I&EC research
Industrial & Engineering Chemistry Research
Article
pubs.acs.org/IECR

Heuristic Approach to Incorporate Timesharing Schemes in Multiperiod Heat Exchanger Network Designs

Efrat Sadeli and Chuei-Tin Chang*

Department of Chemical Engineering, National Cheng Kung University, Tainan 70101, Taiwan, ROC

ABSTRACT: Heat exchanger network (HEN) synthesis has been widely recognized as an effective design method for significant energy saving in industrial processes. Although traditionally its sole objective is to minimize the total annual cost, there are additional implementation issues which must be addressed in realistic multiperiod applications. In particular, if a conventional programming-based procedure is adopted for generating the optimal HEN structure, each embedded match inevitably calls for different heat-transfer areas in different periods. Since the usual practice is to select the largest unit, its operation in one (or more) period may be grossly inefficient because the corresponding overdesign level is simply too high. To circumvent this drawback, a modified mathematical model and a set of timesharing heuristics have been developed in this work to generate practical network configurations that can handle all heat duties. On the basis of the extensive case studies performed so far, it can be observed that this proposed approach is especially effective for multiperiod HEN design problems in which the process conditions vary significantly.

1. INTRODUCTION

Two programming-based approaches are available for generating the single-period heat exchanger network designs, i.e., the sequential and simultaneous strategies.[1] Although the former is simpler, the design results are often suboptimal since the trade-off issues between operating and capital costs cannot be adequately addressed. The latter method was first developed by Yee and Grossmann[2] and Yee et al.[3] for the purpose of achieving minimum total annual cost. In the subsequent studies,[4−8] a modified formulation was proposed for solving the *multiperiod* Heat exchanger network (HEN) design problems. A common feature can almost always be identified in the reported examples, i.e., the process data adopted for different periods do not vary appreciably (e.g., see Table 1). As a result, the period-dependent heat duty of each match in the optimal solution tends to stay within a narrow range. Under this situation, the heat-exchange operation between each pair of hot and cold streams in HEN can usually be facilitated with a single dedicated unit and its specifications can be determined on the basis of the *largest* required area among all periods. This simple design approach was in fact quite successful in the published studies. However, in certain realistic processes, the initial and target temperatures of process streams may change drastically from one period to another and some of them may not even be present at all time. For these systems, the aforementioned practice could end up with grossly inefficient exchangers due to very high overdesign levels. It is thus our belief that, in order to reduce total capital cost as much as possible, the heat-exchange capacities embedded in a HEN should be more efficiently shared by matches which are not present in the same period. Such timesharing opportunities in multiperiod HEN design have never been fully explored before.

Table 1. Process Data of Case I[5]

stream	period	T_{in} (K)	T_{out} (K)	F_{CP} (kW/K)
H1	1	583	323	1.4
	2	593	323	1.8
	3	593	323	1.8
H2	1	723	553	2.0
	2	723	553	2.0
	3	723	553	2.0
C1	1	313	393	3.0
	2	313	393	3.0
	3	313	393	3.0
C2	1	388	553	2.0
	2	383	553	2.4
	3	393	553	1.6

Notice also that, in the published works for multiperiod HEN designs,[4−7] all exchangers were assumed to be *ideal*. In order to produce more accurate estimates of the heat-transfer areas and capital costs of exchangers, it is obviously necessary to incorporate the well-established F_T correction factors in the exchanger sizing procedure. Galli and Cerda[9] tried to address this issue by using a sequential approach for the single-period HENs. Since the trade-off between capital and operating costs could not be properly assessed with a sequential optimization strategy, Ravagnani and Caballero[10] later used a simultaneous approach to produce the optimal heat-exchanger designs in a HEN and, also, Ponce et al.[11] adopted genetic algorithm to solve essentially the same problem. Although successful applications were reported in both studies, it should be noted that they were still concerned with the single-period designs.

The present study is aimed at circumventing the aforementioned drawbacks of the existing works. In other words, our research objectives are (1) to develop an optimization procedure that incorporates realistic exchanger designs in the optimal

Received: September 22, 2011
Revised: March 5, 2012
Accepted: May 24, 2012
Published: May 24, 2012

   dx.doi.org/10.1021/ie202171g | Ind. Eng. Chem. Res. 2012, 51, 7967−7987

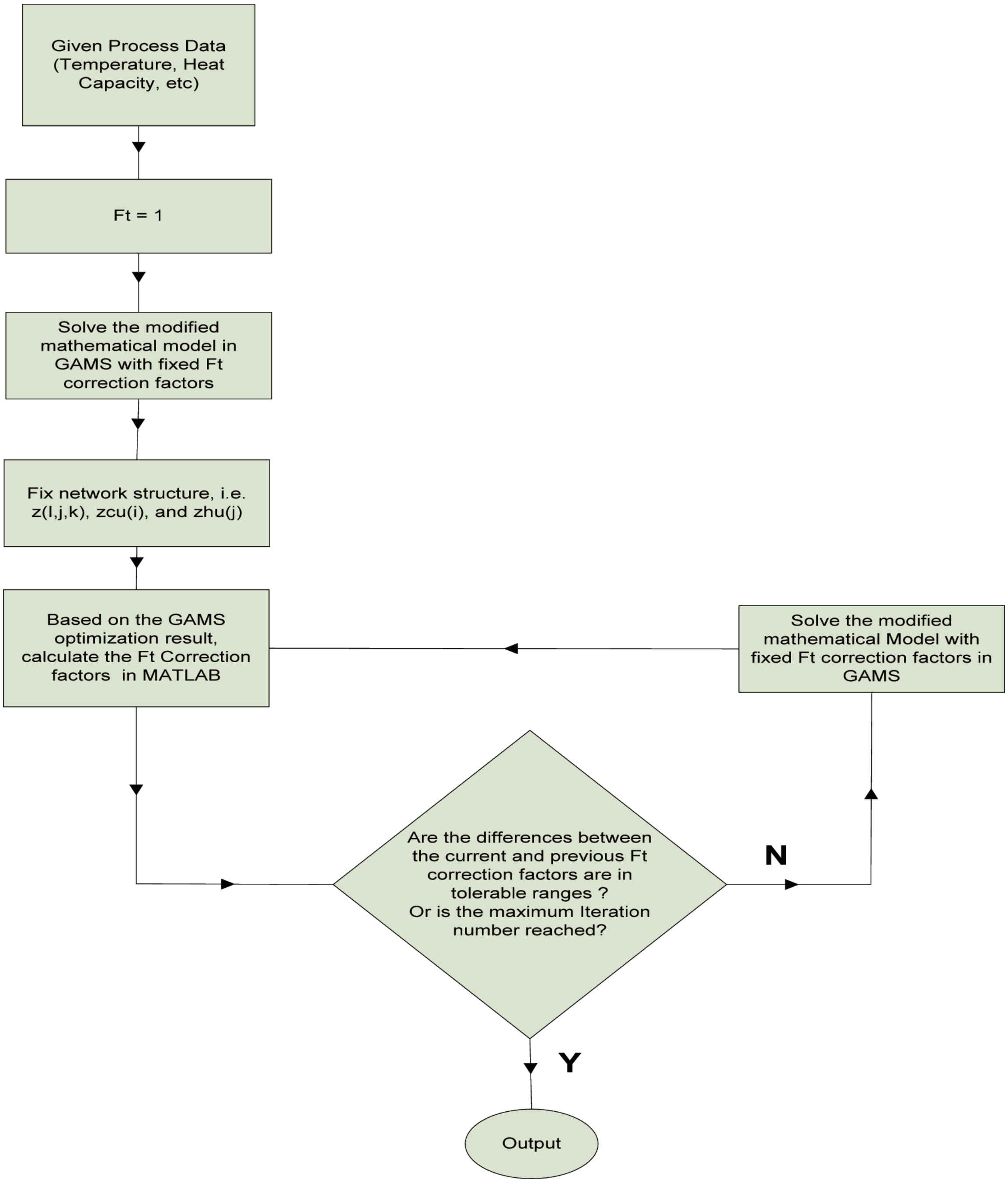


Figure 1. Solution algorithm for generating preliminary multiperiod HEN design.

Table 2. Comparison between the Preliminary Multiperiod HEN Designs Generated with Two Different Models in Case I

	conventional model	modified model
total heat transfer area (m^2)	18.74	19.95
total annual cost (USD/y)	61,099	62,817

Table 3. Preliminary Heat-Transfer Areas Obtained in Case I

match	period-1 area (m^2)	period-2 area (m^2)	period-3 area (m^2)
(1,1,1)	3.985	3.985	3.985
(2,2,1)	4.268	4.268	4.268
(1,*CU*,3)	8.507	8.507	8.507
(2,*CU*,3)	0.291		0.645
(*HU*,2,0)		2.543	

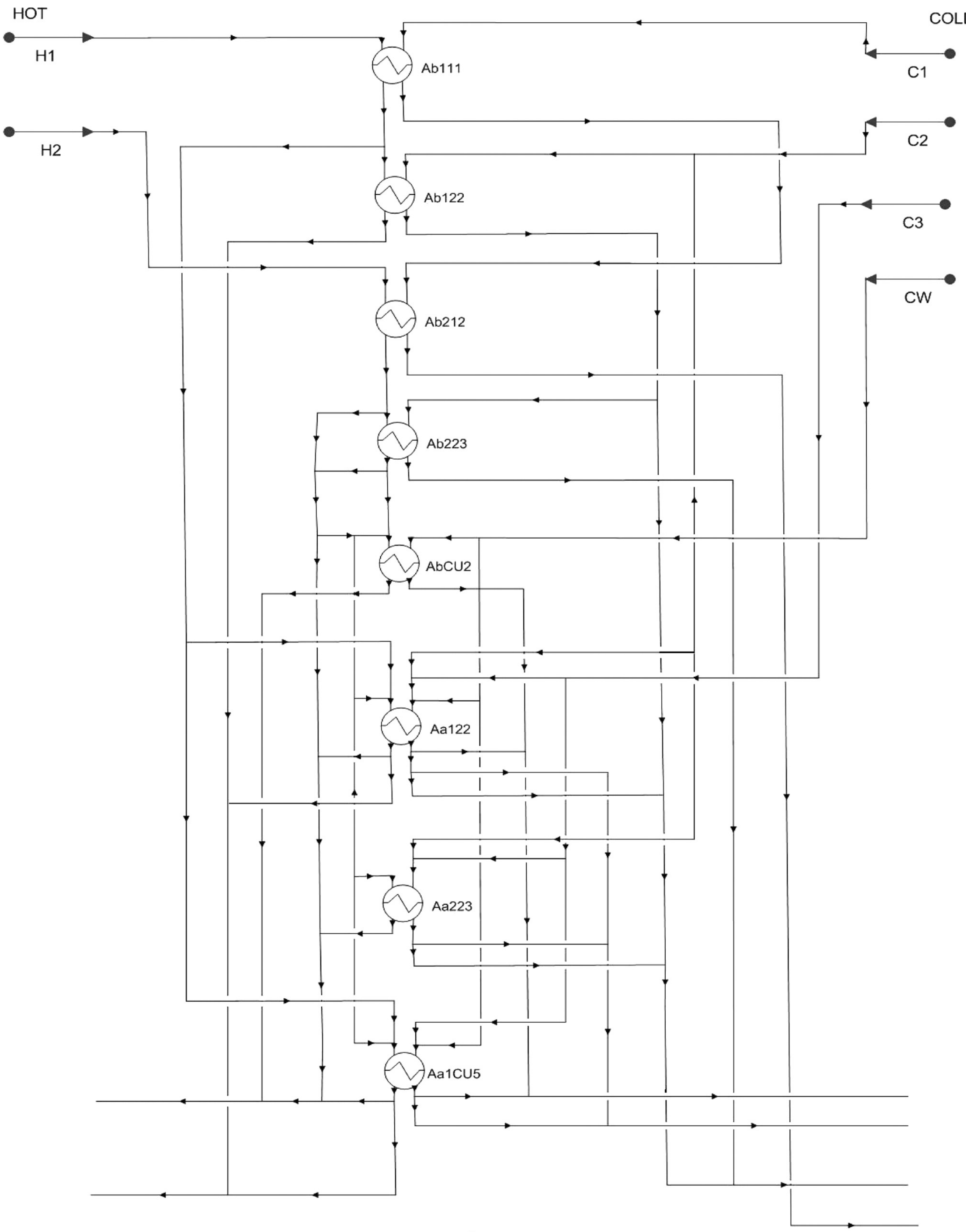
multiperiod HEN structures and (2) to reduce the capital cost of this HEN design by efficient utilization of heat-exchange units in a timesharing scheme. To illustrate the key ideas developed for achieving these objectives, this paper is organized as follows. The modified MINLP model is first described in the next section. Specifically, the mathematic formulas for producing the improved area estimates are presented here and the corresponding constraints in the conventional model are also explicitly given. In section 3, a set of heuristic rules are provided for the purpose of developing the timesharing scheme of a multiperiod HEN design. Two examples are used in this section to illustrate the

dx.doi.org/10.1021/ie202171g | *Ind. Eng. Chem. Res.* 2012, 51, 7967−7987

Table 4. Preliminary Heat-Transfer Areas Obtained in Case II

match	period-1 area (m^2)	period-2 area (m^2)	period-3 area (m^2)
(1,1,1)	14.8	15.7	14.0
(1,2,2)	87.6	54.6	52.3
(2,1,2)	28.2	27.9	30.8
(2,2,3)	214.1	231.4	304.6
(2,3,4)	0.0	143.2	0.0
(1,CU,5)	0.0	0.0	56.6
(2,CU,5)	100.9	108.3	105.2

implementation steps. Since the timesharing opportunities may not be present in every given system, four screening indicators are used in this study to assess the potential for a successful application of the proposed heuristics. These indicators are described in section 4. Two more case studies are then presented in section 5 to demonstrate the effectiveness of the proposed strategies for synthesizing an optimal multiperiod HEN design with timesharing scheme. Finally, the conclusions are outlined in the last section.



