. Thus,

$$Ahu_j = \frac{2fc_j}{U_{j,HU}}\tanh^{-1}\frac{qhu_j}{fc_j(Tin_{HU} + Tout_{HU} - tc_{j,1} - Tout_j)} \qquad j \in CP \quad (41)$$

Since phase change is not expected in any cooler, the corresponding heat-transfer area can be expressed as

$$Acu_i = \frac{2C_{i,CU}^{\min}}{U_{i,CU}(1 - Cr_{i,CU})} \times \tanh^{-1}\frac{qcu_i(1 - Cr_{i,CU})}{C_{i,CU}^{\min}(th_{i,NOK+1} + Tout_i - Tin_{CU} - Tout_{CU})} \qquad i \in HP \quad (42)$$

Finally, notice that eqs 41 and 42 are used in the present study to replace eqs 30 and 31, respectively.

■ ENTROPY GENERATION

Entropy generation in a typical heat exchanger is caused by heat transfer and pressure drop. Neglecting the typically small contribution of the latter factor, the dimensionless entropy generation rate can be expressed as[8]

$$\sigma_{i,j,k} = \frac{\dot{S}_{i,j,k}^{gen}}{C_{i,j,k}^{\min}} = \frac{fhd_{i,j,k}}{C_{i,j,k}^{\min}}\ln\left(\frac{thr_{i,j,k}}{thl_{i,j,k}}\right) + \frac{fcd_{i,j,k}}{C_{i,j,k}^{\min}}\ln\left(\frac{tcl_{i,j,k}}{tcr_{i,j,k}}\right) \qquad i \in HP,\ j \in CP,\ k \in ST \quad (43)$$

The entropy generation rate for a heater can be determined with a similar formula. However, the contribution of the heating utility should approach a limit if there is a phase change and the corresponding inlet and outlet temperatures are identical, i.e.,

$$\sigma_{HU,j} = \lim_{Tout_{HU} \to Tin_{HU}} \frac{f_{HU}}{fc_j} \ln\left(\frac{Tout_{HU}}{Tin_{HU}}\right) + \ln\left(\frac{Tout_j}{tc_{j,1}}\right) = \frac{qhu_j}{Tin_{HU} fc_j} + \ln\left(\frac{Tout_j}{tc_{j,1}}\right) \quad j \in CP \tag{44}$$

On the other hand, the formula for computing the entropy generation rate of a cooler can be produced according to eq 43, i.e.,

$$\sigma_{i,CU} = \frac{fh_i}{C_{i,CU}^{\min}} \ln\left(\frac{Tout_i}{th_{i,NOK+1}}\right) + \frac{\left(\frac{qcu_i}{Tout_{CU} - Tin_{CU}}\right)}{C_{i,CU}^{\min}} \times \ln\left(\frac{Tout_{CU}}{Tin_{CU}}\right) \quad i \in HP \tag{45}$$

Notice also that the minimum entropy generation[8] for a *balanced* counter-flow exchanger can be expressed as

$$\sigma_{i,j,k}^{\min} = \ln \frac{\left(1 + \frac{tcr_{i,j,k}}{thl_{i,j,k}} NTU_{i,j,k}\eta_{i,j,k}\right)\left(1 + \frac{thl_{i,j,k}}{tcr_{i,j,k}} NTU_{i,j,k}\eta_{i,j,k}\right)}{(1 + NTU_{i,j,k}\eta_{i,j,k})^2} \quad i \in HP,\ j \in CP,\ k \in ST \tag{46}$$

In addition to the aforementioned exchanger efficiency, the following performance index is also adopted in this work to evaluate the degree of irreversibility associated with every heat-transfer unit in a HEN design:

$$\phi_{i,j,k} = \frac{\sigma_{i,j,k} - \sigma_{i,j,k}^{\min}}{\sigma_{i,j,k}^{\min}} \tag{47}$$

Finally, it should be noted that the same approach should be applicable for the utility users as well.

■ CASE STUDIES

Three examples are presented in this section to demonstrate the superiority of the proposed model. The process data used in these examples are listed in Tables 1, 2, and 3, respectively.