a

Figure 2. continued

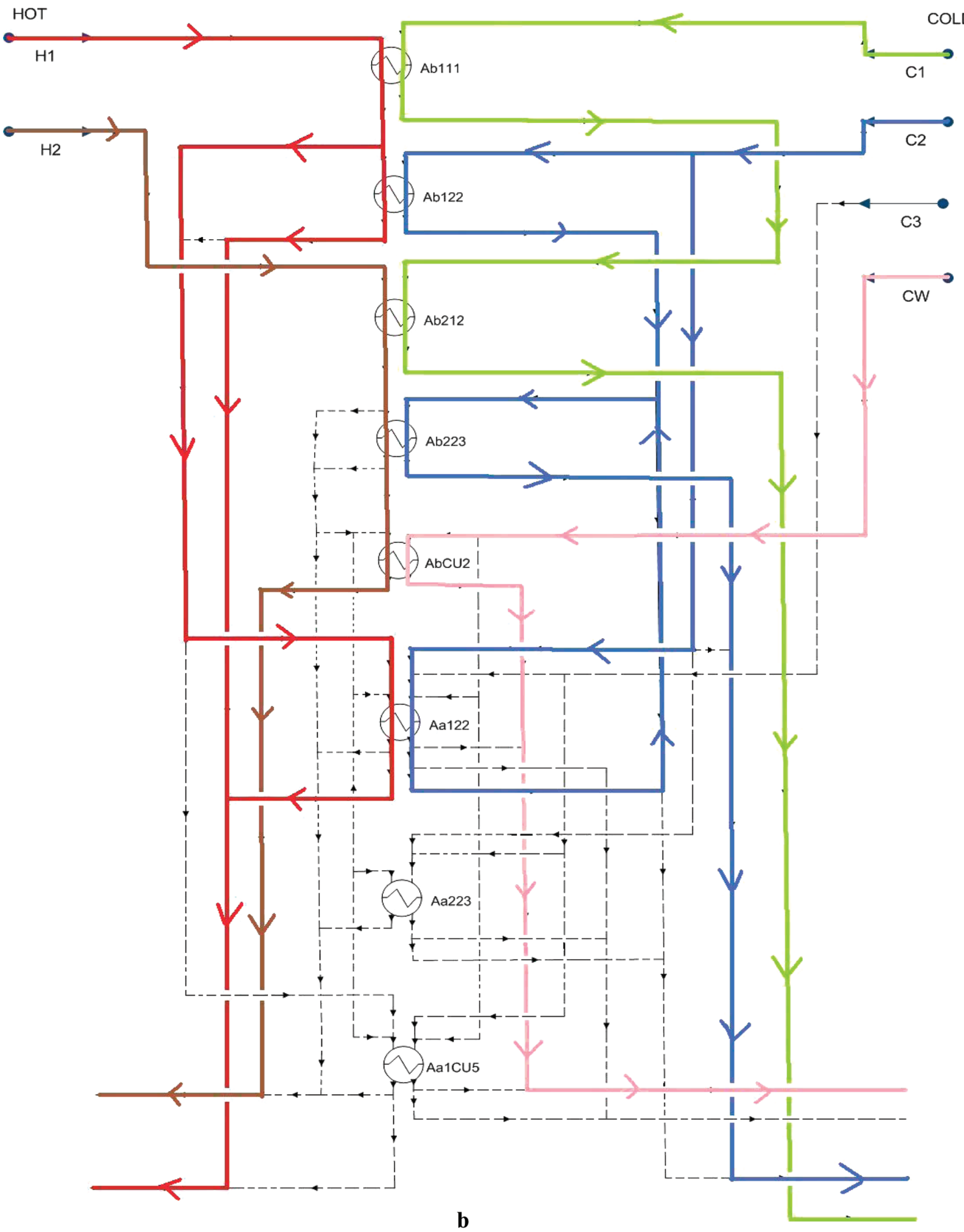


Figure 2. continued

2. MODIFIED MATHEMATICAL PROGRAMMING MODEL

In the present study, the available mathematical programming model[2,8] has been modified to improve the area estimates of exchangers in multiperiod HEN designs. For the sake of completeness, the conventional MINLP model is first repeated below:

1. Overall heat balances:

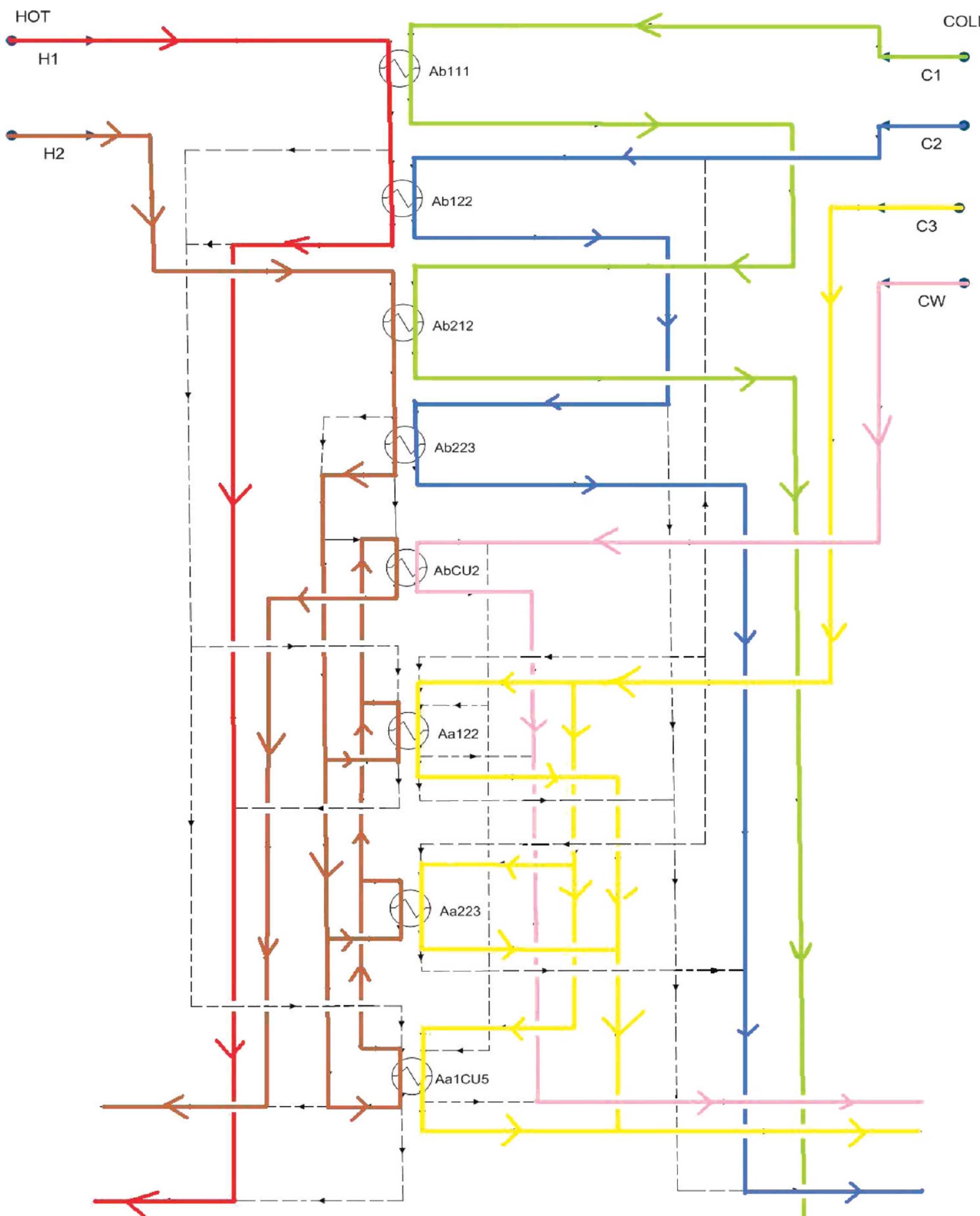
$$[T_{i,p}^{Hin} - T_{i,p}^{Hout}]F_{i,p}^{H} = \sum_{k \in ST} \sum_{j \in CP} q_{i,j,k,p} + q_{i,p}^{CU} \quad i \in HP, p \in PR \tag{1}$$

$$[T_{j,p}^{Cout} - T_{j,p}^{Cin}]F_{j,p}^{C} = \sum_{k \in ST} \sum_{i \in HP} q_{i,j,k,p} + q_{j,p}^{HU} \quad j \in CP, p \in PR \tag{2}$$

2. Stagewise heat balances:

$$[t_{i,k,p}^{H} - t_{i,k+1,p}^{H}]F_{i,p}^{H} = \sum_{j \in CP} q_{i,j,k,p} \quad i \in HP, k \in ST, p \in PR \tag{3}$$

$$[t_{j,k,p}^{C} - t_{j,k+1,p}^{C}]F_{j,p}^{C} = \sum_{i \in HP} q_{i,j,k,p} \quad j \in CP, k \in ST, p \in PR \tag{4}$$



c

Figure 2. continued

3. Heat balance for each exchanger:

$$[t^H_{i,k,p} - ths_{i,j,k,p}]rh_{i,j,k,p}F^H_{i,p} = q_{i,j,k,p} \quad i \in HP, j \in CP, k \in ST, p \in PR \tag{5}$$

$$[tcs_{i,j,k,p} - t^C_{j,k+1,p}]rc_{i,j,k,p}F^C_{j,p} = q_{i,j,k,p} \quad i \in HP, j \in CP, k \in ST, p \in PR \tag{6}$$

4. Sum of split ratio:

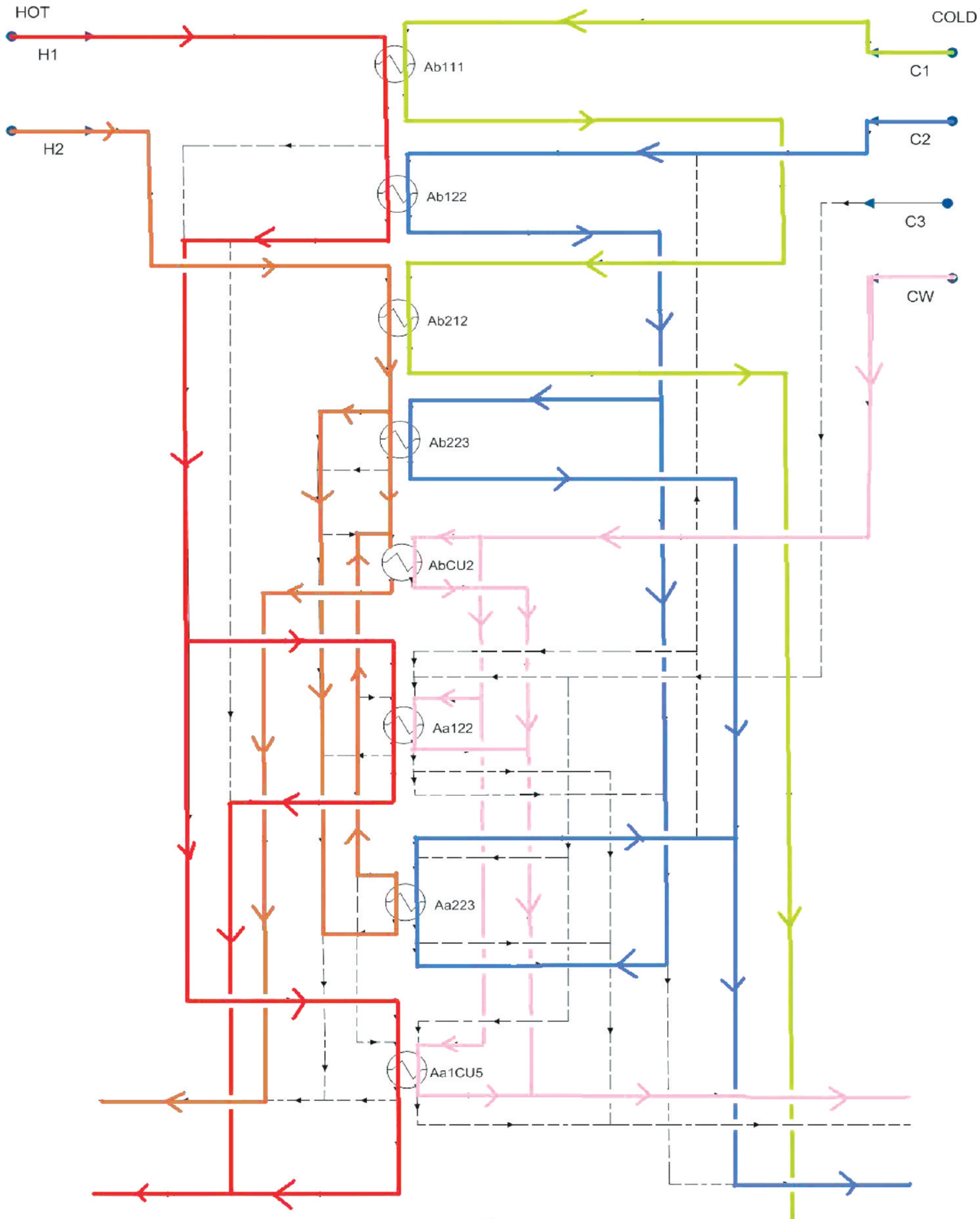
$$\sum_{j \in CP} rh_{i,j,k,p} = 1 \quad i \in HP, k \in ST, p \in PR \tag{7a}$$

$$\sum_{i \in HP} rc_{i,j,k,p} = 1 \quad j \in CP, k \in ST, p \in PR \tag{7b}$$

5. Outlet temperature of each exchanger:

$$\sum_{j \in CP} rh_{i,j,k,p}ths_{i,j,k,p} = t^H_{i,k+1,p} \quad i \in HP, k \in ST, p \in PR \tag{8}$$

dx.doi.org/10.1021/ie202171g | Ind. Eng. Chem. Res. 2012, 51, 7967−7987



d

Figure 2. (a) Process flow diagram of overall HEN design for case II. (b) Process flow diagram of HEN design in period 1 for case II (H1: red; H2: brown; C1: light green; C2: blue; CW: pink). (c) Process flow diagram of HEN design in period 2 for case II (H1: red; H2: brown; C1: light green; C2: blue; C3: yellow; CW: pink). (d) Process flow diagram of HEN design in period 3 for case II (H1: red; H2: brown; C1: light green; C2: blue; CW: pink).

$$\sum_{i \in HP} rc_{i,j,k,p} tcs_{i,j,k,p} = t^{C}_{j,k,p} \quad j \in CP,\ k \in ST,\ p \in PR \tag{9}$$

6. Temperature feasibility constraints:

$$T^{Hin}_{i,p} = t^{H}_{i,1,p} \quad i \in HP,\ p \in PR \tag{10}$$

$$T^{Cin}_{j,p} = t^{C}_{j,NOK+1,p} \quad j \in CP,\ p \in PR \tag{11}$$

$$t^{H}_{i,k,p} \geq t^{H}_{i,k+1,p} \quad i \in HP,\ k \in ST,\ p \in PR \tag{12}$$

$$t^{C}_{j,k,p} \geq t^{C}_{j,k+1,p} \quad j \in CP,\ k \in ST,\ p \in PR \tag{13}$$

Table 5. Screening Indicators

indicator	case I		case II		case III		case IV	
	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2
number of matches satisfying 1	1	0	2		1	1	0	1
number of periods satisfying 2	1	0	2		2	2	0	2
number of matches satisfying 3	1	1	2		5	1	3	0
number of periods satisfying 4	1	2	3		3	2	3	0

Table 6. Process Data of Case III

stream	period	T_{in} (K)	T_{out} (K)	F_{CP} (kW/K)
H1	1	650	370	10
	2	630	375	10.2
	3	645	369	10
H2	1	590	370	20
	2	585	585	20.5
	3	600	600	20.3
H3	1	630	630	30.0
	2	625	350	30.5
	3	610	610	30.3
C1	1	410	650	15
	2	405	654	15
	3	421	650	14.3
C2	1	350	500	13
	2	356	508	13.5
	3	369	502	13

$$T_{i,p}^{Hout} \leq t_{i,NOK+1,p}^{H} \quad i \in HP, p \in PR \tag{14}$$

$$T_{j,p}^{Cout} \geq t_{j,1,p}^{C} \quad j \in CP, p \in PR \tag{15}$$

7. Utility loads:

$$[t_{i,NOK+1,p}^{H} - T_{i,p}^{Hout}]F_{i,p}^{H} = q_{i,p}^{CU} \quad i \in HP, p \in PR \tag{16}$$

$$[T_{j,p}^{Cout} - t_{j,1,p}^{C}]F_{j,p}^{C} = q_{j,p}^{HU} \quad j \in CP, p \in PR \tag{17}$$

8. Heat load constraints:

$$q_{i,j,k,p} - Q_{UP} z_{i,j,k} \leq 0 \quad i \in HP, j \in CP, k \in ST, p \in PR \tag{18}$$

$$q_{i,p}^{CU} - Q_{UP} z_{i}^{CU} \leq 0 \quad i \in HP, p \in PR \tag{19}$$

$$q_{j,p}^{HU} - Q_{UP} z_{j}^{HU} \leq 0 \quad j \in CP, p \in PR \tag{20}$$

9. Approach temperatures:

$$dt_{i,j,k,p} \leq t_{i,k,p}^{H} - tcs_{i,j,k,p} + \Gamma(1 - z_{i,j,k}) \quad i \in HP, j \in CP, k \in ST, p \in PR \tag{21}$$

$$dt_{i,j,k+1,p} \leq ths_{i,j,k,p} - t_{j,k+1,p}^{C} + \Gamma(1 - z_{i,j,k}) \quad i \in HP, j \in CP, k \in ST, p \in PR \tag{22}$$

$$dt_{j,p}^{HU} \leq T^{HU,out} - t_{j,1,p}^{C} + \Gamma(1 - z_{j}^{HU}) \quad j \in CP, p \in PR \tag{23}$$

Table 7. Process Data of Case IV

stream	period	T_{in} (K)	T_{out} (K)	F_{CP} (kW/K)
H1	1	650	370	10.0
	2	630	630	10.2
	3	645	369	10.0
H2	1	630	630	12.5
	2	625	380	13.0
	3	610	610	13.3
H3	1	580	580	9.1
	2	575	370	8.5
	3	578	440	9.8
H4	1	600	480	15.0
	2	590	500	15.1
	3	595	450	15.3
C1	1	320	580	28.0
	2	330	574	29.0
	3	330	560	29.3

Table 8. Comparison between the Preliminary Multiperiod HEN Designs Generated with Two Different Models in Cases III and IV

	conventional model	modified model
case III		
total heat transfer area (m^2)	452.90	439.64
total annualized cost (USD/y)	763,409	761,606
case IV		
total heat transfer area (m^2)	356.07	433.24
total annualized cost (USD/y)	469,701.24	502,621.91

$$dt_{i,p}^{CU} \leq t_{i,NOK+1,p}^{H} - T^{CU,out} + \Gamma(1 - z_{i}^{CU}) \quad i \in HP, p \in PR \tag{24}$$

10. Approach temperature bounds:

$$dt_{i,j,k,p}, dt_{j,p}^{HU}, dt_{i,p}^{CU} \geq \Delta T_{min} \quad i \in HP, j \in CP, k \in ST, p \in PR \tag{25}$$

11. Maximum heat transfer area for stream and utility matches:[8]

(a) For the stream matches

$$A_{i,j,k}^{max} \geq A_{i,j,k,p} \quad i \in HP, j \in CP, k \in ST, p \in PR \tag{26a}$$

where

$$A_{i,j,k,p} = \frac{q_{i,j,k,p}}{\Delta T_{i,j,k,p}^{LMTD}}\left(\frac{1}{h_{i,p}^{H}} + \frac{1}{h_{j,p}^{C}}\right) \tag{26b}$$

To promote computation efficiency, the following formula[16] is adopted in this work to approximate the log-mean temperature difference, i.e.

$$\Delta T_{i,j,k,p}^{LMTD} \approx \left[\frac{1}{2} dt_{i,j,k,p} dt_{i,j,k+1,p}(dt_{i,j,k,p} + dt_{i,j,k+1,p})\right]^{1/3} \tag{26c}$$

dx.doi.org/10.1021/ie202171g | Ind. Eng. Chem. Res. 2012, 51, 7967−7987

Table 9. Final Assignments in Case III

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,2)	164.35	$A^b_{1,1,2}$	164.35	$A^b_{1,1,2}$	164.35	$A^b_{1,1,2}$
(2,1,2)	16.94	$A^a_{2,1,2}$				
(2,2,2)	45.42	$A^a_{2,2,2}$				
(3,2,1)			58.61	$A^a_{2,1,2} + A^a_{2,2,2}$		
(1,*CU*,3)	14.60	$A^b_{1,CU,3} + A^a_{1,CU,3}$	9.99	$A^b_{1,CU,3}$	14.67	$A^b_{1,CU,3} + A^a_{1,CU,3} + A^a_{1,CU,3}{}^*$
(2,*CU*,3)	38.56	$A^a_{2,CU,3}$				
(3,*CU*,3)			63.29	$A^a_{1,CU,3} + A^a_{2,CU,3} + A^a_{1,CU,3}{}^* + A^a_{3,CU,3}$		
(*HU*,1,0)	29.1	$A^b_{HU,1,0} + A^a_{HU,2,0} + A^a_{HU,1,0}$	29.1	$A^b_{HU,1,0} + A^a_{HU,2,0} + A^a_{HU,1,0}$	22.43	$A^b_{HU,1,0}$
(*HU*,2,0)					8.71	$A^a_{HU,2,0}$

(b) For the hot-utility matches

$$A_j^{HU,\max} \geq A_{j,p}^{HU} \quad j\in CP,\, p\in PR \tag{27a}$$

where

$$A_{j,p}^{HU} = \frac{q_{j,p}^{HU}}{\Delta T_{j,p}^{\mathrm{LMTD}-HU}}\left(\frac{1}{h^{HU}} + \frac{1}{h_{j,p}^{C}}\right) \tag{27b}$$

$$\Delta T_{j,p}^{\mathrm{LMTD}-HU} \approx \left\{\frac{1}{2}dt_{j,p}^{HU}(T^{HU,\mathrm{in}} - T_{j,p}^{\mathrm{Cout}})[dt_{j,p}^{HU} + (T^{HU,\mathrm{in}} - T_{j,p}^{\mathrm{Cout}})]\right\}^{1/3} \tag{27c}$$

(c) For the cold-utility matches

$$A_i^{CU,\max} \geq A_{i,p}^{CU} \quad i\in HP,\, p\in PR \tag{28a}$$

where

$$A_{i,p}^{CU} = \frac{q_{i,p}^{CU}}{\Delta T_{i,p}^{\mathrm{LMTD}-CU}}\left(\frac{1}{h^{CU}} + \frac{1}{h_{i,p}^{H}}\right) \tag{28b}$$

$$\Delta T_{i,p}^{\mathrm{LMTD}-CU} \approx \left\{\frac{1}{2}dt_{i,p}^{CU}(T_{i,p}^{H\mathrm{out}} - T^{CU,\mathrm{in}})[dt_{i,p}^{CU} + (T_{i,p}^{H\mathrm{out}} - T^{CU,\mathrm{in}})]\right\}^{1/3} \tag{28c}$$

12. Objective function:

$$\mathrm{TAC} = \sum_{p\in PR}\frac{DP_p}{NP}\sum_{i\in HP} C_{CU} q_{i,p}^{CU} + \sum_{p\in PR}\frac{DP_p}{NP}\sum_{j\in CP} C_{HU} q_{j,p}^{HU} + C_E\Big[\sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST}(A_{i,j,k}^{\max})^\beta + \sum_{j\in CP}(A_j^{HU,\max})^\beta + \sum_{i\in HP}(A_i^{CU,\max})^\beta\Big] \tag{29}$$

Table 10. Assigned Exchanger Areas in Case III

no.	unit	area (m^2)	no.	unit	area (m^2)
1	$A^b_{1,1,2}$	164.35	7	$A^{a*}_{1,CU,3}$	0.09
2	$A^b_{1,CU,3}$	9.99	8	$A^a_{2,CU,3}$	44.35
3	$A^b_{HU,1,0}$	22.43	9	$A^a_{3,CU,3}$	21.55
4	$A^a_{2,1,2}$	19.48	10	$A^a_{HU,1,0}$	2.33
5	$A^a_{2,2,2}$	52.34	11	$A^a_{HU,2,0}$	10.01
6	$A^a_{1,CU,3}$	6.80			

The improved area estimates can be produced by incorporating the F_T correction factors, i.e.

$$A_{i,j,k,p} = \frac{q_{i,j,k,p}}{\Delta T_{i,j,k,p}^{\mathrm{LMTD}} F_T(i,j,k,p)}\left(\frac{1}{h_{i,p}^{H}} + \frac{1}{h_{j,p}^{C}}\right) \quad i\in HP,\, j\in CP,\, k\in ST,\, p\in PR \tag{30}$$

$$A_{j,p}^{HU} = \frac{q_{j,p}^{HU}}{\Delta T_{j,p}^{\mathrm{LMTD}-HU} F_T^{HU}(j,p)}\left(\frac{1}{h^{HU}} + \frac{1}{h_{j,p}^{C}}\right) \quad j\in CP,\, p\in PR \tag{31}$$

$$A_{i,p}^{CU} = \frac{q_{i,p}^{CU}}{\Delta T_{i,p}^{\mathrm{LMTD}-CU} F_T^{CU}(i,p)}\left(\frac{1}{h_{i,p}^{H}} + \frac{1}{h^{CU}}\right) \quad i\in HP,\, p\in PR \tag{32}$$

In other words, these three equations are adopted to replace the corresponding constraints in original model, i.e., eqs 26b, 27b, and 28b, respectively. Notice also that definitions of the symbols used above and also those throughout this paper can all be found in the nomenclature section.