Table 1. Process Data Used in Case I[a]

	FCP (kW/K)	Tin (K)	Tout (K)	h (kW/(m² K))	utility cost ($/(kW yr))
H1	10	650	370	1	
H2	20	590	370	1	
C1	15	410	650	1	
C2	13	350	500	1	
S1		680	680	5	80
W1		300	320	1	15

[a]Data taken from ref 4. Exchange cost = 5500 + 150 × area.

For comparison purposes, the existing and improved models were both solved in each example. The optimization results obtained in all three cases are presented in Tables 4, 5, and 6, respectively, and the corresponding network configurations can be found in Figures 1−12. For comparison purposes, the optimal solutions generated with the *existing* model were used to manually calculate the "actual" exchanger areas, according to eqs 40−42, as well as the corresponding total annual costs (TACs). These data can be found in Tables 4−6. On the other hand, to show that the cost-optimal HEN designs are, at the same time, thermodynamically appealing, the exchanger efficiency ($\eta_{i,j,k}$) and the irreversibility index ($\varphi_{i,j,k}$) were computed on the basis of the optimization results obtained with *both* models. These evaluation results are reported

Table 2. Process Data Used in Case II[a]

	FCP (kW/K)	Tin (K)	Tout (K)	h (kW/(m² K))	utility cost ($/(kW yr))
H1	20	423.15	318.15	2	
C1	13	333.15	393.15	2	
C2	12	293.15	393.15	2	
S1		483.15	483.15	1	80
W1		278.15	288.15	1	20

[a]Data taken from ref 5. Exchange cost = 4000 + 700 × area$^{0.8}$.

Table 3. Process Data Used in Case III[a]

	FCP (kW/K)	Tin (K)	Tout (K)	h (kW/(m² K))	utility cost ($/(kW yr))
H1	22	440	350	2	
C1	20	349	430	2	
C2	7.5	320	368	0.67	
S1		500	500	1	120
W1		300	320	1	20

[a]Data taken from ref 7. Exchange cost = 6600 + 670 × area$^{0.83}$.

Table 4. Optimization Results Obtained in Case I

	Existing Model		Improved Model	
	EMAT = 10 K	EMAT = 5 K	EMAT = 10 K	EMAT = 5 K
total heat-transfer area (m²)	334	345	330	345
hot utility consumption rate (kW)	491	526	490	518
cold utility consumption rate (kW)	2141	2176	2140	2168
total utility cost ($/yr)	71409	74736	71267	73990
total capital investment ($/yr)	83162	79284	81527	78230
total annual cost ($/yr)	154571	154021	152794	152220
average efficiency	0.92	0.89	0.92	0.90
average entropy increase (%)	24.82	24.25	24.90	23.66

Table 5. Optimization Results Obtained in Case II

	Existing Model		Improved Model	
	EMAT = 10 K	EMAT = 5 K	EMAT = 10 K	EMAT = 5 K
total heat-transfer area (m²)	78	92	105	104
hot utility consumption rate (kW)	228	292	0	0
cold utility consumption rate (kW)	348	172	120	120
total utility cost ($/yr)	25200	19632	2400	2400
total capital investment ($/yr)	50192	52869	46049	45905
total annual cost ($/yr)	75392	72502	48449	48305
average efficiency	0.92	0.87	0.93	0.92
average entropy increase (%)	29.83	32.15	16.52	16.96

Table 6. Optimization Results Obtained in Case III

	Existing Model		Improved Model	
	EMAT = 10 K	*EMAT* = 5 K	*EMAT* = 10 K	*EMAT* = 5 K
total heat-transfer area (m^2)	153	165	127	204
hot utility consumption rate (kW)	87	52	105	0
cold utility consumption rate (kW)	87	52	105	0
total utility cost (\$/yr)	12218	7334	14761	0
total capital investment (\$/yr)	78252	80468	68829	73295
total annual cost (\$/yr)	90471	87802	83590	73295
average efficiency	0.93	0.89	0.89	0.83
average entropy increase (%)	33.21	38.64	33.50	35.64