By following the computation procedure suggested by Smith,[12] the correction factor $F_T(i,j,k,p)$ of stream match (i,j,k) in period p can be determined according to the inlet and outlet temperatures of hot process stream (i.e., $t_{i,k,p}^{H}$ and $ths_{i,j,k,p}$) and those of cold process stream (i.e., $t_{j,k+1,p}^{C}$ and $tcs_{i,j,k,p}$). The same procedure is also adopted in this work to compute the correction factor for the coolers, i.e., $F_T^{CU}(i,p)$. The inlet and outlet temperatures of hot process stream i in this case are $t_{i,NOK+1,p}^{H}$ and $T_{i,p}^{H\mathrm{out}}$, while those of the cold utility stream should be $T^{CU,\mathrm{in}}$ and $T^{CU,\mathrm{out}}$. Finally, since the inlet and outlet temperatures of hot utility stream (i.e., steam) in a heater are usually the same, $F_T^{HU}(j,p)$ is always set to be *one* in this work. This practice could be justified by referring to published correlations of the correction factors for $N - 2N$ configurations.[13]

Table 11. Fnal Assignments in Case IV

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	107.11	$A^b_{1,1,1}$			107.11	$A^b_{1,1,1}$
(2,1,1)			138.46	$A^b_{1,1,1} + A^a_{2,1,1}$		
(3,1,1)			86.09	$A^b_{3,1,1}$	86.09	$A^b_{3,1,1}$
(4,1,1)	63.97	$A^b_{4,1,1}$	63.97	$A^b_{4,1,1}$	63.97	$A^b_{4,1,1}$
(HU,1,0)	37.61	$A^b_{HU,1,0}+A^a_{HU,1,0}$	37.61	$A^b_{HU,1,0}+A^a_{HU,1,0}$	19.10	$A^b_{HU,1,0}$

Table 12. Assigned Exchanger Areas in Case IV

no.	unit	area (m^2)	no.	unit	area (m^2)
1	$A^b_{1,1,1}$	107.11	4	$A^b_{HU,1,0}$	19.10
2	$A^b_{3,1,1}$	86.09	5	$A^a_{2,1,1}$	52.12
3	$A^b_{4,1,1}$	63.97	6	$A^a_{HU,1,0}$	24.15

Although the detailed exchanger designs have never been considered in the published works concerning multiperiod HEN synthesis, it should be clear that the total capital cost actually required in any practical application should be significantly higher than that determined with the conventional model. A more realistic optimal heat exchanger network can thus be configured on the basis of better estimates of the heat-transfer areas. The modified mixed integer nonlinear programming (MINLP) model can be solved with a commercial solver, e.g., GAMS, while the calculation procedure suggested by Smith[12] is coded in this work with MATLAB to determine the F_T correction factor of every match in a given HEN design. On the basis of these codes, a solution algorithm has been developed in this study to identify the optimal engineering design for a multiperiod HEN synthesis problem (see Figure 1).

Case I: Preliminary HEN Structure. To show the impacts of incorporating the improved area estimates in designs, let us consider the process data given in Table 1.[5] The capital investments of exchangers in this example are computed with a cost model adopted from the literature,[4,5,7] i.e.

$$C_{HX} = C_E A^{\beta} \tag{33}$$

where A is the heat-transfer area in squared meters and C_E is a cost coefficient (4333 USD/m^2·y). The unit cost of cooling water (C_{CU}) is assumed to be 53.1 USD/kW·y, while that of steam (C_{HU}) is 150.2 USD/kW·y. It is also assumed that ΔT_{min} = 10 K and β = 0.6. The corresponding conventional and modified models were both solved in the present example. The corresponding capital costs, utility costs, and total annual costs are compared in Table 2. Since the actual capital cost is higher than that obtained with the conventional approach, there is an incentive to try to reduce the TAC with a timesharing scheme. To facilitate further discussion in the next section, the area requirements of every match in all three periods are also identified from the solution obtained with the modified model and presented in Table 3.

3. TIMESHARING HEURISTICS

As mentioned previously, the HEN designs generated with the programming-based approach may suffer a serious drawback, i.e., the optimal heat transfer area for the same match varies considerably from period to period. Two intuitive approaches can be taken to cope with this problem.

- First of all, one can simply select a single exchanger with the *largest* area required for that match and try to adjust the heat duty via an elaborate control scheme. This is the implied assumption adopted in most of the published studies.[4−7] Although this approach is computationally feasible in design stage, the resulting process conditions may not be realizable in practical applications if the required heat-transfer area in certain period(s) is much smaller than that of the selected exchanger. It should also be noted that, even for the feasible cases, the temperature control system may be extremely sensitive to external disturbances.
- On the other hand, one can also adopt a base exchanger with the *smallest* area and add auxiliary ones when necessary. For example, let us consider match (2,*CU*,3) in Table 3. Since the smallest heat-transfer area in this case is 0.291 m^2 in period 1, the additional heat-transfer area needed for match (2,*CU*,3) in period 3 should be 0.354 m^2. In order to fulfill the heat-exchange requirement in every period, a simple-minded strategy would be to install two corresponding heat exchangers in parallel. The base exchanger should be used in both the first and third periods, while the auxiliary unit is needed only in period 3. Although this configuration is obviously operable, it is clear that the overall capital cost is higher than that of the single-exchanger option mentioned above.

In order to identify a multiperiod HEN design which is both operable and cost-effective with a hybrid approach, the following heuristic rules are used in this work to evolve from a preliminary design obtained by solving the modified mathematical programming model:

Rule 1. Timesharing opportunities should be identified independently in two distinct groups of matches. The first set of heat exchangers are shared by matches between the hot process streams and cold process/utility streams (type 1) in different time periods, while a separate second set of exchangers should be adopted to facilitate the remaining matches (type 2). This rule is adopted on the grounds that, in practice, the above two groups of matches are usually facilitated with different types of heat-exchange equipment.

Rule 2. If a single match is realized with more than one *smaller* heat exchanger, an appropriate overdesign margin must be introduced in sizing these units. According to Bennett et al.[14] and Edwards,[15] this margin should be at least 15%. On the other hand, if the heat duty of a match is carried out with a *larger* exchanger, then its heat-transfer area should not exceed 15% of that actually required.

Rule 3. The *base exchangers* must be identified first on the basis of a given preliminary HEN design. Each should be used in more than one period for a match between the same pair of hot and cold streams and also in the same stage. The base exchanger should be sized according to rule 2 so as to avoid using unnecessarily large area in any period. To this end, let us consider the difference between the required area for match (i,j,k) in period p (denoted as $A^p_{i,j,k}$) and the minimum area (denoted as $A^{min}_{i,j,k}$) among all periods, i.e.

$$\Delta^P_{i,j,k} = A^P_{i,j,k} - A^{min}_{i,j,k} \tag{34}$$

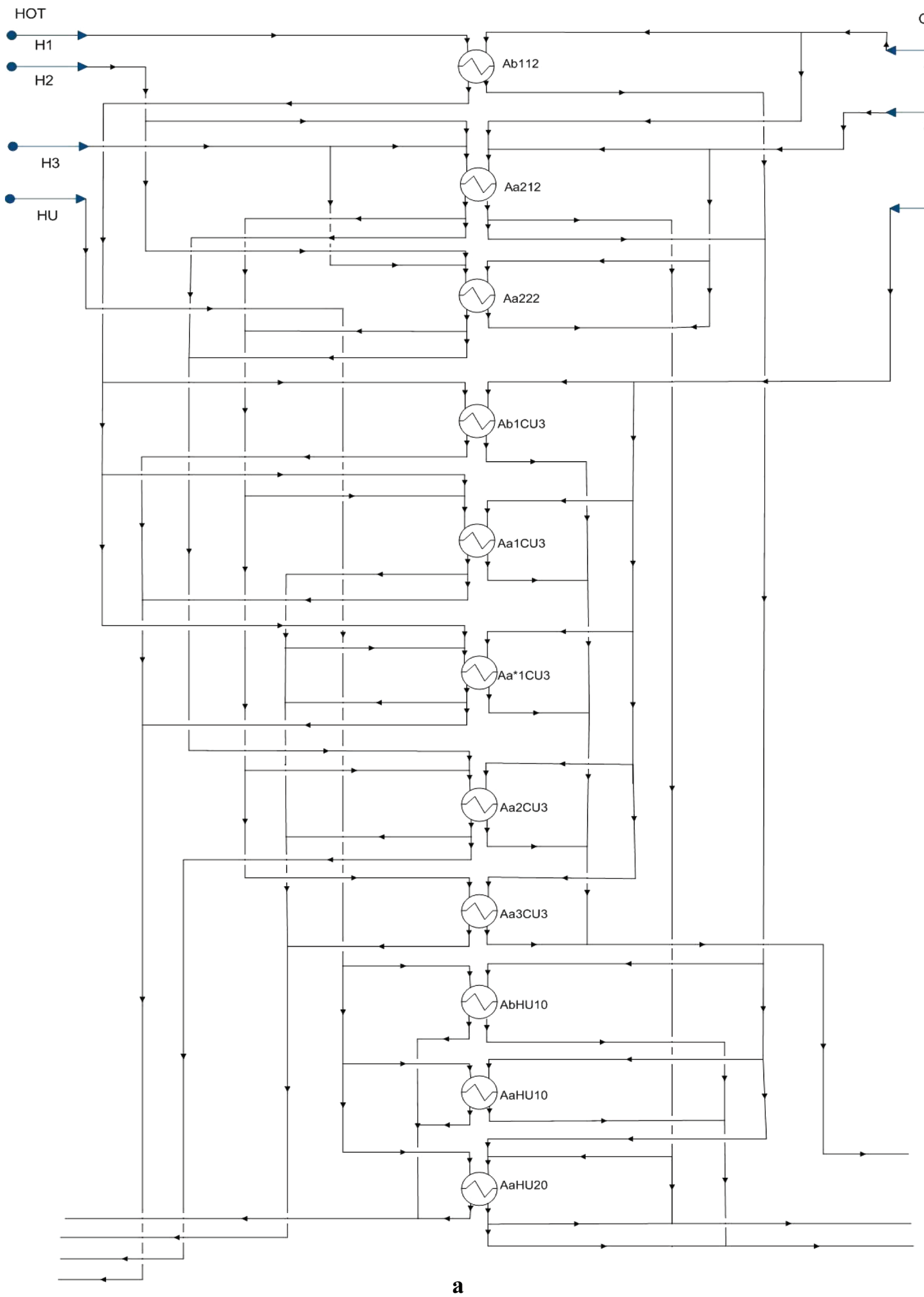


Figure 3. continued

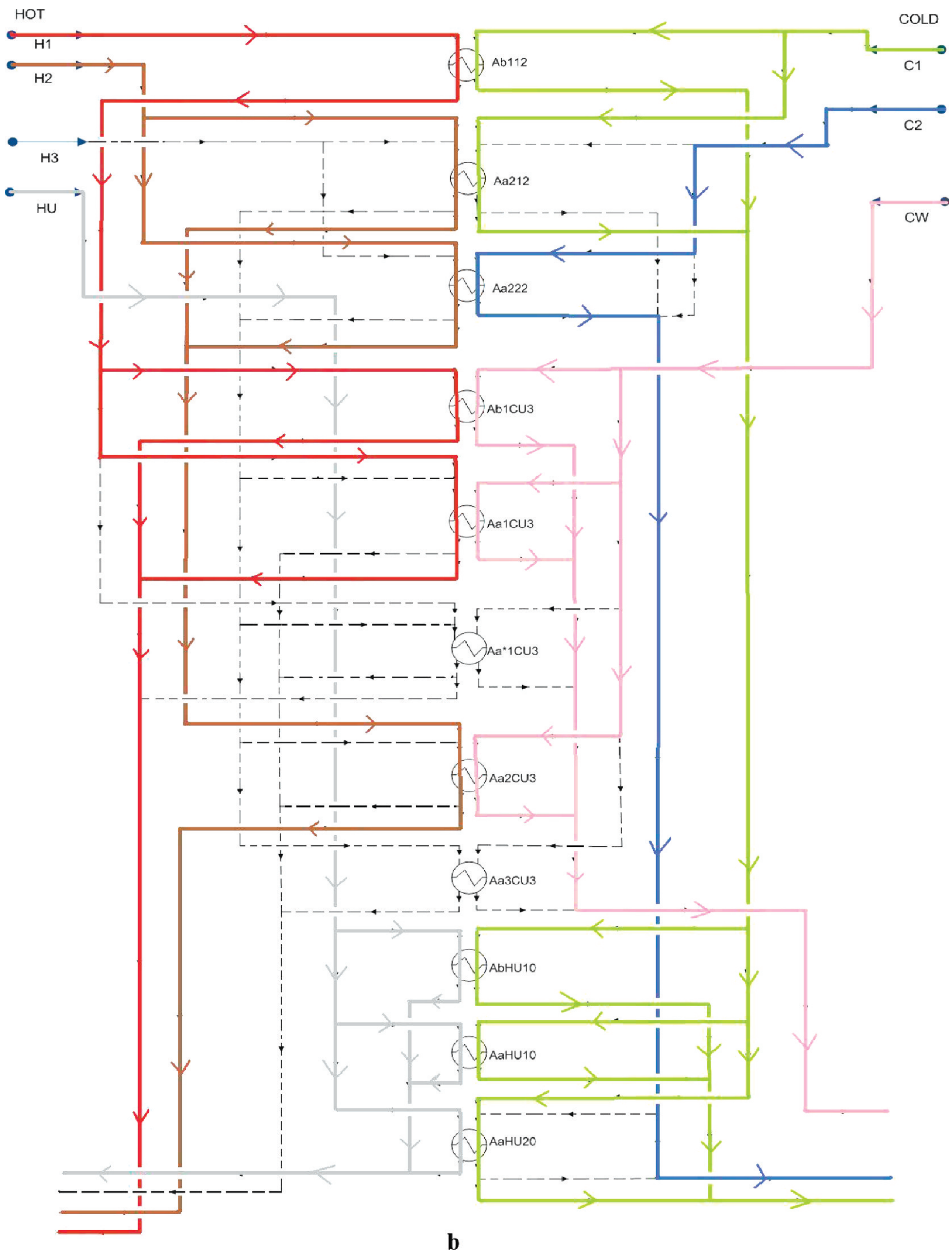
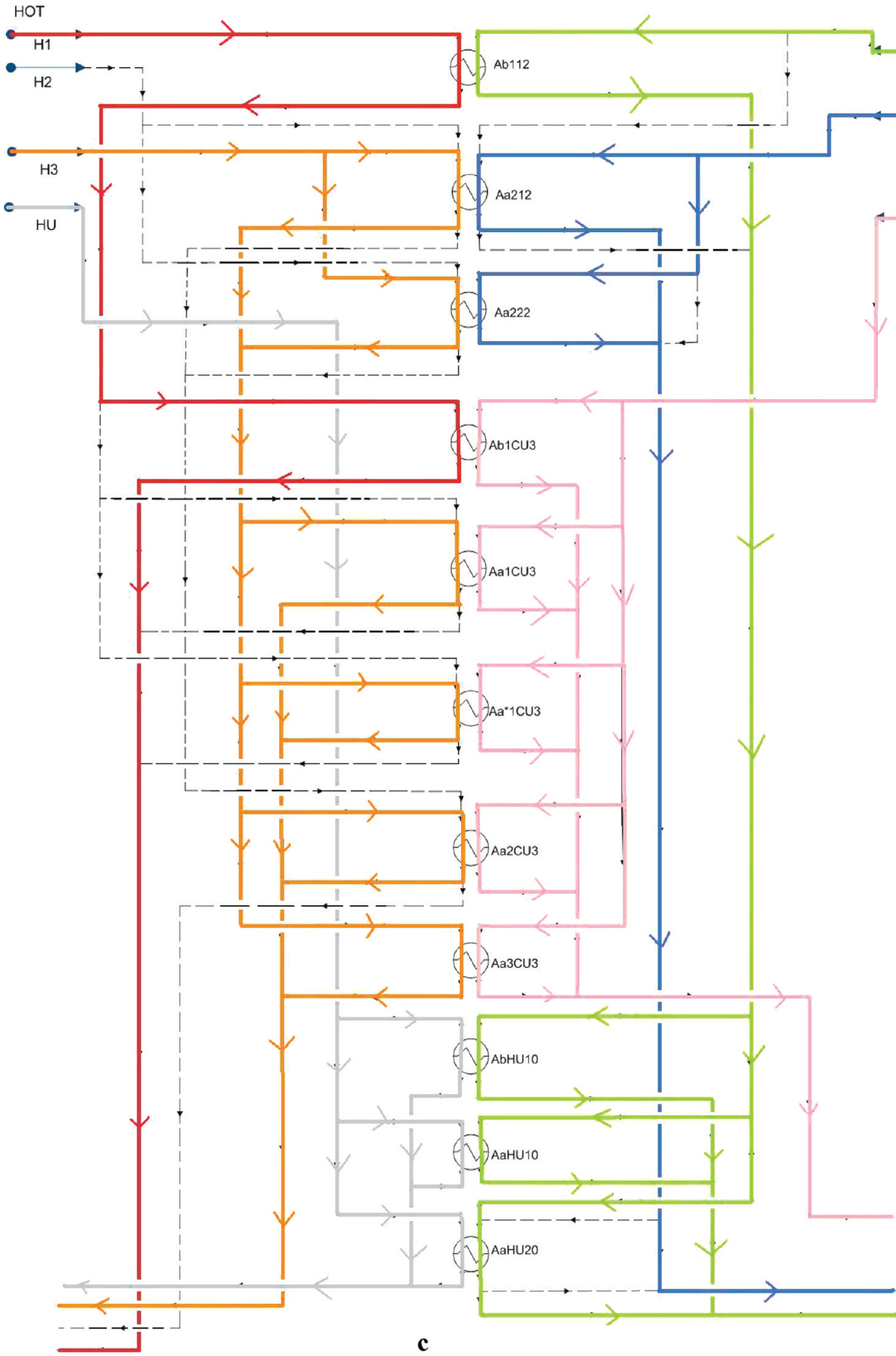