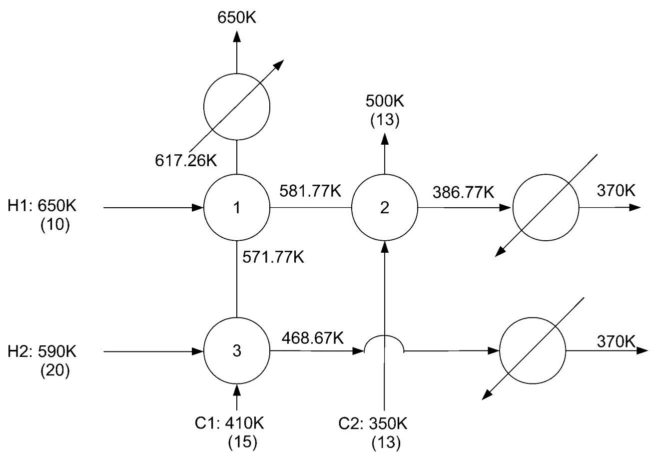

Figure 1. Optimal HEN design obtained with existing model in Case I (EMAT = 10 K).

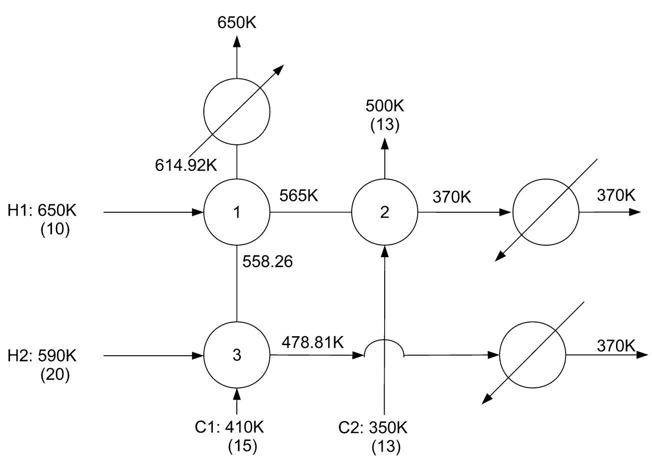

Figure 2. Optimal HEN design obtained with existing model in Case I (EMAT = 5 K).

in Tables 7–9. A brief analysis of all HEN designs is given below.

Case I. It can be observed from Table 4 that, by embedding efficiency constraints into the MINLP model, a smaller total heat-transfer area is required in the improved HEN design and the TAC drops from 154751 \$/yr to 152794 \$/yr if EMAT is set to be 10 K, and from 154021 \$/yr to 152220 \$/yr if EMAT = 5 K. Notice also that the total utility cost is also reduced from

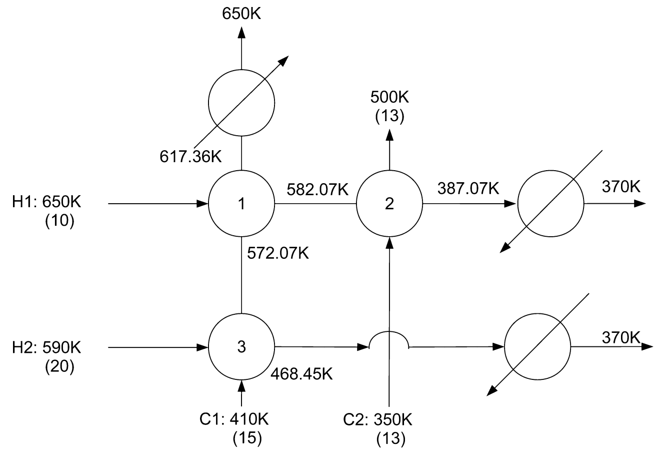

Figure 3. Optimal HEN design obtained with improved model in Case I (EMAT = 10 K).

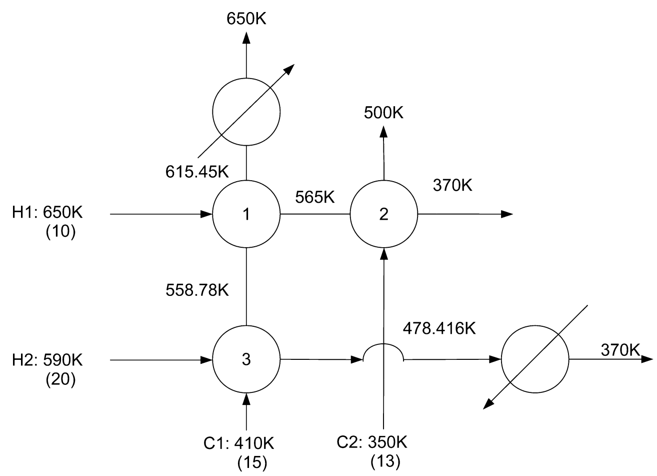

Figure 4. Optimal HEN design obtained with improved model in Case I (EMAT = 5 K).

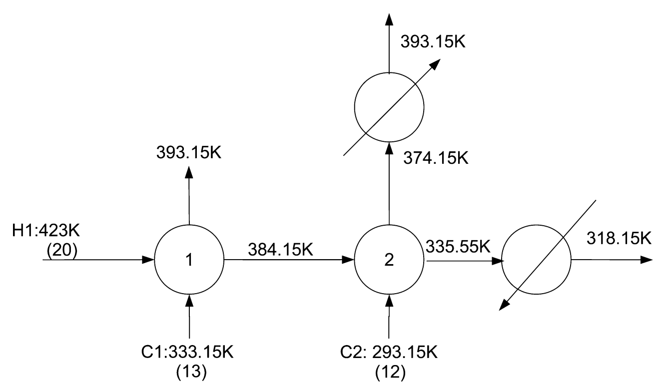

Figure 5. Optimal HEN design obtained with existing model in Case II (EMAT = 10 K).

71409 \$/yr to 71267 \$/yr (EMAT = 10 K), and from 74736 \$/yr to 73990 \$/yr (EMAT = 5 K). Based on the performance indices reported in Table 7, one can see that the entropy increases of the heat-transfer operations can generally be reduced slightly but the exchanger efficiencies remain unchanged. Therefore, the insignificant cost saving can be explained by the fact that the efficiency and irreversibility indices obtained with both the existing and improved models are almost identical.

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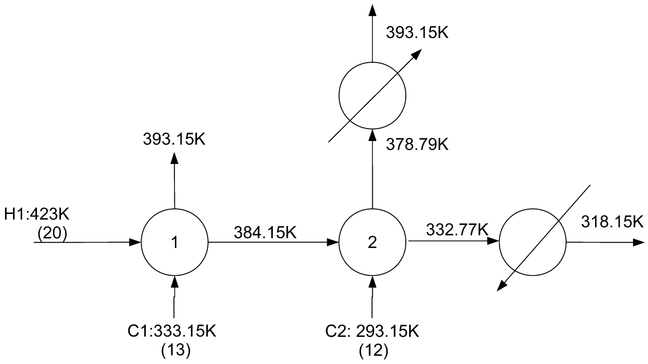

Figure 6. Optimal HEN design obtained with existing model in Case II (EMAT = 5 K).

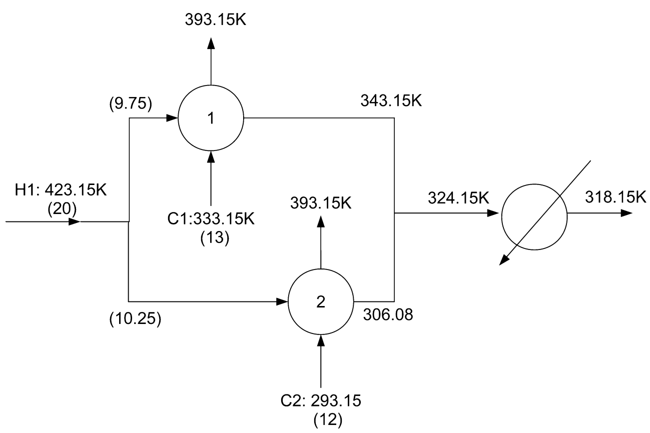

Figure 7. Optimal HEN design obtained with improved model in Case II (EMAT = 10 K).