Figure 3. continued



c

Figure 3. continued

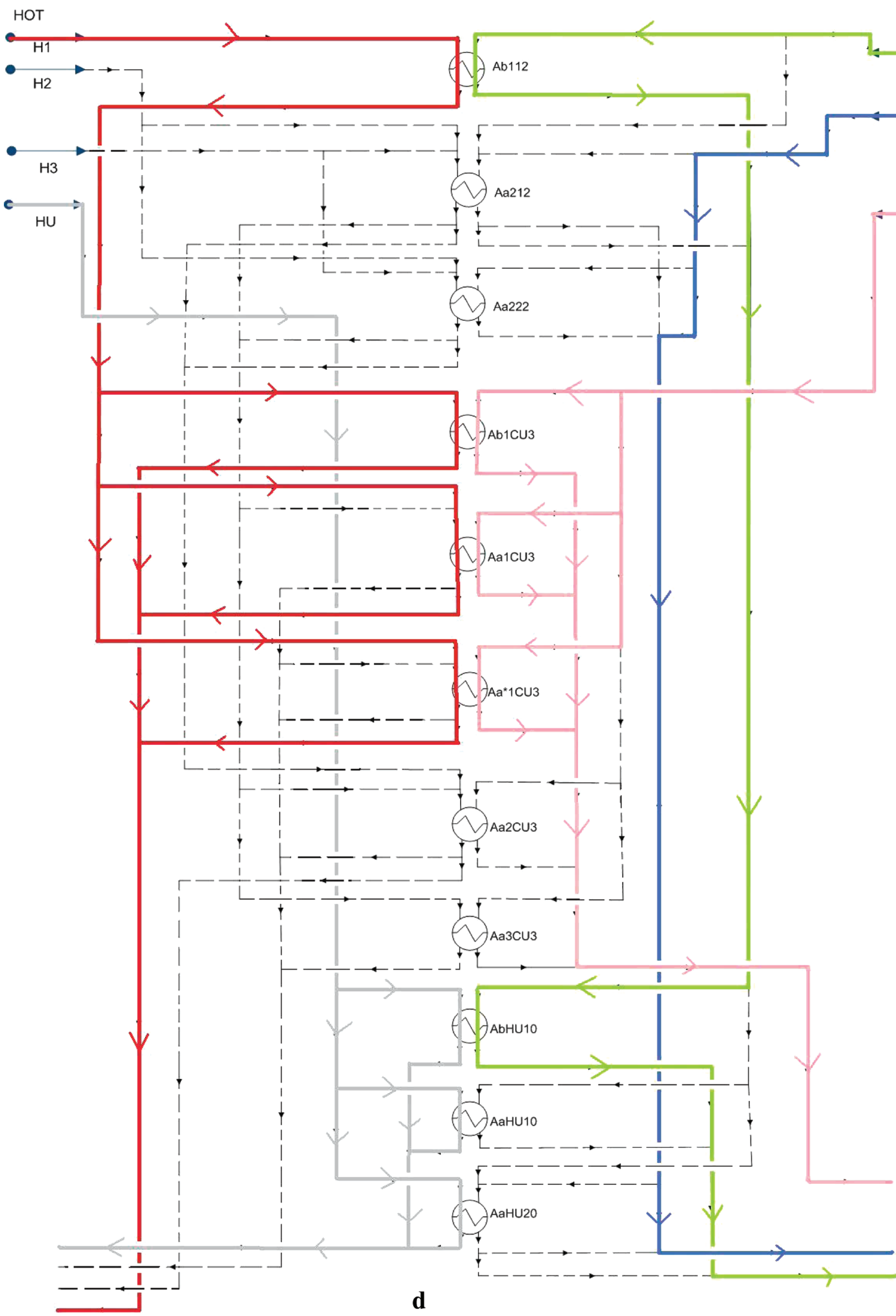


Figure 3. (a) Process flow diagram of overall HEN design for case III. (b) Process flow diagram of HEN design in period 1 for case III (H1: red; H2: brown; ST: gray; C1: light green; C2: blue; CW: pink). (c) Process flow diagram of HEN design in period 2 for case III (H1: red; H3: orange; ST: gray; C1: light green; C2: blue; CW: pink). (d) Process flow diagram of HEN design in period 3 for case III (H1: red; ST: gray; C1: light green; C2: blue; CW: pink).

In order to comply with rule 2, the maximum difference should not exceed 15% of the minimum heat transfer area, i.e.

$$\Delta_{i,j,k}^{P} \leq 0.15 A_{i,j,k}^{\min} \tag{35}$$

This practice can be succinctly summarized as

$$A_{i,j,k}^{b} = A_{i,j,k}^{\min} + \max_{p} \Delta_{i,j,k}^{p} \tag{36}$$

where $A_{i,j,k}^{b}$ denotes the heat transfer area of base exchanger for match (i,j,k). Notice that $\Delta_{i,j,k}^{p}$ here is subject to the constraint specified in eq 35. Notice also that the same principle could be applied to the utility users

Rule 4. After identifying the base exchangers according to rule 3, the remaining heat duties should be considered one at a time in ascending order and each satisfied with one or more auxiliary exchanger. For each assignment, the precedence order given below should be followed so as to yield the smallest possible exchanger number in HEN. If there is more than one option, the configuration requiring the minimum total heat-transfer area should be selected.

1. A single available base exchanger should be selected if the following criterion can be satisfied

$$A_{i,j,k}^{p} \leq A_{i',j',k'}^{b} \tag{37}$$

where, (i,j,k) and (i',j',k') represent different matches.

2. A combination of *available* base and auxiliary exchangers should be chosen to satisfy the following constraint:

$$1.15 A_{i,j,k}^{p} \leq \sum_{i',j',k'} A_{i',j',k'}^{b} + \sum_{i'',j'',k''} A_{i'',j'',k''}^{a} \tag{38}$$

where $A_{i'',j'',k''}^{a}$ is the heat-transfer area of auxiliary exchanger for match (i'',j'',k'') in period $p'' \neq p$. If more than one set can be found, then the one with the fewest exchangers should be chosen.

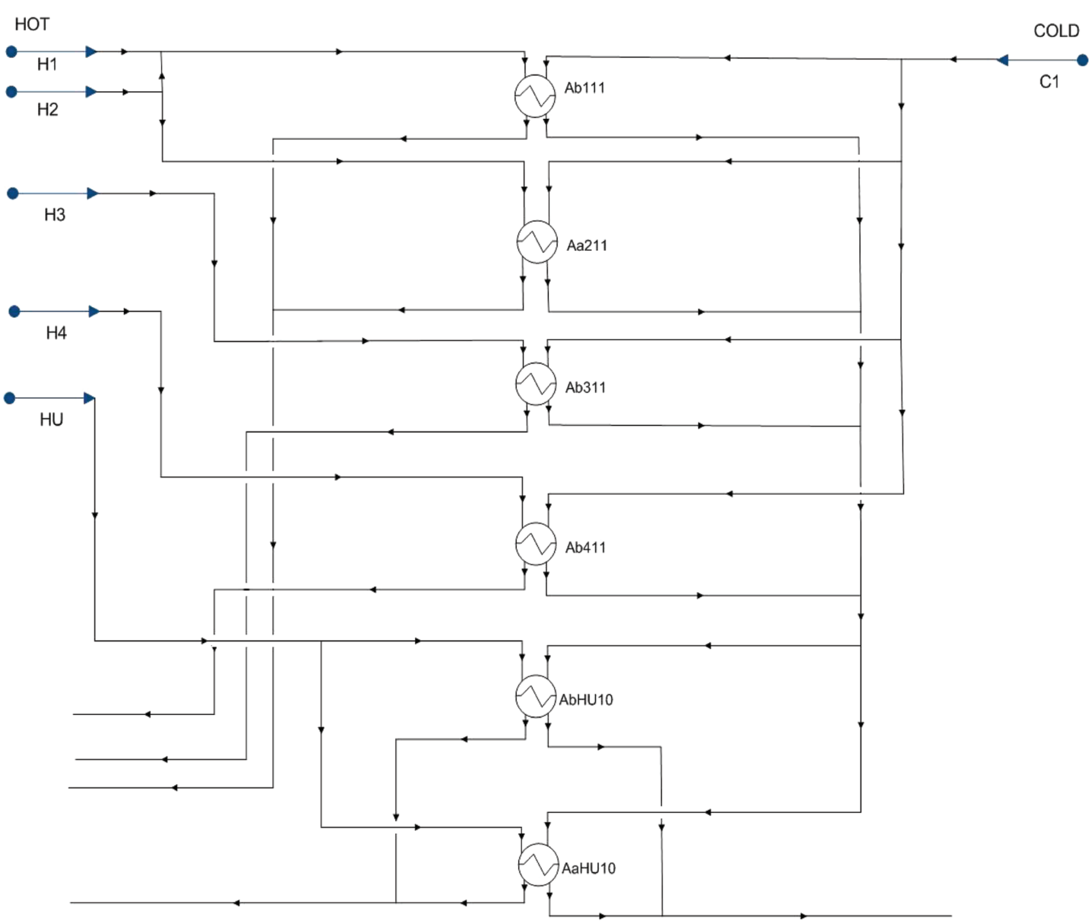
3. The heat-transfer area of a new auxiliary heat exchanger should be determined according to the formula given below:

$$A_{i,j,k}^{a} = 1.15 A_{i,j,k}^{p} - \sum_{i',j',k'} A_{i',j',k'}^{b} - \sum_{i'',j'',k''} A_{i'',j'',k''}^{a} \tag{39}$$

To illustrate the implementation steps of the aforementioned heuristic rules, let us consider the following two examples:

Case I: The Timesharing Scheme. The preliminary multiperiod HEN design of the first example is given in Table 3. By using the proposed heuristic rules, the final assignments of base and auxiliary exchangers and their areas can be identified (see Tables A.5 and A.6 in Appendix A). A comparison between the key performance indices of the conventional and the proposed timesharing schemes is given as follows:

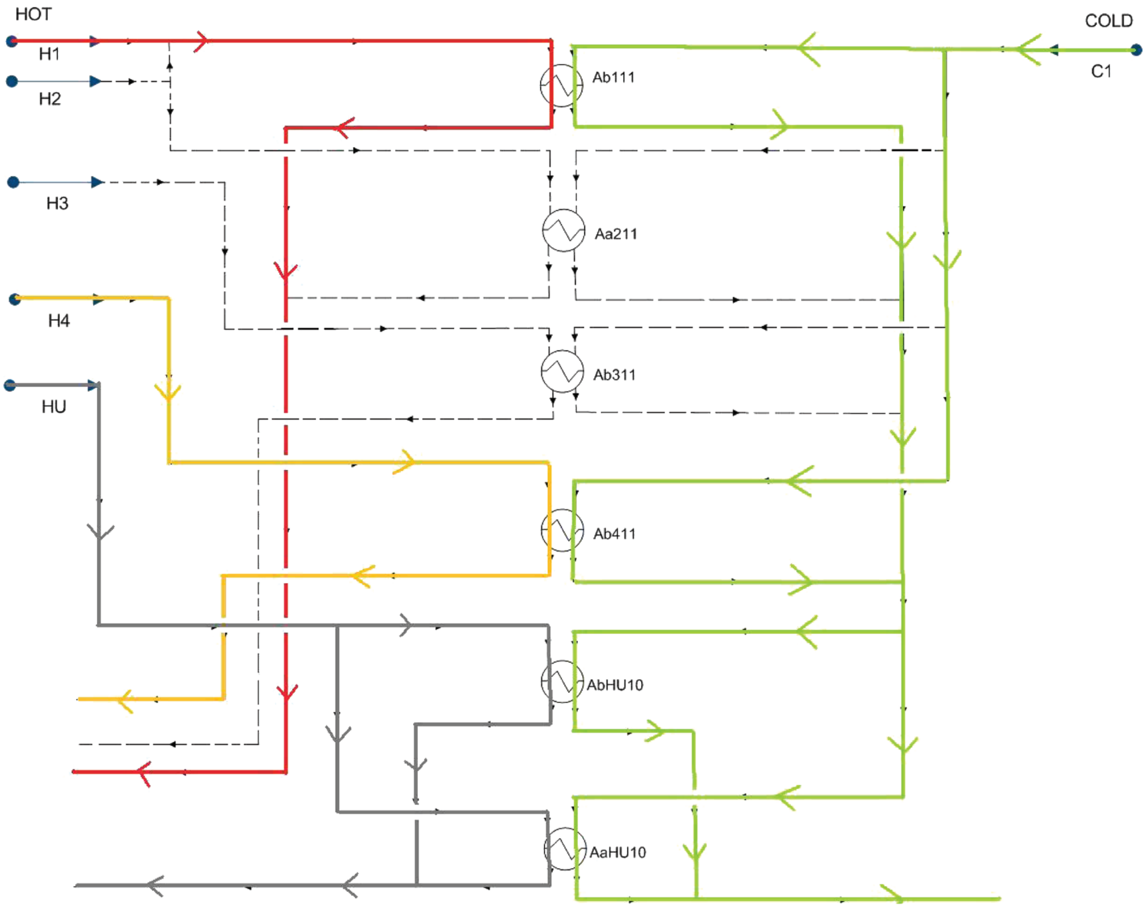
- Five heat exchangers are embedded in the conventional scheme. The total heat transfer area is 20 m^2, and the corresponding annualized capital cost is \$46,855 USD/y.



a

Figure 4. continued

dx.doi.org/10.1021/ie202171g | Ind. Eng. Chem. Res. 2012, 51, 7967−7987



b

Figure 4. continued

- Six heat exchangers are used in the timesharing scheme. A total heat transfer area of 20.4 m^2 is needed, and the required annual cost is \$48,940 USD/y.

Clearly the proposed approach cannot used to produce more economical design in this case. This is due to the fact that, since the process data for different periods are almost the same in this case, the corresponding area requirements of every stream match in the preliminary design are identical (see Table 3).

Case II: Timesharing Scheme. The preliminary multiperiod HEN design of the second example is presented in Table 4. By using the proposed heuristic rules, the final assignments of base and auxiliary exchangers and their areas can be identified (see Tables A.13 and A.14 in Appendix A). The comparison between the conventional and the proposed schemes can be found below:

- Seven heat exchangers are used in the conventional scheme. The total heat transfer area is 746.8 m^2, and the corresponding capital cost is \$459,918 USD/y.
- Eight heat exchangers must be utilized in the timesharing scheme, which requires a total heat transfer area of 624.8 m^2 and an annual cost of \$434,579 USD/y.

It can also be observed that, since the required areas of each match in the three periods under consideration differ significantly, the proposed heuristic approach becomes quite effective in the present case. Finally, notice that the proposed timesharing scheme can be realized with the network structure given in Figure 2.

4. SCREENING INDICATORS

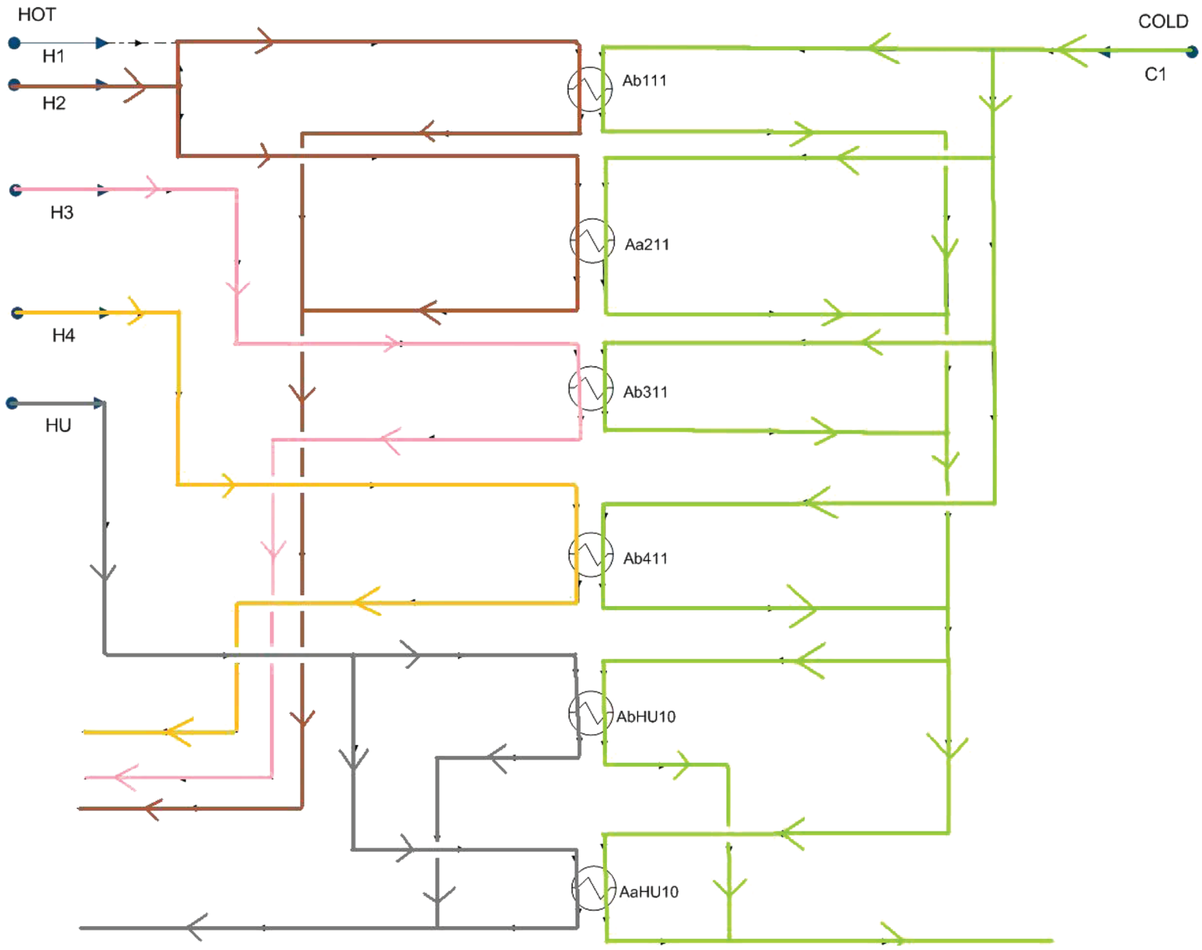
From the above discussion, it is clear that the proposed timesharing procedure may not always be effective. In most cases, the following four conditions among type-1 matches, i.e., matches between hot process streams and cold process/utility streams, and type-2 matches, i.e., matches between cold stream and hot utility, should be identified respectively:

1. Matches requiring auxiliary exchangers.
2. Periods in which the requirements for auxiliary exchangers are present.
3. Matches that are empty in at least one period.
4. Periods in which the number of empty matches is larger than zero.

They can be utilized as screening indicators for a potential successful application. For example, the screening indicators for cases I and II can be determined easily (see Table 5), and it can be observed that these indicators can indeed be used to predict the effectiveness of timesharing strategy.

5. CASE STUDIES

Two additional examples (cases III and IV) are presented here to demonstrate the effectiveness of the proposed strategies to generate the preliminary multiperiod HEN design and the corresponding timesharing scheme. Let us consider the process data given in Tables 6 and 7. The cost model in eq 33 is again used for determining the capital investments of heat exchangers. The optimization results obtained by solving the conventional



c

Figure 4. continued

model and the modified model (with embedded F_T correction factors) are compared in Table 8. The screening indicators of the latter design can be found in Table 5. Notice that these high indicator values imply that further cost saving may be achieved with timesharing strategies. The final assignments and the corresponding heat-transfer areas for case III are given in Tables 9 and 10, while the results for case IV are presented in Tables 11 and 12. The timesharing schemes obtained in cases III and IV are depicted in Figures 3 and 4, respectively. The important features of case III design and those of the corresponding conventional scheme are compared below:

- Nine heat exchangers are adopted in the conventional scheme. A total heat transfer area of 439.6 m^2, and an annual capital cost of \$370,082 USD/y are needed in this case.
- Eleven exchangers are installed in the timesharing scheme, while the corresponding total heat transfer area and total annual cost should be 353.7 m^2 and \$318,748 USD/y, respectively.
- The total heat transfer area is reduced by 19.54% and a saving of 13.87% in the total capital cost can be realized.

Also, the important features of case IV design and those of the corresponding conventional scheme are compared as follows:

- Five heat exchangers are adopted in the conventional scheme. A total heat transfer area of 433.24 m^2 and an annual capital cost of \$502,621 USD/y are needed in this case.
- Six exchangers are required in the timesharing scheme, while the corresponding total heat transfer area and total annual cost should be 352.54 m^2 and \$482,111 USD/y, respectively.
- The total heat transfer area is decreased by 18.62% and a saving of 4.08% in the total capital cost can be achieved with the proposed approach.

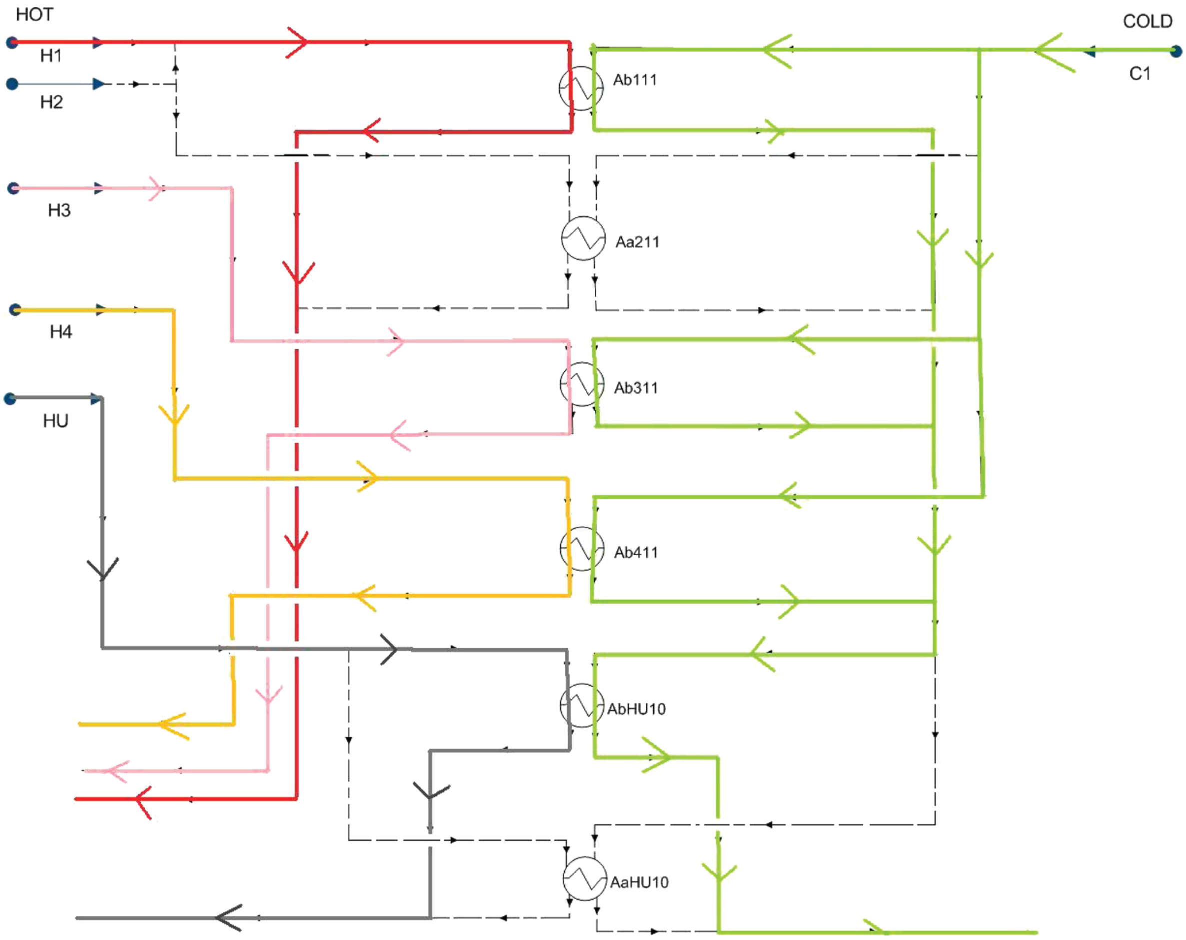
6. CONCLUSIONS

The contributions of this study can be summarized as follows:

- A modified mathematical model with embedded F_T correction factors has been successfully developed to synthesize the engineering design of any multiperiod heat exchange network
- A set of heuristic rules for creating a timesharing scheme for any given HEN design have also been conjectured and tested
- Extensive case studies have been carried out to confirm the validity of the proposed approach

■ APPENDIX A: APPLICATION EXAMPLES OF THE HEURISTIC RULES

The detailed implementation procedure of proposed heuristic rules is illustrated here by using the preliminary designs given in Tables 3 (case I) and 4 (case II), respectively.



d

Figure 4. (a) Process flow diagram of overall HEN design for case IV. (b) Process flow diagram of HEN design in period 1 for case IV (H1: red; H4: orange; ST: gray; C1: light green). (c) Process flow diagram of HEN design in period 2 for case IV (H2: brown; H3: pink; H4: orange; ST: gray; C1: light green). (d) Process flow diagram of HEN design in period 3 for case IV (H1: red; H3: pink; H4: orange; ST: gray; C1: light green).