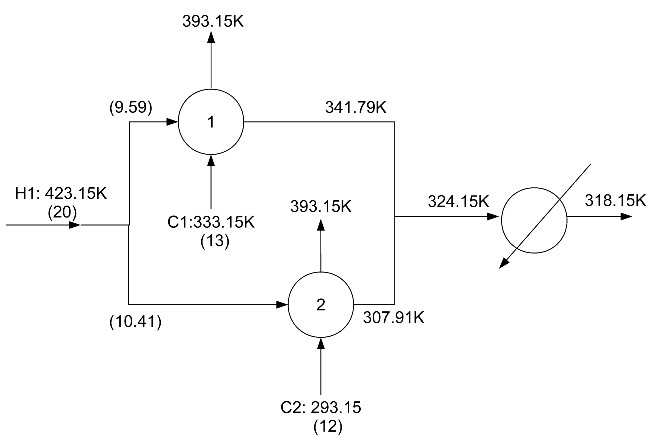

Figure 8. Optimal HEN design obtained with improved model in Case II (EMAT = 5 K).

Case II. A drastic reduction in TAC—from 75392 \$/yr to 48449 \$/yr (EMAT = 10 K) and from 72502 \$/yr to 48305 \$/yr—is brought about in this case (see Table 5) by reducing the utility consumption, which is mainly the result of structural changes. This conclusion can be easily reached by comparing Figures 5–8. Specifically, a stream-splitting structure on hot stream H1 is favored in HEN designs generated with the improved model, while this configuration cannot be obtained with the existing model. Such structural variation is believed to be a direct consequence of the area function (eq 40) and entropy generation function (eq 43). Notice that the exchanger

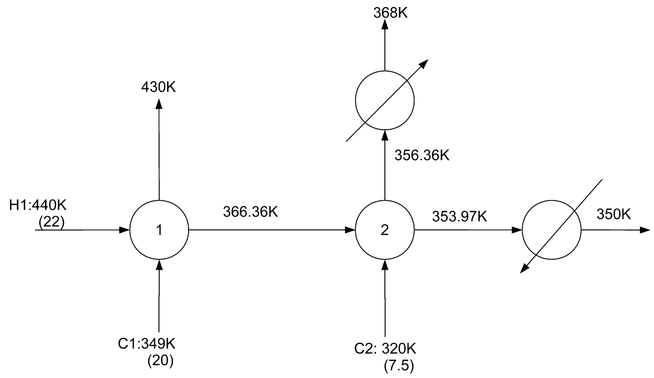

Figure 9. Optimal HEN design obtained with existing model in Case III (EMAT = 10 K).

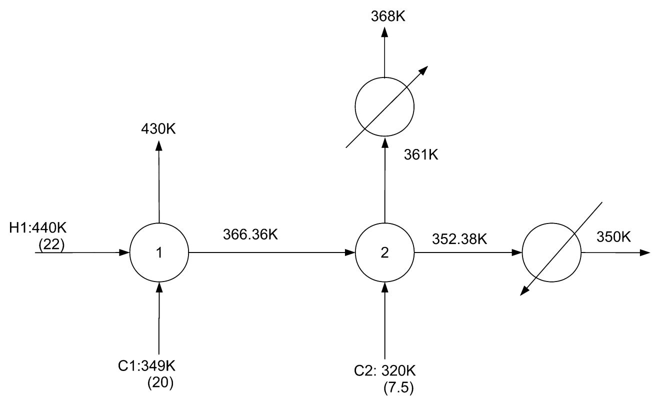

Figure 10. Optimal HEN design obtained with existing model in Case III (EMAT = 5 K).

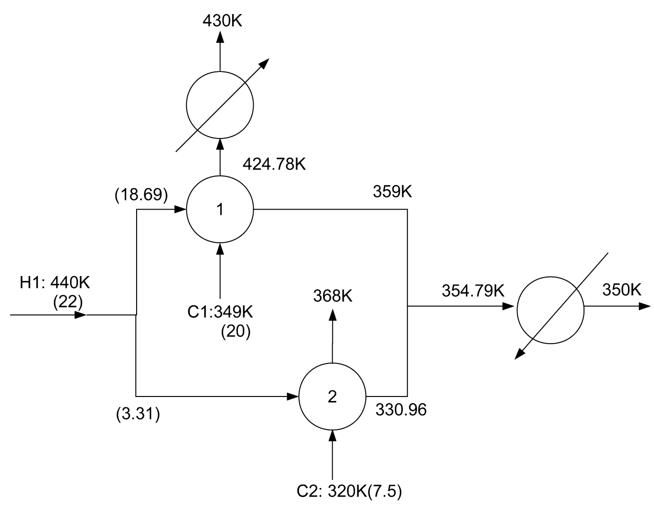

Figure 11. Optimal HEN design obtained with improved model in Case III. (EMAT=10K).

area and the entropy generation rate would both be minimized when Cr approaches unity. With stream splitting, the Cr ratios can be made higher (approaching 1) in HEN designs obtained with the improved model.

Case III. Let us first consider the designs produced by setting EMAT = 10 K. From Table 6, it can be observed that the total heat-transfer area is reduced from 153 m^2 to 127 m^2 with the proposed model, which results in a significant decrease in the TAC (from 90471 \$/yr to 83590 \$/yr). The stream

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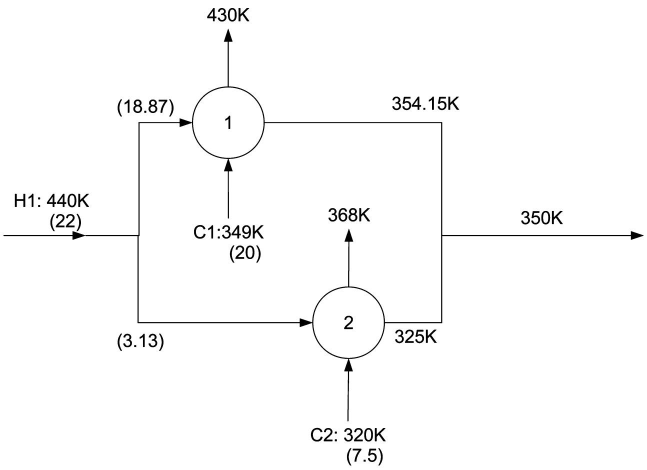

Figure 12. Optimal HEN design obtained with improved Model in Case III (EMAT = 5 K).

Table 7. Performance Indices of HEN Designs Obtained in Case I

exchange No.	$\eta_{i,j,k}$		$\varphi_{i,j,k}$ (%)	
	EMAT = 10 K	*EMAT* = 5 K	*EMAT* = 10 K	*EMAT* = 5 K
	Existing Model			
1	0.90	0.82	30	32.43
2	0.95	0.90	20.17	19.53
3	0.90	0.95	24.27	20.80
	Improved Model			
1	0.90	0.82	30	30.56
2	0.95	0.90	20.30	19.53
3	0.90	0.95	24.39	20.88

Table 8. Performance Indices of HEN Designs Obtained in Case II

exchange No.	$\eta_{i,j,k}$		$\varphi_{i,j,k}$ (%)	
	EMAT = 10 K	*EMAT* = 5 K	*EMAT* = 10 K	*EMAT* = 5 K
	Existing Model			
1	0.98	0.98	22.63	22.63
2	0.86	0.76	37.04	41.67
	Improved Model			
1	0.91	0.89	21.74	23.60
2	0.94	0.96	11.30	10.33

Table 9. Performance Indices of HEN Designs Obtained in Case III

exchange No.	$\eta_{i,j,k}$		$\varphi_{i,j,k}$ (%)	
	EMAT = 10 K	*EMAT* = 5 K	*EMAT* = 10 K	*EMAT* = 5 K
	Existing Model			
1	0.98	0.98	10.61	10.61
2	0.89	0.80	55.81	66.67
	Improved Model			
1	0.99	0.96	5.10	3.74
2	0.79	0.70	61.90	67.54

splitting structures in Figure 11 raise the Cr ratios of heat exchangers 1 and 2 to levels slightly larger than those shown in Figure 9. This feature also appeared previously in Case II.