A.1. Case I

(1) Determine the minimum heat transfer area ($A_{i,j,k}^{min}$) of each match (Table A.1).

Table A.1. Minimum Heat-Transfer Area of Every Match in Case I

match	(1,1,1)	(2,2,1)	(1,*CU*,3)	(*HU*,2,0)
$A_{i,j,k}^{min}$ (m^2)	3.985	4.268	8.507	0.291

(2) Determine the heat-transfer area of every base exchanger according to eqs 34−36 (Table A.2).

Table A.2. Heat-Transfer Areas of Base Exchangers in Case I

match	base area (m^2)
(1,1,1)	3.985
(2,2,1)	4.268
(1,*CU*,3)	8.507
(*HU*,2,0)	0.291

(3) Assign base exchangers to the corresponding matches in suitable periods according to eqs 35−37. These basic assignments are given in Table A.3. Notice that a symbol "+" denotes that additional auxiliary exchanger(s) is needed.

(4) Assign a combination of base and auxiliary exchangers to every remaining match according to eqs 37−39.

It can be observed from Table A.3 that only two matches need further assignments (indicated by plus sign). According to rule 4, the match with smaller area requirement, i.e., (2,*CU*,3) in period 3, should be considered first.

- There is only one available base heat exchanger, i.e., $A_{2,CU,3}^{b}$ = 0.291, and no auxiliary exchangers at this point. Thus, a new auxiliary exchanger should be assigned according to eq 39, i.e.,

$$A_{2,CU,3}^{a} = A_{2,CU,3}^{3} \times 1.15 - A_{2,CU,3}^{b}$$
$$= 0.645 \times 1.15 - 0.291$$
$$= 0.451$$

The resulting intermediate assignments can be found in Table A.4.

- The last assignment should be for match (*HU*,2,0) in period 2. Since no suitable exchangers can be identified from Table A.4, a new auxiliary exchanger should be used according to eq 38, i.e.,

$$A_{HU,2,0}^{a} = 1.15 \times A_{HU,2,0}{}^{2} - A_{HU,2,0}^{b}$$
$$= 1.15 \times 2.543$$
$$= 2.925$$

dx.doi.org/10.1021/ie202171g | Ind. Eng. Chem. Res. 2012, 51, 7967−7987

Table A.3. Basic Assignments in Case I

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	3.985	$A^b_{1,1,1}$	3.985	$A^b_{1,1,1}$	3.985	$A^b_{1,1,1}$
(2,2,1)	4.268	$A^b_{2,2,1}$	4.268	$A^b_{2,2,1}$	4.268	$A^b_{2,2,1}$
(1,*CU*,3)	8.507	$A^b_{1,CU,3}$	8.507	$A^b_{1,CU,3}$	8.507	$A^b_{1,CU,3}$
(2,CU,3)	0.291	$A^b_{2,CU,3}$	0.0		0.645	$A^b_{2,CU,3}$ +
(*HU*,2,0)	0.0		2.543	+	0.0-	

Table A.4. Intermediate Assignments in Case I

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	3.985	$A^b_{1,1,1}$	3.985	$A^b_{1,1,1}$	3.985	$A^b_{1,1,1}$
(2,2,1)	4.268	$A^b_{2,2,1}$	4.268	$A^b_{2,2,1}$	4.268	$A^b_{2,2,1}$
(1,*CU*,3)	8.507	$A^b_{1,CU,3}$	8.507	$A^b_{1,CU,3}$	8.507	$A^b_{1,CU,3}$
(2,CU,3)	0.291	$A^b_{2,CU,3}$	0.0		0.645	$A^b_{2,CU,3} + A^a_{2,CU,3}$
(*HU*,2,0)	0.0		2.543	+	0.0-	

Table A.5. Final Assignments in Case I

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	3.985	$A^b_{1,1,1}$	3.985	$A^b_{1,1,1}$	3.985	$A^b_{1,1,1}$
(2,2,1)	4.268	$A^b_{2,2,1}$	4.268	$A^b_{2,2,1}$	4.268	$A^b_{2,2,1}$
(1,*CU*,3)	8.507	$A^b_{1,CU,3}$	8.507	$A^b_{1,CU,3}$	8.507	$A^b_{1,CU,3}$
(2,*CU*,3)	0.291	$A^b_{2,CU,3}$	0.0		0.645	$A^b_{2,CU,3} + A^a_{2,CU,3}$
(*HU*,2,0)	0.0		2.543	$A^a_{HU,2,0}$	0.0-	

Table A.6. Assigned Exchanger Areas in Case I

no.	unit	area (m^2)	no.	unit	area (m^2)
1	$A^b_{1,1,1}$	3.985	4	$A^b_{2,CU,3}$	0.291
2	$A^b_{2,2,1}$	4.268	5	$A^a_{2,CU,3}$	0.451
3	$A^b_{1,CU,3}$	8.507	6	$A^a_{HU,2,0}$	2.925

Table A.7. Minimum Heat-Transfer Area of Every Match in Case II

match	(1,1,1)	(1,2,2)	(2,1,2)	(2,2,2)	(2,CU,3)
$A^{min}_{i,j,k}$ (m^2)	14.0	52.3	27.9	214.1	100.9

The final assignments are specified in Table A.5, and the corresponding heat-transfer areas can be found in Table A.6.

A.2. Case II

(1) Determine the minimum heat transfer area ($A^{min}_{i,j,k}$) of each match (Table A.7).

(2) Determine the heat-transfer area of every base exchanger according to eqs 34−36 (Table A.8).

(3) Assign base exchangers to the corresponding matches in suitable periods on the basis of eqs 35−37. These basic assignments are given in Table A.9. Notice that a symbol "+" denotes that additional auxiliary exchanger(s) is needed.

(4) Assign a combination of base and auxiliary exchangers to every remaining match according to eqs 37−39.

It can be observed from Table A.9 that four matches need further assignments. They should be considered in such a way that the one with the smallest remained area requirement is treated first.

- On the basis of the aforementioned principle, match (1,2,2) in period 1 should be considered first and there are no available base and auxiliary exchangers at this point.

Table A.8. Heat-Transfer Areas of Base Exchangers in Case II

match	base area (m^2)
(1,1,1)	15.7
(1,2,2)	54.6
(2,1,2)	30.8
(2,2,2)	231.4
(2,CU,3)	108.3

Thus, a new auxiliary exchanger should be assigned according to eq 39, i.e.

$$A^a_{1,2,2} = A^1_{1,2,2} \times 1.15 - A^b_{1,2,2}$$
$$= 87.6 \times 1.15 - 54.6$$
$$= 46.14$$

The resulting assignments can be found in Table A.10.

- Match (1,*CU*,5) in period 3 should be considered next. From Table A.10, it is clear that the available heat exchangers is the auxiliary exchanger for match (1,2,2) in period 1 (with area $A^a_{1,2,2}$). Notice that eq 38 must be satisfied. Due to insufficient heat transfer area, a new auxiliary heat exchanger needs to be installed according to eq 39, i.e.

$$A^a_{1,CU,5} = A^3_{1,CU,5} \times 1.15 - A^a_{1,2,2}$$
$$= 56.6 \times 1.15 - 46.14$$
$$= 18.95$$

The resulting intermediate assignments can be found in Table A.11.

- The third auxiliary assignment should be concerned with match (2,2,3) in period 3 since the required extra area is smaller than that of match (2,3,4) in period 2. From Table A.11, it is clear that there are no available exchangers. Therefore a new auxiliary exchanger should

Table A.9. Basic Assignments in Case II

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	14.8	$A^b_{1,1,1}$	15.7	$A^b_{1,1,1}$	14.0	$A^b_{1,1,1}$
(1,2,2)	87.6	$A^b_{1,2,2}$ +	54.6	$A^b_{1,2,2}$	52.3	$A^b_{1,2,2}$
(2,1,2)	28.2	$A^b_{2,1,2}$	27.9	$A^b_{2,1,2}$	30.8	$A^b_{2,1,2}$
(2,2,3)	214.1	$A^b_{2,2,2}$	231.4	$A^b_{2,2,2}$	304.6	$A^b_{2,2,2}$ +
(2,3,4)	0.0		143.2	+	0.0	
(1,CU,5)	0.0		0.0		56.6	+
(2,CU,5)	100.9	A^b_{CU2}	108.3	A^b_{CU2}	105.2	A^b_{CU2}

Table A.10. Intermediate Assignments in Case II-1

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	14.8	$A^b_{1,1,1}$	15.7	$A^b_{1,1,1}$	14.0	$A^b_{1,1,1}$
(1,2,2)	87.6	$A^b_{1,2,2} + \boldsymbol{A^a_{1,2,2}}$	54.6	$A^b_{1,2,2}$	52.3	$A^b_{1,2,2}$
(2,1,2)	28.2	$A^b_{2,1,2}$	27.9	$A^b_{2,1,2}$	30.8	$A^b_{2,1,2}$
(2,2,3)	214.1	$A^b_{2,2,3}$	231.4	$A^b_{2,2,3}$	304.6	$A^b_{2,2,3}$ +
(2,3,4)	0.0		143.2	+	0.0	
(1,*CU*,5)	0.0		0.0		56.6	+
(2,*CU*,5)	100.9	A^b_{CU2}	108.3	A^b_{CU2}	105.2	A^b_{CU2}

Table A.11. Intermediate Assignments in Case II-2

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	14.8	$A^b_{1,1,1}$	15.7	$A^b_{1,1,1}$	14.0	$A^b_{1,1,1}$
(1,2,2)	87.6	$A^b_{1,2,2} + \boldsymbol{A^a_{1,2,2}}$	54.6	$A^b_{1,2,2}$	52.3	$A^b_{1,2,2}$
(2,1,2)	28.2	$A^b_{2,1,2}$	27.9	$A^b_{2,1,2}$	30.8	$A^b_{2,1,2}$
(2,2,3)	214.1	$A^b_{2,2,3}$	231.4	$A^b_{2,2,3}$	304.6	$A^b_{2,2,3}$ +
(2,3,4)	0.0		143.2	+	0.0	
(1,CU,5)	0.0		0.0		56.6	$\boldsymbol{A^a_{1,2,2} + A^a_{1,CU,5}}$
(2,CU,5)	100.9	A^b_{CU2}	108.3	A^b_{CU2}	105.2	A^b_{CU2}

Table A.12. Intermediate Assignments in Case II-3

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	14.8	$A^b_{1,1,1}$	15.7	$A^b_{1,1,1}$	14.0	$A^b_{1,1,1}$
(1,2,2)	87.6	$A^b_{1,2,2} + \boldsymbol{A^a_{1,2,2}}$	54.6	$A^b_{1,2,2}$	52.3	$A^b_{1,2,2}$
(2,1,2)	28.2	$A^b_{2,1,2}$	27.9	$A^b_{2,1,2}$	30.8	$A^b_{2,1,2}$
(2,2,3)	214.1	$A^b_{2,2,3}$	231.4	$A^b_{2,2,3}$	304.6	$A^b_{2,2,3} + \boldsymbol{A^a_{2,2,3}}$
(2,3,4)	0.0		143.2	+	0.0	
(1,*CU*,5)	0.0		0.0		56.6	$\boldsymbol{A^a_{1,2,2} + A^a_{1,CU,5}}$
(2,*CU*,5)	100.9	A^b_{CU2}	108.3	A^b_{CU2}	105.2	A^b_{CU2}

Table A.13. Final Assignments in Case II

match	required area in period 1 (m^2)	area assignment in period 1	required area in period 2 (m^2)	area assignment in period 2	required area in period 3 (m^2)	area assignment in period 3
(1,1,1)	14.8	$A^b_{1,1,1}$	15.7	$A^b_{1,1,1}$	14.0	$A^b_{1,1,1}$
(1,2,2)	87.6	$A^b_{1,2,2} + \boldsymbol{A^a_{1,2,2}}$	54.6	$A^b_{1,2,2}$	52.3	$A^b_{1,2,2}$
(2,1,2)	28.2	$A^b_{2,1,2}$	27.9	$A^b_{2,1,2}$	30.8	$A^b_{2,1,2}$
(2,2,3)	214.1	$A^b_{2,2,3}$	231.4	$A^b_{2,2,3}$	304.6	$A^b_{2,2,3} + \boldsymbol{A^a_{2,2,3}}$
(2,3,4)	0.0		143.2	$\boldsymbol{A^a_{1,2,2} + A^a_{2,2,3} + A^a_{1,CU,5}}$	0.0	
(1,*CU*,5)	0.0		0.0		56.6	$\boldsymbol{A^a_{1,2,2} + A^a_{1,CU,5}}$
(2,*CU*,5)	100.9	A^b_{CU2}	108.3	A^b_{CU2}	105.2	A^b_{CU2}

be used and its area ($A^a_{2,2,3}$) can be determined according to eq 39, i.e.

$$A^a_{2,2,3} = A^3_{2,2,3} \times 1.15 - A^b_{2,2,3} = 304.6 \times 1.15 - 231.4 = 118.9$$

The resulting assignments are specified in Table A.12.