Next, let us consider the case in which EMAT = 5 K. With a lower-temperature approach, the improved HEN design requires no utilities but its total heat-transfer area is larger than that generated with an existing model (see Figures 10 and 12). Notice that the lower exchanger efficiencies achieved in the improved structure should not be regarded as the direct results of the area function in eq 40. Although an additional small bypass may enhance efficiency, the corresponding TAC could also be raised to a higher value. In addition, any small bypass is usually not preferred in practice, because of operational difficulties and financial penalties. Notice that, by slightly sacrificing the exchanger efficiency, the TAC and the entropy generation rate are both kept at satisfactory levels with the improved model.

CONCLUSIONS

This work is the first attempt to incorporate efficiency considerations in the HEN synthesis procedure. Specifically, the traditional mixed-integer nonlinear programming (MINLP) model is modified by inserting additional constraints for computing the heat-transfer areas more accurately, to generate improved designs. This proposed model can generally be used to produce more-realistic cost-optimal structures with high efficiency and low irreversibility. The effectiveness of the proposed approach is clearly demonstrated in the examples presented in this paper.

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Notes

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NOMENCLATURE

A = heat-transfer area in exchanger
Ahu = heat-transfer area in heater
Acu = heat-transfer area in cooler
C^{min} = minimum heat-capacity flow rate, i.e., C^{min} = min(*fhd*,*fcd*)
C^{max} = maximum heat-capacity flow rate, i.e., C^{max} = max(*fhd*,*fcd*)
Cr = heat-capacity flow ratio, i.e., $Cr = C^{min}/C^{max}$
CP = set of all cold process streams
Fa = fin analogy number; $Fa = NTU(1 - Cr)/2$ for counter-flow exchangers
h = heat-transfer coefficient
hc = heat-transfer coefficient of cold fluid
hh = heat-transfer coefficient of hot fluid
hck = heat-transfer coefficient of cold fluid in a match
hhk = heat-transfer coefficient of hot fluid in a match
HP = set of all hot process streams
NTU = number of transfer units, i.e., $NTU = UA/C^{min}$
q = heat-transfer rate in an exchanger
qhu = heat-transfer rate in a heater
qcu = heat-transfer rate in a cooler
fh = heat-capacity flow rate of a hot stream
fc = heat-capacity flow rate of a cold stream
fhd = heat-capacity flow rate of a split branch of hot stream
fcd = heat-capacity flow rate of a split branch of cold stream
ST = set of all stages
Tin = inlet temperature of a process stream

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Tout = outlet temperature of a process stream
th = temperature of hot stream
tc = temperature of cold stream
thl = temperature of hot stream at the inlet of an exchanger
thr = temperature of hot stream at the outlet of an exchanger
tcr = temperature of cold stream at the inlet of an exchanger
tcl = temperature of cold stream at the outlet of an exchanger
dtl = temperature approach at the hot end of an exchanger
dtr = temperature approach at the cold end of an exchanger
dthu = temperature approach at the cold end of a heater
dtcu = temperature approach at the hot end of a cooler
Δt = temperature difference
z = binary variable used to denote the existence of a match between process streams
zhu = binary variable used to denote the existence of a match between a hot utility and a cold stream
zcu = binary variable used to denote the existence of a match between a hot stream and a cold utility
U = overall heat-transfer coefficient
η = heat-exchanger efficiency
Ω = upper bound for heat exchange
Γ = upper bound for temperature difference
C_F = fixed charge for capital investment of an heat exchanger
C_A = cost coefficient for heat-transfer area
C_{HU} = unit cost for hot utility
C_{CU} = unit cost for cold utility
$\dot{S}^{\mathrm{gen}}$ = entropy production rate
σ = dimensionless entropy generation rate, i.e., $\sigma = \dot{S}^{\mathrm{gen}}/C^{\mathrm{min}}$
σ^{min} = dimensionless minimum entropy generation rate
φ = irreversibility index

Subscripts and Superscripts

i = hot stream
j = cold stream
k = stage
HU = heating utility
CU = cooling utility
NOK = total number of stages
β = exponent for area cost
LMTD = log-mean temperature difference
AMTD = arithmetic mean temperature difference
EMAT = minimum approach temperature
ideal = ideal process
— = reference value

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