- The last auxiliary assignment should be for match (2,3,4) in period 2. Three available exchangers can be identified from Table A.12, i.e., the auxiliary exchanger of match (1,2,2) in period 1 (with area $A^a_{1,2,2}$), the auxiliary exchanger of match (2,2,3) in period 3 (with area $A^a_{2,2,3}$), and the auxiliary exchanger of match (1,*CU*,3) in period 3.

In order to comply with eq 38, those three exchangers can be used. The final assignments are given in Table A.13 and the corresponding heat-transfer areas are presented in Table A.14.

Table A.14. Assigned Exchanger Areas in Case II

no.	unit	area (m^2)	no.	unit	area (m^2)
1	$A^b_{1,1,1}$	15.7	5	A^b_{CU2}	108.3
2	$A^b_{1,2,2}$	54.6	6	$A^a_{1,2,2}$	46.14
3	$A^b_{2,1,2}$	30.8	7	$A^a_{1,CU,5}$	18.95
4	$A^b_{2,2,3}$	231.4	8	$A^a_{2,2,3}$	118.9

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: ctchang@mail.ncku.edu.tw.

Notes

The authors declare no competing financial interest.

■ NOMENCLATURE

A = heat transfer area (m^2)
A_{min} = heat transfer area if $F_T = 1$ (m^2)
$A^b_{i,j,k}$ = base heat exchanger heat transfer area of a match between hot stream i and cold stream j at stage k (m^2)
$A^a_{i,j,k}$ = auxiliary heat exchanger heat transfer area of a match between hot stream i and cold stream j at stage k (m^2)
$A^{min}_{i,j,k}$ = minimum heat transfer area of a match between hot stream i and cold stream j at stage k (m^2)
$A^{max}_{i,j,k}$ = maximum heat transfer area of a match between hot stream i and cold stream j at stage k (m^2)
$A^{HU,max}_j$ = maximum heat transfer area of a match between cold stream j and hot utility (m^2)
$A^{CU,max}_i$ = maximum heat transfer area of a match between hot stream i and cold utility (m^2)
$A_{i,j,k,p}$ = heat transfer area of match between hot stream i and cold stream j at stage k in period p (m^2)
$A^{HU}_{j,p}$ = heat transfer area of match between cold stream j and hot utility (m^2)
$A^{CU}_{i,p}$ = heat transfer area of a match between hot stream i and cold utility (m^2)
$A^p_{i,j,k}$ = heat transfer area of match between hot stream i and cold stream j at stage k in period p (m^2)
C_{HX} = annualized capital cost of heat exchanger (USD/y)
C_E = area cost coefficient for heat exchanger (USD/m^2·y)
C_{CU} = unit cost of cold utility (USD/kW·y)
C_{HU} = unit cost of hot utility (USD/kW·y)
DP_p = duration of period (dimensionless)
$dt_{i,j,k,p}$ = temperature difference for match of hot stream i and cold stream j at stage k in period p (K)
$dt^{HU}_{j,p}$ = temperature difference for match of cold stream j and hot utility in period p (K)
$dt^{CU}_{i,p}$ = temperature difference for match of hot stream i and cold utility in period p (K)
F_{CP} = heat capacity flow rate of hot or cold stream (kW/K)
$F^H_{i,p}$ = heat capacity flow rate of hot stream in period p before split (kW/K)
$F^C_{j,p}$ = heat capacity flow rate of cold stream in period p before split (kW/K)
F_T = correction factor of shell-and-tube heat exchanger (dimensionless)
$F_T(i,j,k,p)$ = correction factor of shell-and-tube heat exchanger for match of hot stream i and cold stream j at stage k in period p (dimensionless)
$F^{HU}_T(j,p)$ = correction factor of shell-and-tube heat exchanger for match of cold stream j and hot utility in period p (dimensionless)
$F^{CU}_T(i,p)$ = correction factor of shell-and-tube heat exchanger for match of hot stream i and cold utility in period p (dimensionless)
h = individual film heat-transfer coefficient of cold or hot stream (kW/m^2·K)
$h^H_{i,p}$ = individual film heat-transfer coefficient of hot stream i in period p (kW/m^2·K)
$h^C_{j,p}$ = individual film heat-transfer coefficient of cold stream j in period p (kW/m^2·K)
h^{HU} = individual film heat-transfer coefficient of hot utility (kW/m^2·K)
h^{CU} = individual film heat-transfer coefficient of cold utility (kW/m^2·K)
HEN = heat exchanger network
MINLP = mixed integer nonlinear programming
N = number of shell (dimensionless)
$N_{S,min}$ = minimum value of the numbers of 1−2 shells in shell-and-tube heat exchangers (dimensionless)
NOK = number of stages (dimensionless)
NP = number of periods (dimensionless)
P = thermal effectiveness of 1−2 shell-and-tube heat exchanger (dimensionless)
Q_{UP} = upper bound on heat exchange capacity (kW)
$q_{i,j,k,p}$ = heat exchanged between hot stream i and cold stream j in stage k in period p (kW)
$q^{CU}_{i,p}$ = heat exchanged between hot stream i and cold utility in period p (kW)
$q^{HU}_{j,p}$ = heat exchanged between cold stream j and hot utility in period p (kW)
R = heat capacity ratio of $N - 2N$ shell-and-tube heat exchanger (dimensionless)
$R^{HU}_{j,p}$ = heat capacity ratio of $N - 2N$ shell-and-tube heat exchanger for match of cold stream j and hot utility in period p (dimensionless)
r_H = hot stream split ratio (dimensionless)
r_C = cold stream split ratio (dimensionless)
$rh_{i,j,k,p}$ = split ratio of hot stream in a match between hot stream i with cold stream j at stage k in period p (dimensionless)
$rc_{i,j,k,p}$ = split ratio of cold stream in a match between hot stream i with cold stream j at stage k in period p (dimensionless)
T_{in} = inlet temperature of cold or hot stream (K)
T_{out} = outlet temperature of cold or hot stream (K)
$T^{Hin}_{i,p}$ = inlet temperature of hot stream i in period p (K)
$T^{Hout}_{i,p}$ = outlet temperature of hot stream i in period p (K)
$t^H_{i,k,p}$ = inlet temperature of hot stream in a match between hot stream i and cold stream j at stage k in period p (K)
$t^H_{i,k+1,p}$ = outlet temperature of hot stream in a match between hot stream i and cold stream j at stage k in period p (K)
$t^H_{i,NOK+1,p}$ = temperature of hot stream at stage $NOK + 1$ in period p (K)
$ths_{i,j,k,p}$ = outlet temperature of hot stream in a match between hot stream i and cold stream j at stage k in period p if stream splitter is present (K)
$T^{Cin}_{j,p}$ = inlet temperature of cold stream j in period p (K)
$T^{Cout}_{j,p}$ = outlet temperature of cold stream j in period p (K)

$t^C_{j,k,p}$ = inlet temperature of cold stream in a match between hot stream i and cold stream j at stage k in period p (K)
$t^C_{j,k+1,p}$ = outlet temperature of cold stream in a match between hot stream i and cold stream j at stage k in period p (K)
$t^C_{j,NOK+1,p}$ = temperature of cold stream in at stage NOK + 1 in period p (K)
$tcs_{i,j,k,p}$ = outlet temperature of cold stream in a match between hot stream i and cold stream j at stage k in period p if stream splitter is present (K)
$T^{CU,\mathrm{in}}$ = inlet temperature of cold utility (K)
$T^{CU,\mathrm{out}}$ = outlet temperature of cold utility (K)
$t_{h,\mathrm{in}}$ = inlet temperature of hot stream (K)
$t_{h,\mathrm{out}}$ = outlet temperature of hot stream (K)
$t_{c,\mathrm{in}}$ = inlet temperature of cold stream (K)
$t_{c,\mathrm{out}}$ = outlet temperature of cold stream (K)
$t_{h,1}$ = inlet temperature of hot stream (K)
$t_{h,2}$ = outlet temperature of hot stream (K)
$t_{c,1}$ = inlet temperature of cold stream (K)
$t_{c,2}$ = outlet temperature of cold stream (K)
TAC = total annual cost (USD/y)
$z_{i,j,k}$ = existence of match between hot stream i and cold stream j in stage k(dimensionless)
z_i^{CU} = existence of match between hot stream i and cold utility (dimensionless)
z_j^{HU} = existence of match between cold stream j and hot utility (dimensionless)
Γ = upper bound for temperature difference
$\Delta T^{\mathrm{LMTD}}_{i,j,k,p}$ = log mean temperature difference for match of hot stream i and cold stream j at stage k in period p (K)
$\Delta T^{\mathrm{LMTD}-HU}_{j,p}$ = log mean temperature difference for match between cold stream j and hot utility in period p (K)
$\Delta T^{\mathrm{LMTD}-CU}_{i,p}$ = log mean temperature difference for match between hot stream i and cold utility in period p (K)
β = heat exchanger area exponent factor (dimensionless)
$\Delta^P_{i,j,k}$ = difference between the $A^P_{i,j,k}$ and $A^{app}_{i,j,k}$ (m^2)

Indices

i = hot process stream or hot utility
j = cold process stream or cold utility
k = stage number or temperature interval
p = period of operation
P = period of operation for $\Delta^P_{i,j,k}$

Sets

CP = set of a cold process stream j
HP = set of a hot process stream i
PR = set of a operation period, p = 1, ..., NP
ST = set of a stage in the superstructure, k = 1, ..., NOK

■ REFERENCES

(1) Biegler, L. T.; Westerberg, A. L.; Grossmann, I. E. *Systematic Methods of Chemical Process Design*; Prentice Hall: Upper Saddle River, NJ, 1997.
(2) Yee, T. F.; Grossmann, I. E. Simultaneous optimization models for heat integration – II. Heat exchanger network synthesis. *Comput. Chem. Eng.* **1990**, *14* (10), 1165–1184.
(3) Yee, T. F.; Grossmann, I. E.; Kravanja, Z. Simultaneous optimization models for heat integration - I. Area and energy targeting and modeling of multi-stream exchangers. *Comput. Chem. Eng.* **1990**, *14* (10), 1151–1164.
(4) Aaltola, J. Simultaneous synthesis of flexible heat exchanger network. *Appl. Therm. Eng.* **2002**, *22*, 907–918.
(5) Chen, C. L.; Hung, P. S. A Novel Strategy for Synthesis of Flexible Heat-Exchange Networks. *J. Chin. Inst. Chem. Eng.* **2005**, *36* (5), 421–432.
(6) Ma, X.; Yao, P. J.; Luo, X.; Roetzel, W. Synthesis of multi-stream heat exchanger network for multi-period operation with genetic/simulated annealing algorithms. *Appl. Therm. Eng.* **2008**, *28*, 809–823.
(7) Fraser, D. M.; Isafiade, A. J. Interval based MINLP superstructure synthesis of heat exchanger networks for multi-period operations. *Chem. Eng. Res. Des.* **2010**, *88*, 1329–1341.
(8) Verheyen, W.; Zhang, N. Design of flexible heat exchanger network for multi-period operation. *Chem. Eng. Sci.* **2006**, *61*, 7730–7753.
(9) Galli, M. R.; Cerda, J. Synthesis of heat exchanger networks featuring a minimum number of constrained-size shells of 1–2 type. *Appl. Therm. Eng.* **2000**, *20*, 1443–1467.
(10) Ravagnani, M. A. S. S.; Caballero, J. A. Optimal heat exchanger network synthesis with the detailed heat transfer equipment design. *Comput. Chem. Eng.* **2007**, *31*, 1432–1448.
(11) Ponce-Ortega, J. M.; Gonzalez, M. S.; Jimenez-Gutierrez, A. Synthesis of multipass heat exchanger networks using genetic algorithms. *Comput. Chem. Eng.* **2008**, *32*, 2320–2332.
(12) Smith, R. *Chemical Process: Design and Integration*; John Wiley & Sons, Ltd.: New York, 2005.
(13) Perry, R. H.; Green, D. W. *Perry's Chemical Engineers' Handbook*, 7th ed.; McGraw-Hill: New York, 1999.
(14) Bennett, C. A.; Kistler, R. S.; Lestina, T. G.; King, D. C. Improving Heat Exchanger Designs. *Chem. Eng. Prog.* **2007**, *2007*, 40–45.
(15) Edwards, J. E. *Design and rating shell and tube heat exchangers*; P&ID Design Ltd: Teesside, UK, 2008.
(16) Chen, J. J. J. Comments on improvements on a replacement for the logarithmic mean. Letters to the editor. *Chem. Eng. Sci.* **1987**, *42* (10), 2488–2489.